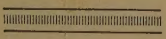
MODERN SCHOOL MATHEMATICS

BOOK

TWO

SCHORLING · CLARK · SMITH

MODERN-SCHOOL MATHEMATICS



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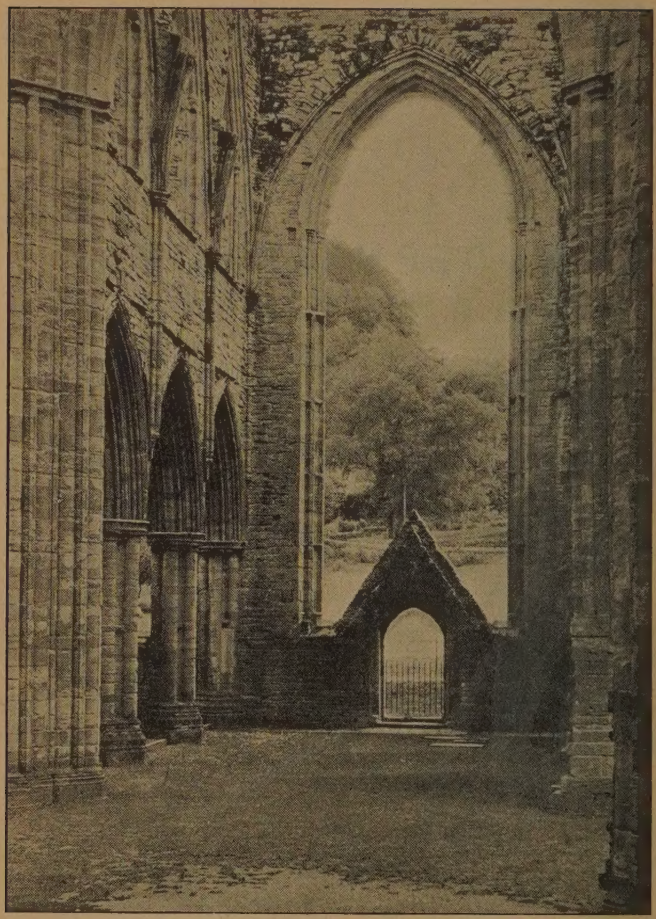

Martha E. Bonham

LOOKING OUT FROM TINTERN ABBEY,
IN MONMOUTHSHIRE, ENGLAND

MODERN-SCHOOL MATHEMATICS

BOOK TWO

By Raleigh Schorling

HEAD OF DEPARTMENT OF MATHEMATICS
THE UNIVERSITY HIGH SCHOOL
AND PROFESSOR OF EDUCATION
UNIVERSITY OF MICHIGAN

and

John R. Clark

THE LINCOLN SCHOOL OF
TEACHERS COLLEGE
COLUMBIA UNIVERSITY

With the coöperation of

Rolland R. Smith

SPECIALIST IN MATHEMATICS
PUBLIC SCHOOLS
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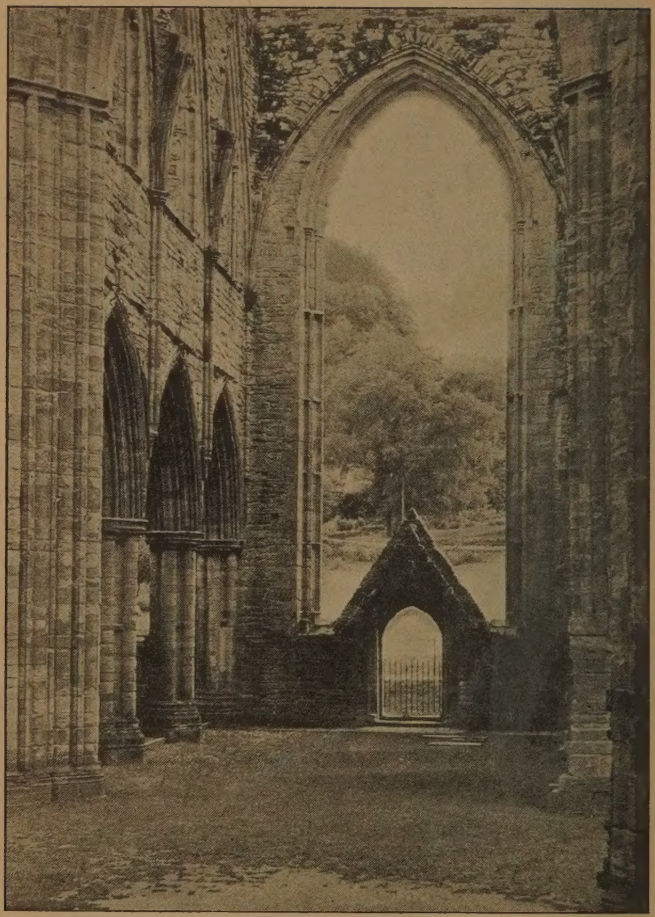

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PREFACE

A modern curriculum cannot become static or final; and certainly this should be true in the teaching of mathematics, where so much progress is being made. Improvements in industries come frequently, and they are brought about as a result of painstaking and expensive research. Progress in education must take the same road. The work here presented represents an effort of this kind which it is hoped will be acceptable to the thousands of school people who have been generous in their judgments of the earlier editions.

The first edition of this series of textbooks was published at a time when the mathematics program to follow the arithmetic of the elementary school was passing through a transition period that brought great changes. The series made an outstanding contribution, and today the general pattern has been written into all of the best courses of study. The series served to fix the frame of reference for the modern curriculum in mathematics.

The many differences of opinion as to what should be taught in these grades, which were held a decade ago, have largely disappeared. Today well-trained teachers of mathematics are generally agreed that a basic exploratory course in mathematics is needed. They want it to include the simple and significant concepts of arithmetic, geometry, and algebra, and they want it organized so that these can be taught to show their natural interrelations. Finally, the course must provide continued growth in the fundamentals of arithmetic. This basic platform of the earlier editions has apparently become the accepted pattern for these grades, and the reader will find this program the core of the present edition.

But the last decade has brought many important improvements. We now have a clearer idea as to what phases

of arithmetic are really important in common life, and we are more nearly agreed as to what mathematical concepts should be emphasized to obtain that broad understanding that lends power to later work. The schools have been forced to give more and more attention to ways and means of teaching pupils who exhibit enormous differences in abilities and cultural backgrounds. There is greater emphasis on an activity program which tends to conceive of the life and work of the people of a community as the real textbook for children. Finally, more of the materials are now arranged so that teacher and pupils can work together in true laboratory fashion to discover the great universal truths of mathematics. These will serve to illustrate the new features of the present edition that have come into wide acceptance in recent years.

We would, therefore, summarize the special provisions of this book and of the other volumes in the series as follows:

I. *Increase in skill in computation.* One effect of the recent reorganization of arithmetic in the lower grades is a reduction in the emphasis given to arithmetic skills. It has become of primary importance, therefore, that pupils in the later grades should extend and increase their mastery of the skills involved in simple computation.

Teachers have long recognized the difficulty of developing in pupils dependable habits of computation. For years the authors have been experimenting with a new program of drill. The essential features of the program employed in this book include *provision for individualizing the progress* of pupils toward their mastery of the skills. *Diagnostic tests* reveal to the pupil his specific weaknesses or needs. *Specially designed remedial practice,* correlated with the diagnostic tests, makes for the most economical learning or maintenance of the needed skills. *Norms of achievement* assist in adapting the program to pupils or groups of pupils of varying abilities.

II. *Enlarged understanding of the concepts and principles of mathematics.* The authors have sought to make mathematics more meaningful, better understood. Meanings grow out of associations and relations with pupils' experiences. The concepts (e.g., ratio, formula, area, fraction, per cent, and the like) have been built out of a careful selection of varied, meaningful pupil experiences. It is of primary importance that pupils' understanding of the concepts and principles of mathematics be extended. Pupils not only need to add, subtract, multiply, and divide, but they must know more about when to use these operations. The possession of skills is of little value without clear understanding of the concepts and principles of mathematics.

III. *A wider range of application of mathematics* to other subjects and to life needs. The subject matter deals with a variety of everyday projects, taken chiefly from pupil experience: Boy Scout activities, classroom activities, making pictures, going on picnics, forming clubs, visiting shops, housing and building projects, etc. The aim has been to make the material fresh, stimulating, and closely related to modern pupil life. In the past the school subjects have been too unrelated to each other and insufficiently related to their uses. In this book the authors have constantly sought to teach not only mathematics but the uses of mathematics.

IV. *More power in mathematical reasoning* (problem solving). One of the major purposes of teaching mathematics is to enable boys and girls to solve the problems which arise in their everyday experiences. This requires not only the skills and the concepts heretofore referred to, but ability in thinking, analysis, and reasoning.

Pupils do their best thinking when they are confronted with a challenging, thought-provoking problem situation. Growth in power to reason results from success in reasoning in a variety of situations. Experience has shown that effec-

tive ways of challenging the pupil to reason are to have him complete a paragraph, choose the better of alternate ways of working, plan a program, etc. Illustrations of this kind of stimulation to reasoning are found on pages 35, 67, and 81. The reader's attention is particularly invited to the great variety of good problems.

V. *Understanding of measurement.* Measuring is one of the commonest acts of daily life. In almost every kind of work — preparing meals, making clothing, buying and selling, and building — we must measure. Progress in civilization has gone hand in hand with progress in the ability to measure more and more accurately. Particularly in an industrial society is measurement a fundamental process. Far too often pupils graduate from high school without discovering these fundamental facts. Pupils seldom understand the procedures used, and less frequently have command of the skills involved. In this book pupils employ various units of measure, use many kinds of scales, and are led to appreciate the nature and importance of measurement.

VI. *Familiarity with the properties of the common geometrical forms* (intuitive geometry). Few things, if any, are as interesting to pupils in the early "teens" as the body of materials constituting intuitive geometry. Its concreteness, obvious practical value, and ease of teaching make it an appropriate topic for this course. In these books geometry is studied after the manner of the laboratory. Pupils make inferences, and check their inferences by measuring and weighing them; in short, they learn and practice the experimental method. Thus they come to appreciate better the concepts of length, area, volume, position, shape, size, etc. In the development of his culture man has constantly been confronted with the necessity of dealing with the *form, size,* and *position* of objects. A knowledge of the properties of

the commonly experienced geometric forms has been and will continue to be necessary.

VII. *Methods of expressing mathematical relationships.* In a sense, mathematics deals primarily with relationships. In the laboratory, in the shop, in commerce, one quantity changes with another. Man is constantly trying to determine the relations between variables which change together. In *Modern-School Mathematics* the authors emphasize this notion of the dependence of one quantity upon another. It is this concept, which is repeatedly presented throughout the series, that knits together the several topics of the course.

Graphs and formulas are the language which educated people use to express vividly and concisely quantitative relationships. In this series, both the graphic and the formula methods are used consistently. They take on meaning because they grow out of concrete situations. The formula emerges from intuitive geometry. Through it arithmetic, geometry, and algebra are unified.

VIII. *Adaptation of materials to meet a wider range of individual needs.* One of the marked tendencies in the modern school is toward greater individualization of instruction. What are the practical provisions in this textbook that make the teacher's task easier? First of all, verbal problems are on three levels of difficulty (e.g., see pages 31–33). In using these differentiated lists some teachers prefer to let the pupil select the list appropriate for him. Second, there are three norms for each of the many standardized tests, representing different goals which pupils of varying ability may aspire to reach. Finally, in recognition of the fact that this text will be used by classes of widely varying ability, the authors have enriched the content so that teachers of such groups will have an ample amount of well-balanced teaching material. Teachers of slower groups may omit that enriching material without detriment to the

unity or sequence of the course. This enriching material is on the starred pages, which it is presumed the teacher will use at his discretion.

For numerous valuable suggestions that have been incorporated in this edition, the authors wish to acknowledge their indebtedness to Caroline Hatton, Arthur S. Otis, William C. Krathwohl, and Vera Sanford, and to the many teachers who have used the earlier editions and have criticized proof of the present edition, including especially E. A. Bond, Bellingham, Washington; Nellie Loss, Flint, Michigan; Amanda Loughren, Elizabeth, New Jersey; Eva MacEwen, Charlotte, North Carolina; Mary A. Potter, Racine, Wisconsin; Nanette Roche, Baltimore, Maryland; Frank D. Smigelsky, Syracuse, New York; B. M. Trescott, Upper Darby, Pennsylvania; Anna Whitney, Yakima, Washington; and May L. Wilt, Morgantown, West Virginia. It is only by coöperative thinking and research that substantial advances can be effected.

R. S.
J. R. C.
R. R. S.

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In writing this book the authors have tried to make mathematics an interesting and worth-while study for you. You will find in it many problems about activities of boys and girls of your own age and about things that you do from day to day.

There is much arithmetic to do, but this is because arithmetic is one of the most useful of all the school subjects in the everyday life of people. Something new that will be interesting to you is geometry (meaning "earth-measure").

This has to do with circles, squares, and triangles, and with making designs and measuring.

One of the marks of a person who is successful in his work is his ability to think things out for himself. This book is written with the special thought in the authors' mind of helping you learn to do this. Whenever you meet a new kind of problem, try to discover for yourself how to solve it. Then read carefully the explanation in the book. As you practice doing this you will more and more easily discover for yourself good ways to solve problems.

The publishers have tried to make the book attractive. Do not write in it. If you take care of it and keep it clean, you will find greater satisfaction in using it.

MODERN-SCHOOL MATHEMATICS

Book Two

CHAPTER I

EXPRESSING RELATIONS BETWEEN NUMBERS

A Table to Show Relationship

Underwood & Underwood

1. The Boy Scouts of Allwood are learning camp cookery. Do you think they would need to use mathematics in camp cookery? If so, in what way?

2. About how long do you think it would take to roast a 10-pound piece of beef?

In studying the roasting of meats, they found a table showing how long to roast beef. A part of the table is given on the next page.

TABLE I. TIME-TABLE FOR ROASTING BEEF

Weight (lb.)	3	4	5	6	7	8	9	10	12
Time (hr.) well done	$1\frac{1}{2}$	2	$2\frac{1}{2}$	3	$3\frac{1}{2}$	4			
Time (hr.) rare . . .	1	$1\frac{1}{3}$	$1\frac{2}{3}$	2	$2\frac{1}{3}$				

1. The table above shows that a 3-pound roast of beef, to be well done, should be cooked $1\frac{1}{2}$ hours. How long should a 4-pound roast be cooked? a 5-pound roast? a 10-pound piece? a 12-pound piece?

2. What relation exists between the numbers showing the weight and the time required for roasting when the meat is to be well done?

3. How long should a piece of beef weighing 3 pounds be cooked to be rare? a 4-pound piece? a 5-pound piece? a 10-pound piece? a 12-pound piece?

4. What relation exists between the weight and the time when the roast is to be rare?

Jack Owen is in the camp cookery class. He showed his mother the time-table they had used for the roasting of beef and asked her if she used such a table.

Mrs. Owen said: "No, I don't use a table for the roasting of meats; I work by rule. My rule for roast beef is: *Allow one-half hour of roasting for each pound of beef.* Somebody drew up that table in your book from such a rule."

Mrs. Owen's rule may be stated differently thus:

The time in hours needed to roast any number of pounds of beef well done equals one-half the number of pounds.

5. Which statement of the rule do you like better?

6. From the table make up the rule which you think Mrs. Owen would use if she wished to have her roast rare.

7. Which is easier to remember, a rule or a table? Which makes computation unnecessary?

1. Here is a time-table for the roasting of pork. What numbers are missing? (*Never write in your book.*)

TABLE II. TIME-TABLE FOR ROASTING PORK

Weight (in pounds)	3	4	5	6	7	8	10
Time (in minutes)	75	100	125	150			
Time (in hours)	$1\frac{1}{4}$	$1\frac{2}{3}$	$2\frac{1}{12}$				

2. With the help of Table II, complete this rule which Mrs. Owen uses for the roasting of pork: Allow ? minutes of roasting for each pound of pork.

3. What fractional part of an hour must be allowed for roasting each pound of pork?

4. Here is a time-table for the roasting of shoulder of veal. What numbers are missing?

TABLE III. TIME-TABLE FOR ROASTING SHOULDER OF VEAL

Weight (in pounds)	3	4	5	6	7	8	10
Time (in minutes)	105	140	175				
Time (in hours)	$1\frac{3}{4}$	$2\frac{1}{3}$	$2\frac{11}{12}$				

5. Make up a rule by which you can tell the number of *minutes* needed to roast a shoulder of veal if you know its weight.

From the preceding exercises we learn that the weight of a roast and the length of time for cooking it bear a definite relation to each other. This ***relationship*** can be expressed by a table or as a simple arithmetic rule.

6. Make up a rule by which you can tell the number of *hours* needed to roast a shoulder of veal if you know its weight.

7. Make up a table showing the cost of 3 to 10 pounds of pork when pork is selling at 20¢ a pound.

1. We often abbreviate an arithmetic rule by using letters to stand for numbers. Can you write this rule more briefly?

The time in hours needed to roast any number of pounds of beef well done equals one-half the number of pounds.

2. You might abbreviate it in these ways:

(a) time (in hours) $= \frac{1}{2}$ weight (in pounds)

(b) time $= \frac{1}{2}$ number of pounds

(c) $h = \frac{1}{2}\ p$

The last statement of the rule, $h = \frac{1}{2}\ p$, is called a ***formula***. What does h stand for? What does p stand for?

3. Which of these statements expresses the time in minutes needed to roast a piece of beef well done?

(a) $m = \frac{1}{2}\ p$ (b) $m = 30\ p$ (c) $m = \dfrac{30\ p}{60}$

4. Read the formula $m = 30\ p$ as a sentence.

5. The formula $h = \frac{1}{3}\ p$ expresses a rule for computing the time in *hours* required for the roasting of beef (rare). Read the formula $h = \frac{1}{3}\ p$ as a sentence.

6. Make up a formula expressing a rule for computing the time in *minutes* required for the roasting of beef (rare). (Let m = time in minutes and p = weight in pounds.)

7. The formula $m = 25\ p$ expresses a rule for computing the time in *minutes* required for the roasting of pork. Read the formula $m = 25\ p$ as a sentence.

8. With the help of Table II on page 3 make up a formula expressing a rule for computing the time in *hours* required for the roasting of pork. (Let h = time in hours and p the weight in pounds.)

9. Here are the formulas written by four pupils as solutions of Ex. 8 above:

(a) $h = 25\ p \div 60$ (b) $h = \dfrac{25\ p}{60}$ (c) $h = \dfrac{5\ p}{12}$ (d) $h = \dfrac{5}{12}\ p$

All of these formulas are correct. Explain each. What advantage have formulas (c) and (d) over the others?

1. The expression $20\,p$ means 20 times any number for which p may stand. What does $8\,p$ mean? $10\,p$?

2. What does $20\,p$ equal when p equals 5? when $p = 2\frac{1}{2}$? $3\frac{1}{4}$?

3. What is the value of $5\,a$ when the value of a is 6 inches? when the value of a is 2.5 inches? 3.4 inches?

4. Using the formula $m = 12\,p$, find the value of m when the value of p is 8; 12; 20; $2\frac{1}{2}$; $3\frac{1}{4}$; $6\frac{2}{3}$.

5. Using the formula $q = \frac{1}{2}\,p$, find the value of q when the value of p is 2; 8; 1; $12\frac{1}{2}$; 20.4; 14.8.

6. What is the value of f in the formula $f = \frac{1}{3}\,y$, when the value of y is 6? 12? 20? 2? $4\frac{1}{2}$? 9.6?

7. What is the value of t in the formula $t = \frac{5}{12}\,p$, when the value of p is 12? 24? 36? 60? 6? 4? 8?

8. Using the formula $p = 4\,s$, tell what numbers are missing from this table: (*Never write in your book.*)

If s is.....	1	2	3	..	6.5	8.25	..	..	0
Then p is.....	4	8	..	16	..	..	40	100	..

9. Tell what numbers are missing from this table:

Length (in inches)	12	18	..	6	8	..	0	..	20
Length (in feet)..	..	..	$2\frac{1}{2}$	..	..	$3\frac{1}{4}$	..	10	..

10. Using the formula $f = \frac{1}{12}\,i$, find the value of f when the value of i is 12; 18; 30; 6; 8; 39; 0; 120; 20. (Do you see any similarity between Ex. 10 and Ex. 9?)

11. Make a table containing 6 pairs of numbers for the formula $y = 3\,f$. (Let f be 1, 2, 3, 6, 8, and 10.)

12. Make a table containing 6 pairs of numbers for the formula $f = 3\,y$. (Let y be 1, 2, 4, 8, $10\frac{1}{2}$, and $12\frac{1}{2}$.)

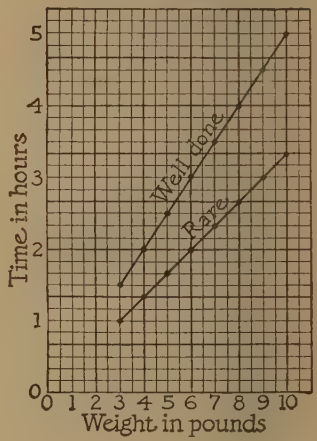
1. What does this graph show about the time required for roasting beef?

2 What is measured on the horizontal scale? on the vertical scale?

3. What does each unit square on the vertical scale represent?

4. Use the graph to find the time required to cook the following roasts of beef well done: a 4-pound roast; a 5-pound roast; an 8-pound roast; a 10-pound roast.

GRAPH SHOWING TIME FOR ROASTING BEEF



5. Note that each point on the graph gives us a *pair of numbers that go together*. One of these numbers expresses weight, the other shows time.

TIME-TABLE FOR ROASTING BEEF (WELL DONE)

Weight (in pounds)...	3		5		7	8	9	
Time (in hours).....	$1\frac{1}{2}$	2		3	$3\frac{1}{2}$			5

Using the graph, tell the numbers that are missing from the table above. (*Remember never to write in your book.*)

6. Use the graph to make a table for roasting beef (rare).

We have just seen how the time for roasting various quantities of meat *depends* upon the weight. This *relationship* may be expressed: (1) by means of a *table;* (2) as a complete sentence, the arithmetic *rule;* (3) by a *formula;* and (4) by means of a *graph*. All four methods are common. You will find it necessary to make use of each of these methods on the next few pages.

1. Mary Studebaker took some packages to the postal station in Baltimore, to mail them to friends in Indianapolis, 575 miles from Baltimore. Before telling Mary the postage, the postal clerk weighed each package and looked at a table, a part of which is printed below.

How much would Mary have to pay for sending a 3-pound package to Indianapolis? a 5-pound package? a 7-pound package?

2. What numbers are missing from the table below?

By Ewing Galloway, N. Y.

PARCEL POST RATES IN THE FOURTH ZONE (300 TO 600 MILES)

Weight (in pounds)	1	2	3	4	5	6	7	8	9	10	11	12
Cost (in cents)	8	12	16	20	24	28	32					

3. Make up a rule for finding the postage on a parcel of any weight sent into the fourth zone. Test your rule.

4. Is this rule correct? To find the number of cents due on a parcel to be sent into the fourth zone, multiply the number of pounds in the weight of the parcel by 4 and then add 4.

5. Make a formula which shows the relation between the postage and the weight of any parcel to be sent into the fourth zone. All you need to do is to abbreviate the rule in Ex. 4. Use c for the cost and w for the weight.

6. Is this formula a correct answer for Ex. 5? $c = 4w + 4$.

7. The interest on different sums of money for 1 year, at 6%, is shown in the following table:

Principal....	$1	2	5	10	25	60	75	100	1000
Interest at 6%	$.06	.12	.30	.60					

(*a*) What numbers are missing from the table?

(*b*) State a rule by which you can find the interest on *any* principal for 1 year at 6%.

(*c*) Write a formula which shows the relation between the interest for 1 year at 6% and the principal. (Use the letters i and p.)

8. John Steele joined a school Radio Club, which charged an initiation fee of 50 cents and dues of 10 cents a month.

Number of months	1	2	3	5	8	10	20	25	30
Cost (in cents)...	60	70	80						

(*a*) What are the missing numbers?

(*b*) How would you find what it cost John to belong to the Radio Club for any number of months?

(*c*) Write a formula which shows the relation between the cost in cents and the number of months during which John continued his membership. (Use c and m.)

(*d*) What is c, if m is 16? if m is 12?

9. (*Optional.*) Ruby Jones began working for $8 a week and received each week an increase of 20 cents.

(*a*) Make a table for finding Ruby's wages for the 1st week, the 2d week, etc., to the 10th week.

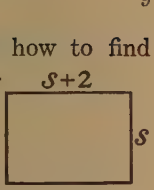
(*b*) Write a word rule for finding Ruby's wages in dollars for the tenth, or twelfth, or *any other* week.

(*c*) Can you make a formula which shows the relation between amount of wages in dollars (w) and the number of the week (use t)? (HINT. $w = 7.80 + ?$)

1. When Arthur Brown was asked to tell how to find the perimeter of a rectangle like the one illustrated here, he worked out a formula like this:

$p = s + (s+2) + s + (s+2)$

$p = 4s + 4$



Explain his formula and then study the table below. If there are any errors in the table, give the correct numbers.

If s is . . .	1	2	3	4	5	10	25	36
then p is.	8	12	16	20	24	56	104	150

2. Is there a value of p for every value of s? What value of p corresponds to the value 4 for s? the value 5? Show that a table is made up of *corresponding* pairs of numbers.

3. Paul wrote this table on the blackboard:

If x is	1	5	20	50
then y is. .	6	10	25	55

He said: "I have a secret formula that shows the relation between x and y. Is it $x + y = 6$, or is it $x = y + 4$, or what is it?" Mary Louise was the only pupil who discovered the formula that Paul used in making the table. Can you find it?

4. Match these tables and formulas:

TABLE 1

If x is.	0	2	5	10
then y is. . . .	4	8	14	24

TABLE 2

If x is.	0	2	5	10
then y is. . . .	0	4	10	20

TABLE 3

If x is.	2	5	10	20
then y is. . . .	0	6	16	36

FORMULA A

$y = 2x$

FORMULA B

$y = 2x + 4$

FORMULA C

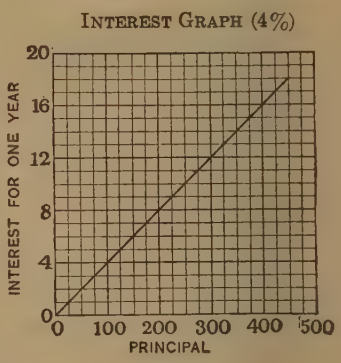
$y = 2x - 4$

This graph shows the interest which one may expect each year on money loaned at 4% interest. Can you read the graph?

1. What does the horizontal scale on this graph show?

2. What does each small unit on the horizontal scale represent?

3. Complete: One small unit on the vertical scale represents __?__ dollar.



4. Use the graph to find how much interest you would get per year on \$300; \$250; \$200; \$150; \$325; \$75.

5. Use the graph to tell how much a man would need to lend at 4% to get \$8 interest per year; \$10 interest; \$6 interest; \$2 interest; \$7 interest; \$11 interest.

6. With the help of the graph tell what numbers are missing from this table.

Principal (in dollars) .	100	125	175	225	350	400
Interest (1 yr., 4%) ..	4					

7. Read, supplying the missing words:

(*a*) The interest __?__ as the principal changes.

(*b*) To find the interest, multiply the __?__ by $\frac{4}{100}$.

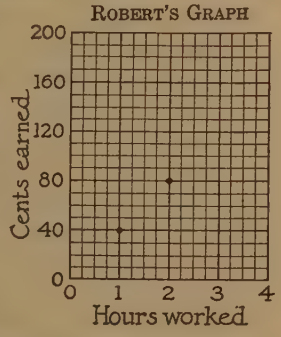
(*c*) If the horizontal scale of the above graph were extended to include \$1000, then the vertical scale would have to be extended to include __?__ dollars.

Do you see that a graph shows vividly the relation between numbers that change together according to some rule?

Robert Dale earns 40 cents an hour by working in a shop. One day he wrote the formula, $c = 40\,h$, to represent his earnings, in cents, for any number of working hours. Was this correct? What does h represent? What does c represent? Robert wished to make a graph showing the relation between the number of hours he worked and the amount of money he earned. He first made the following table:

Hours worked...	1	2	3	4	5	6	7	8	$8\frac{1}{2}$
Cents earned....	40	80	120						

1. Copy and complete Robert's table. How many pairs of numbers could you include in such a table?



2. On a sheet of graph paper he marked a scale for hours worked and another scale for cents earned. One small unit on the horizontal scale represents ? minutes. One small unit on the vertical scale represents . . .

3. He *plotted* (placed) a point which corresponded to the pair of numbers 1, 40 (1 on the hour scale and 40 on the cents-earned scale). Which point represents this pair of numbers?

4. He next plotted a point for the pair 2, 80. Show where this point is on the graph.

5. On a large piece of graph paper make a copy of Robert's work. Plot points corresponding to the other pairs of numbers in the table. Do the points lie in a straight line? If so, draw a straight line through them. This is the *graph of the formula*, $c = 40\,h$.

6. Use your finished graph to tell how much Robert would earn in $3\frac{1}{2}$ hours; in $4\frac{1}{4}$ hours; in $6\frac{3}{4}$ hours; in 0 hours.

1. (*a*) Oranges sell at 30 cents a dozen. Show that the formula for the cost of any number of dozens is $c = 30\,d$.

(*b*) Copy and complete this table:

Number of dozens....	1	2	3	4	5	6	8	10
Cost in cents.....	30							

(*c*) Construct a graph of the formula $c = 30\,d$. What will be represented by each unit on the horizontal scale? on the vertical scale? Plot on the graph a point which corresponds to each pair of numbers in your table made in (*b*) above. Draw a straight line through these eight points, and extend it.

(*d*) Use your graph to find the cost of $1\frac{1}{2}$ doz. oranges; $2\frac{1}{2}$ doz.; 5 doz.; 12 doz.

(*e*) Use your graph to tell how many dozen oranges you can get for \$1.80; for \$1.05; for \$3.00.

2. Sugar costs 6 cents a pound. Show that the formula for the cost is $c = 6\,p$. Make a table showing the cost of 1, 2, and 3 pounds of sugar. Construct a graph to show the relation between cost and number of pounds.

3. Ray shovels snow at 36 cents an hour. Write the formula, make a table, and draw the graph to show the relation between number of hours he works and amount he earns.

4. Use the graph drawn for Ex. 3 to find how much Ray earns in $\frac{1}{2}$ hour; in 15 minutes; in $2\frac{1}{2}$ hours.

5. Use the graph made for Ex. 3 to find how long Ray has to work to earn 30 cents; to earn \$1.10; to earn 80 cents.

6. Edgar works in a drug store for 60 cents an hour. Write the formula, make the table, and draw the graph to show the relation between time worked and amount earned.

7. Use the graph made for Ex. 6 to find out what Edgar earns in $1\frac{1}{2}$ hours; in $2\frac{1}{4}$ hours; in 3 hours and 20 minutes.

8. How long would he need to work to earn \$1.50? \$2.00?

We have seen that some statements may be expressed much more briefly by *using letters to represent numbers.*

Compare these two ways:

The statement in words: When 7 is added to a certain number, the result is 12.

The "shorthand" way: $n + 7 = 12$

It is clear that the number is 5. Do you find the "shorthand" way just as understandable as the longer way?

If we are to learn how to state a relation between numbers in the shortest possible way, we need practice in the use of the "shorthand" of mathematics. Let us look at two very simple illustrations:

(1) I am thinking of a certain number; if 12 is added to it, the result will be 20. The briefest way to write this is: $n + 12 = 20$.

(2) If 9 is subtracted from a certain number, the remainder is 4. A short way of expressing this is: $n - 9 = 4$.

Write each of the following statements in the short mathematical form used above: (Let n represent the unknown number.)

1. Helen says: "I am thinking of a number. If 4 is added to it, the result will be 16."

2. If 9 is added to a number, the result will be 16.

3. If 8 is subtracted from a certain number, the result is 17.

4. 7 and some number equals 13.

5. A certain number minus 6 equals 2.

6. 5 more than a certain number equals 12.

7. A certain number less 2 is 3.

8. If Grace gives Rose 8 tulip bulbs, Grace will then have 20 bulbs left.

In the preceding exercises you have been translating or changing statements into ***equations***. *An equation states that two number expressions are equal.*

Translate each of these statements into an equation:

1. The sum of a certain number and 6 is 20.

2. Margaret found 7 stamps. She then had 15.

3. Three times some number is 24. (Remember that $3 \times n$ is written as $3\ n$.)

4. Five times a certain number is 36.

5. If a certain number is increased by 60, the result will be 100.

6. The number of cents in my pocket, plus 90, equals 100.

7. Two times a certain number, plus 1, equals 11.

8. Three times a certain number, minus 1, equals 14.

9. If 7 is taken from a certain number, the result will be 5.

10. A teacher wrote $10 - x = 6$ on the blackboard and asked what it meant. Edith said: "If 10 is subtracted from a number, the result is 6." Priscilla said: "If a number is subtracted from 10, the result is 6." Which statement was correct?

11. Which of the two statements below refers to the equation: $2\ m = 18$?

(*a*) The sum of 2 and a certain number is 18.
(*b*) The product of 2 and a certain number is 18.

12. Write in mathematical shorthand:

(*a*) The sum of 2 and a certain number is 18.
(*b*) A number increased by 7 equals 9.
(*c*) A number decreased by 6 equals 3.
(*d*) 7 less a certain number equals 1.

1. What number does x stand for in the equation $x + 5 = 13$? Think: "Since the sum of x and 5 is 13, then x is $13 - 5$, or 8." Check: $8 + 5 = 13$.

x (addend)
$+ 5$ (addend)
13 (sum)

What number does the letter in each of the following stand for?

2. $w + 3 = 12$	5. $x + 7 = 16$	8. $p + 9 = 15$
3. $y + 4 = 11$	6. $y + 8 = 15$	9. $n + 4\frac{1}{2} = 6$
4. $p + 5 = 14$	7. $n + 9 = 17$	10. $p + 2\frac{1}{4} = 5$

Finding the number which the letter in an equation stands for is called ***solving the equation.***

11. To help solve the equation $y + 7\frac{1}{2} = 26$, Arthur made this diagram.

Does the diagram help you to decide how to find y? $y =$ ___?___.

12. Use Arthur's method to help solve these equations:

(*a*) $y + 19 = 34$	(*d*) $x + 12\frac{1}{2} = 20$	(*g*) $n + 240 = 500$
(*b*) $p + 28 = 47$	(*e*) $y + 6\frac{1}{4} = 15$	(*h*) $p + 156.5 = 200$
(*c*) $n + 29 = 48$	(*f*) $p + 5.5 = 10$	(*i*) $x + 16.25 = 50$

To solve an equation like $n + 7 = 27$, many persons think: "Subtract 7 from each side of the equation; then $n = 20$." This method uses the following principle:

$n + 7$	$= 27$
$n + 7 - 7$	$= 27 - 7$
n	$= 20$

The same number may be subtracted from both sides of an equation. (This is the Subtraction Law.)

13. Tell how to use the Subtraction Law to solve the following:

(*a*) $n + 11 = 30$	(*d*) $p + 21\frac{1}{2} = 30\frac{1}{4}$	(*g*) $y + 2\frac{5}{8} = 8\frac{1}{4}$
(*b*) $y + 18\frac{1}{4} = 50$	(*e*) $n + 12.8 = 20$	(*h*) $n + 8\frac{1}{4} = 10\frac{1}{8}$
(*c*) $x + 2.4 = 10.2$	(*f*) $x + 156 = 257$	(*i*) $p + 4\frac{1}{8} = 7$

14. In which of the equations on this page did you know one of the addends and the sum of two addends?

15. If $a + b = c$, what does a equal? What does b equal?

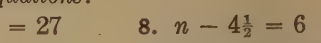
1. What number does m stand for in the equation $m - 7 = 9$? Think: "Since m decreased by 7 equals 9, then m equals 9 plus 7, or 16." Check: $16 - 7 = 9$.

m (minuend)
$-$ 7 (subtrahend)
9 (difference)

Solve and check each of these equations:

2. $m - 4 = 7$	5. $p - 18 = 27$	8. $n - 4\frac{1}{2} = 6$
3. $n - 8 = 9$	6. $x - 16 = 29$	9. $p - 2\frac{1}{4} = 8\frac{1}{2}$
4. $x - 5 = 12$	7. $p - 28 = 47$	10. $x - 1\frac{1}{4} = 8\frac{3}{4}$

11. To help solve the equation $x - 8 = 13$, George used this diagram. Does the diagram help you to decide how to find x?



Use George's method to help solve these equations:

(*a*) $y - 8 = 15$	(*c*) $p - 12\frac{1}{2} = 20$	(*e*) $n - 2\frac{3}{4} = 1\frac{1}{2}$
(*b*) $n - 15 = 18$	(*d*) $a - 22.5 = 28.5$	(*f*) $c - 8.4 = 8.6$

To solve an equation like $y - 18 = 13$, many persons think: "Add 18 to both sides of the equation; then $y = 31$." How do you like this method? It uses the following principle:

$y - 18$	$= 13$
$y - 18 + \mathbf{18}$	$= 13 + \mathbf{18}$
y	$= 31$

The same number may be added to both sides of an equation. (This is the Addition Law.)

12. Tell how to use the Addition Law to solve:

(*a*) $p - 12 = 15\frac{1}{2}$	(*d*) $n - 12.8 = 7$	(*g*) $n - \frac{1}{8} = \frac{3}{4}$
(*b*) $x - 8\frac{1}{2} = 2\frac{1}{2}$	(*e*) $y - 8 = 18.4$	(*h*) $y - \frac{3}{8} = 1\frac{5}{16}$
(*c*) $y - 2.4 = 8.2$	(*f*) $p - 40 = 20$	(*i*) $m - 1\frac{1}{8} = \frac{3}{4}$

13. Which of these could be solved by using (1) the Addition Law? (2) the Subtraction Law? Solve each.

(*a*) $x + 4 = 6$	(*d*) $p - 10 = 20$	(*g*) $a + 4\frac{1}{2} = 6$
(*b*) $y + 5 = 8$	(*e*) $p + 10 = 20$	(*h*) $b - 6\frac{1}{2} = 8\frac{3}{4}$
(*c*) $n - 1 = 8$	(*f*) $x - \frac{1}{2} = 1\frac{1}{2}$	(*i*) $c + 27.1 = 36$

Write as equations and solve:

1. A certain number decreased by 8 equals 15.

2. I am thinking of a number. If 9 is subtracted from it, the result is 19.

3. 12 is the sum of a certain number and 7. What is the number?

4. If $8\frac{3}{8}$ is subtracted from a certain number, the remainder is $27\frac{5}{8}$. What is the number?

5. What number increased by 13.5 equals 30.5?

6. After $3\frac{3}{4}$ yards of cloth were sold from a piece, $6\frac{1}{4}$ yards were left. How many yards were there in the original piece?

7. The sum of two addends is 26; one of them is 17.6. What is the other addend?

Solve and check each of the following equations:

8. $y + 8\frac{1}{4} = 10\frac{1}{8}$
9. $y - 8\frac{1}{4} = 10\frac{1}{8}$
10. $p + 3\frac{5}{8} = 7$
11. $p - 3\frac{5}{8} = 7$
12. $x + 3.5 = 7.5$
13. $x - 3.5 = 7.5$
14. $20 = y + 12.25$
15. $20 = y - 12.25$
16. $31.5 = x + 12.5$
17. $31.5 = x - 12.5$
18. $18 = 8\frac{1}{4} + y$
19. $18 = y - 8\frac{1}{4}$
20. $\frac{1}{4} = x + \frac{1}{8}$
21. $\frac{1}{4} = x - \frac{1}{8}$
22. $12\frac{3}{8} = y + 3\frac{1}{4}$
23. $12\frac{3}{8} = y - 3\frac{1}{4}$
24. $n - \frac{1}{2} = \frac{1}{8}$
25. $n + \frac{1}{2} = \frac{15}{16}$
26. $y + .25 = .75$
27. $y - .25 = .75$
28. $w + 8\frac{1}{4} = 10$
29. $z - \frac{3}{4} = \frac{3}{8}$

30. In solving the equation $16 = x - 12$, Jane got 4 for the value of x. Was that right? If not, what mistake did she make?

1. In the expression $3 \times 4 = 12$, the 3 and the 4 are called the *factors* and the 12 is the *product*. In $3y = 12$, 3 and y are the factors. What is the product?

2. Since $3 \times 4 = 12$, then $3 = 12 \div 4$ and also $4 = 12 \div 3$. In general we say: *Either of the two factors equals the product divided by the other factor.*

3. In equations like $3x = 12$, we find the unknown factor, x, by dividing the product, 12, by the other factor, 3. What does x equal? (If you know one of the factors and the product of the two factors, how do you find the other factor?)

4. Solve and check each of these equations:

(*a*) $3y = 15$ (*c*) $6c = 40$ (*e*) $2x = 10.8$
(*b*) $4a = 20$ (*d*) $9y = 60$ (*f*) $3y = 21.9$

5. To solve an equation like $2\frac{1}{2}n = 20$, you think: "The unknown factor, n, equals the product, 20, divided by the known factor, $2\frac{1}{2}$."

(*a*) $2\frac{1}{2}x = 40$ (*c*) $2\frac{1}{3}x = 14$ (*e*) $2.1b = 10.5$
(*b*) $\frac{7}{8}y = 35$ (*d*) $\frac{2}{3}y = 26$ (*f*) $1.5c = 6$

To solve an equation like $2\frac{1}{2}p = 20$, many persons think: "Divide both sides of the equation by $2\frac{1}{2}$; then $p = 8$." They use the principle:

Both sides of an equation may be divided by the same number. (This is the Division Law.)

$$2\tfrac{1}{2}p = 20$$

$$\frac{2\frac{1}{2}p}{2\frac{1}{2}} = \frac{20}{2\frac{1}{2}}$$

$$p = 8$$

6. Tell how to use the Division Law to solve:

(*a*) $3\frac{1}{2}n = 42$ (*c*) $1.5x = 10.5$ (*e*) $.5n = 40$
(*b*) $4\frac{1}{3}y = 26$ (*d*) $1.06n = 530$ (*f*) $.8y = 16$

7. Tell which law is used in solving each of these equations. Then solve and check each.

(*a*) $n + 4 = 7$ (*d*) $x + 3\frac{1}{2} = 35$ (*g*) $2\frac{1}{2}n = 24$
(*b*) $n - 4 = 7$ (*e*) $x - 3\frac{1}{2} = 35$ (*h*) $n + 2\frac{1}{2} = 24$
(*c*) $4n = 7$ (*f*) $3\frac{1}{2}x = 35$ (*i*) $n - 2\frac{1}{2} = 24$

1. *Mary:* I'm thinking of a number. If I multiply it by 3 and add 1, the result is 16. Guess the number.

William: Suppose we represent the number you are thinking of by x. When you multiply it by 3, you get $3x$. When you add 1, you have $3x + 1$. Then $3x + 1 = 16$. Before you added 1, you had 15; so $3x = 15$ and $x = 5$. Do you want me to prove it?

Mary: Yes, prove it.

William: Well, 3 times 5 is 15, and 1 is 16.

2. William made up this one: I'm thinking of a number. If I multiply it by 4 and then subtract 1, the result is 19.

Can you find what number William was thinking of?

Let x represent the number.	CHECK
Then 4 times the number is $4x$.	$(4 \times 5) - 1 \doteqdot 19$
And $4x - 1 = 19$.	$20 - 1 \doteqdot 19$
So $4x = 19 + 1$, or 20.	$19 = 19$
And $x = \frac{20}{4} = 5$.	The sign $\doteqdot$ means "should equal."

3. In solving the equation $5y - 8 = 42$, George arranged his work this way:

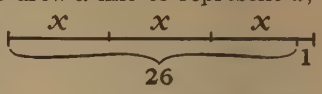
		CHECK
$5y - 8 = 42$		
$5y$	$= 42 + 8$	$(5 \times 10) - 8 \doteqdot 42$
$5y$	$= 50$	$50 - 8 \doteqdot 42$
y	$= \frac{50}{5}$	$42 = 42$
y	$= 10$	

Is his solution correct?

Solve and check each of these equations:

4. $2x + 1 = 17$	12. $7y - 1 = 34$	20. $7p + 4 = 46$
5. $3x - 1 = 14$	13. $7b + 2 = 37$	21. $8y + 20 = 44$
6. $4y + 3 = 35$	14. $8n - 3 = 77$	22. $12x - 50 = 10$
7. $4y - 2 = 38$	15. $8x + 5 = 21$	23. $15y + 20 = 50$
8. $5p + 2 = 37$	16. $9b + 1 = 37$	24. $20x - 40 = 60$
9. $5p - 1 = 39$	17. $9p - 1 = 44$	25. $25n - 4 = 96$
10. $6x - 1 = 11$	18. $8x - 10 = 6$	26. $2\frac{1}{2}x - 4 = 6$
11. $6x + 2 = 38$	19. $9y + 12 = 30$	27. $3\frac{1}{2}y - 8 = 27$

1. Arthur wished to make a diagram to represent the equation $3x - 1 = 26$. (1) He drew a line to represent x, and then made the line 3 times this length to represent $3x$. (2) He marked off a small segment at the right to represent 1, so that the remainder would represent $3x - 1$, or 26. Then he saw that $3x = 26 + 1 = 27$, and $x = 27 \div 3 = 9$.



2. Use his method to picture the equation $4y - 3 = 13$.

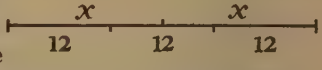
3. Make diagrams to illustrate the solution of each of the following equations:

(*a*) $3p - 2 = 12$ (*d*) $2x - 3 = 8\frac{1}{2}$ (*g*) $9x + 2 = 20$

(*b*) $4y - 5 = 85$ (*e*) $3p - 1 = 18$ (*h*) $8y + 3 = 25$

(*c*) $5x - 3 = 24$ (*f*) $7y - 2 = 14$ (*i*) $7p + 2 = 45$

To make a diagram for the equation $\frac{2x}{3} = 12$, (1) draw a line to represent $2x$, as shown here; (2) divide the line into thirds and label each of the three parts 12. How long is the whole line ($2x$)? What is x?

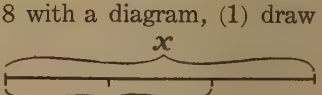


4. Use this method to illustrate each of these equations:

(*a*) $\frac{2x}{3} = 8$ (*c*) $\frac{3x}{4} = 6$ (*e*) $\frac{3y}{4} = 2\frac{1}{4}$ (*g*) $\frac{3n}{4} = 100$

(*b*) $\frac{3x}{4} = 9$ (*d*) $\frac{2x}{3} = 4\frac{1}{2}$ (*f*) $\frac{2p}{3} = 8\frac{1}{3}$ (*h*) $\frac{2y}{3} = 12\frac{1}{2}$

To solve the equation $\frac{2}{3}x = 8$ with a diagram, (1) draw a segment to represent x; (2) mark off 2 thirds of the segment and label that part 8. What is 1 third of x? What is x?



5. Use this method in solving these equations:

(*a*) $\frac{2}{3}x = 12$ (*c*) $\frac{2}{5}x = 4$ (*e*) $\frac{4}{5}x = 16$ (*g*) $\frac{2}{3}p = 10$

(*b*) $\frac{3}{4}x = 9$ (*d*) $\frac{5}{6}y = 10$ (*f*) $\frac{1}{4}m = 5\frac{1}{2}$ (*h*) $\frac{3}{5}n = 9$

1. Solve the equation $\frac{x}{4} = 7\frac{1}{2}$.

A fourth of the number is $7\frac{1}{2}$. $\left(\frac{x}{4}\text{ is sometimes written } \frac{1}{4}x.\right)$

Hence the whole number is $4 \times 7\frac{1}{2}$, or $\underline{\quad ? \quad}$. Check. $\frac{30}{4} = 7\frac{1}{2}$.

Solve and check each of these equations:

2. $\frac{x}{3} = 5$ **5.** $\frac{p}{20} = 3.5$ **8.** $\frac{y}{20} = 4\frac{1}{10}$

3. $\frac{y}{4} = 8$ **6.** $\frac{y}{40} = 2.25$ **9.** $\frac{n}{16} = \frac{5}{8}$

4. $\frac{1}{5}n = 2$ **7.** $\frac{1}{20}n = 8\frac{3}{4}$ **10.** $\frac{1}{8}y = \frac{3}{4}$

11. Solve the equation $\frac{3x}{4} = 15$.

$3x$ divided by 4 is 15.
Hence $3x$ is 4 times 15, or 60.
So $x = \frac{1}{3}$ of 60, or 20.

Check. $\frac{3 \times 20}{4} = \frac{60}{4} = 15$.

$\left(\frac{3x}{4}\text{ may be written as } \frac{3}{4}x.\right)$

Solve and check each of these equations:

12. $\frac{2y}{3} = 8$ **15.** $\frac{4y}{3} = 8$ **18.** $\frac{5n}{3} = 10$

13. $\frac{3}{5}x = 9$ **16.** $\frac{3}{10}n = 6$ **19.** $\frac{5p}{4} = 2\frac{1}{2}$

14. $\frac{4p}{3} = 10$ **17.** $\frac{5y}{2} = 20$ **20.** $\frac{3}{4}x = 7\frac{1}{2}$

To solve the equation $\frac{2y}{3} = 6$, some persons think:

"Multiply each side of the equation by 3; then $2y = 18$ and $y = 9$." They use the following principle:

Both sides of an equation may be multiplied by the same number. (This is the Multiplication Law.)

21. Solve Exs. 12 to 20 by using this law.

$$\frac{2y}{3} = 6$$
$$3 \times \frac{2y}{3} = 3 \times 6$$
$$2y = 18$$
$$y = 9$$

We have learned that the same number may be added to or subtracted from both sides of an equation and that both sides may be multiplied by or divided by the same number.

1. Use the four equation laws stated above to solve these equations. Tell what you do to both sides of each equation.

(*a*) $\frac{x}{2} = 3.5$	(*e*) $\frac{y}{3} = .4$	(*i*) $\frac{x}{4} = 7.8$
(*b*) $2y = 3.6$	(*f*) $4x = 6$	(*j*) $5p = 8.5$
(*c*) $n - 2 = 3\frac{1}{2}$	(*g*) $y - 1.2 = 3$	(*k*) $y - 3\frac{1}{2} = 17\frac{1}{2}$
(*d*) $y + 8\frac{1}{2} = 20\frac{1}{8}$	(*h*) $n + 3.8 = 10$	(*l*) $x + 7.6 = 20$

2. In solving the equation $2n + 5 = 17$, first use the Subtraction Law (subtract 5 from both sides). Then apply the Division Law (divide both sides by 2).

$$\begin{aligned} 2n + 5 &= 17 \\ 2n &= 12 \\ n &= 6 \end{aligned}$$

3. Solve each of these equations, as in Ex. 2.

(*a*) $2x + 9 = 27$	(*c*) $4p + 8.5 = 40.5$	(*e*) $6p + 2\frac{1}{4} = 20\frac{1}{4}$
(*b*) $3y + 4 = 103$	(*d*) $5n + 8.2 = 20$	(*f*) $2\frac{1}{2}x + 1 = 26$

4. In solving the equation $2n - 5 = 17$, first use what law? then what?

5. To solve the equation $\frac{1}{2}x + 5 = 17\frac{1}{2}$, first use what law? then what? $x = ?$

$$\begin{aligned} 2n - 5 &= 17 \\ 2n &= 22 \\ n &= 11 \end{aligned}$$

6. Which laws would you use in solving each of these?

(*a*) $2x + 8 = 12.4$	(*d*) $\frac{1}{4}y - 1 = 7\frac{1}{2}$	(*g*) $2.5x + 10 = 50$
(*b*) $3y - 2 = 4.9$	(*e*) $\frac{1}{3}n + 8 = 12$	(*h*) $3.6y - 8 = 100$
(*c*) $\frac{1}{2}x + 1 = 7$	(*f*) $2y + 7\frac{1}{2} = 10$	(*i*) $.8p - 12 = 4$

7. Which law is used first in the accompanying solution of the equation $5y - 8 = 2y + 20$? then which? then which?

$$\begin{aligned} 5y - 8 &= 2y + 20 \\ 5y &= 2y + 28 \\ 3y &= 28 \\ y &= 9\frac{1}{3} \end{aligned}$$

8. Solve and check these equations:

(*a*) $8y - 10 = 5y + 2$

(*b*) $6x + 5 = 2x + 29$

1. Tell how you would think in solving the equation $3.6x = 18$.

Think: "The unknown factor, x, equals the product, 18, divided by the known factor, 3.6." Find x.

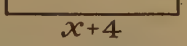
Solve and check each of these equations:

2. $2.5x = 20$	**8.** $.8c = 10$	**14.** $.03y = .60$
3. $1.6x = 8$	**9.** $.06x = 60$	**15.** $.04p = 1.25$
4. $1.2y = 60$	**10.** $.04s = 25$	**16.** $.05x = .50$
5. $.8n = 20$	**11.** $.04y = 80$	**17.** $.08y = 12$
6. $.6p = 18$	**12.** $.02y = 10$	**18.** $.06p = 24$
7. $.3p = 12$	**13.** $.05p = 2.5$	**19.** $.08p = 4$

20. If .8 of the regular price of an article is \$20, what is the regular price? Think: "$.8$ r.p. $= \$20$; r.p. $= \frac{\$20}{.8} = ?$" (You might think: "Let x represent the regular price; then $.8x = 20$; $x = \frac{20}{.8} = \$25$.")

21. If .04 of the day's sales of a store equals \$12, what is the amount of the day's sales?

22. What is the width of the accompanying rectangle? the length? Write an equation stating that the perimeter is 32. Solve the equation.



23. Make a sketch of a rectangle. Let w represent its width. If its length is 4 more than its width, which of these expressions represent its length?

(*a*) $w + 4$ (*b*) $4 + w$ (*c*) $4w$ (*d*) $w - 4$

What expression represents its perimeter? Write an equation stating that its perimeter is 84. Solve the equation.

24. The length of a given rectangle is 10 more than its width. Its perimeter is 80. What are its dimensions?

1. Miss Miller wished to find out whether her class really knew how to use the Multiplication Law. She wrote on the blackboard the equation shown here.

$$\frac{y}{5} = \frac{3}{4}$$

She then asked her pupils to tell her by what number she should multiply to find the value of y. The following answers were given:

(*a*) Multiply each side by 5. (*b*) Multiply each side by 4. (*c*) Multiply each side by the denominator of the fraction containing the unknown.

Choose the best one of the three answers given above. Tell why you think it is the best.

2. Solve and check each of these equations:

(*a*) $\frac{x}{4} = \frac{7}{2}$ (*e*) $\frac{x}{3} = \frac{7}{12}$ (*i*) $\frac{y}{100} = \frac{3}{4}$ (*m*) $\frac{3}{4} = \frac{x}{20}$

(*b*) $\frac{y}{4} = \frac{9}{2}$ (*f*) $\frac{3}{4} = \frac{a}{3}$ (*j*) $\frac{y}{100} = \frac{2}{5}$ (*n*) $\frac{7}{8} = \frac{y}{24}$

(*c*) $\frac{a}{9} = \frac{5}{3}$ (*g*) $\frac{x}{3} = \frac{5}{3}$ (*k*) $\frac{3}{5} = \frac{n}{100}$ (*o*) $\frac{5}{16} = \frac{n}{40}$

(*d*) $\frac{n}{10} = \frac{6}{15}$ (*h*) $\frac{a}{8} = \frac{2}{3}$ (*l*) $\frac{1}{5} = \frac{n}{100}$ (*p*) $\frac{y}{2} = 2\frac{1}{4}$

3. Write as an equation: A certain number, n, divided by 3 equals $\frac{2}{3}$. Solve the equation.

4. One third of a certain number, increased by $2\frac{1}{2}$, is 7. What is the number?

5. If you multiply both sides of the equation $\frac{3x}{5} = 2.1$ by 5, what equation is obtained? Solve it.

6. If you multiply both sides of the equation $\frac{2n}{3} = \frac{4}{5}$ by 15, what equation is obtained? Solve it.

7. Can you solve the equation $\frac{2x}{5} = \frac{2}{3}$?

Solve and check each of these equations:

	(*a*)	(*b*)	(*c*)
1.	$x + 5 = 13$	$2x - 3 = 27$	$2.4y = 12$
2.	$x - 3 = 12$	$3y - 4 = 26$	$\frac{1}{3}n = 3.6$
3.	$y + 8 = 17$	$4p - 1 = 99$	$5\frac{1}{2}y = 17\frac{1}{2}$
4.	$y - 8 = 17$	$\frac{1}{4}n = 2\frac{1}{8}$	$3\frac{1}{4}p = 65$
5.	$\frac{2}{3}n = 18$	$\frac{3}{4}n = 21$	$\frac{4}{5}y = 28$
6.	$x + x = 18$	$\frac{1}{4}n = 6$	$1.6x = 8$
7.	$\frac{h}{5} = 2.2$	$\frac{y}{3} = 8\frac{1}{3}$	$\frac{b}{4} = 6\frac{1}{4}$
8.	$2x + 1 = 41$	$y + y = 30$	$3.8n = 15.2$
9.	$3y + 5 = 35$	$2x + 12 = 20$	$5.6n = 112$
10.	$4p + 8 = 100$	$2\frac{1}{2}n = 40$	$\frac{1}{2}n + 1 = 10$

11. George had x marbles; James had $x + 5$ marbles. How many marbles had both boys? (Does $x + x + 5 = 2x + 5$?)

12. If you knew that George and James together (Ex. 11) had 25 marbles, how many did George have? James?

13. *Mabel:* "I am thinking of a certain number. If it is multiplied by $2\frac{1}{2}$ and then 5 is added, the result is 20. Guess the number."

14. *Grace:* "I am thinking of a certain number. If I divide by 2 and then subtract 3, the result is 7. Guess the number."

15. *Henry:* "A bottle and a cork together cost \$1.10. The cork cost x cents; the bottle cost a dollar more than the cork. Can you tell what each cost?"

16. Write 5 equations that you would solve by using the Subtraction Law; the Addition Law; the Division Law; the Multiplication Law.

Help No. I. Using Equations in Problem Solving

In the early chapters of this book you will be given some special help in problem solving. In Help No. I you will see how equations can be useful in solving problems. Can you explain each of the solutions given below?

1. I am thinking of a number. Three times the number, less 2, equals 14. Find the number.

Solution		Check
Let n	= the number	$(3 \times 5\frac{1}{3}) - 2 \stackrel{?}{=} 14$
then $3n - 2$	$= 14$	$16 - 2 \stackrel{?}{=} 14$
$3n$	$= 16$	$14 = 14$
n	$= 5\frac{1}{3}$ (the number)	

2. Tom said: "Here is a quarter. It is $\frac{5}{8}$ of my money." How much money did Tom have?

Solution		Check
Let c	= the number of cents Tom had	$\frac{5}{8} \times 40 \stackrel{?}{=} 25$
then $\frac{5}{8}c$	$= 25$	$25 = 25$
c	$= 25 \div \frac{5}{8}$	
c	$= 40$ (number of cents Tom had)	

3. George sold Christmas cards. He was allowed to keep $\frac{2}{5}$ of the amount of his sales. He wondered how many dollars' worth of cards he would have to sell to earn $8. Can this problem be solved as Ex. 2 was solved?

4. Harry has 20 feet of wire fencing. With it he wants to fence a rectangular yard for his rabbit. The yard will be so placed that its width must be 4 feet. How long can Harry make his yard? See the solution on page 27.

Solution	Check
Let l = no. ft. in length	$(2 \times 6) + (2 \times 4) \stackrel{?}{=} 20$
$2l$ = no. ft. in 2 lengths	$12 + 8 \stackrel{?}{=} 20$
8 = no. ft. in 2 widths	$20 = 20$
$2l + 8 = 20$ (perimeter)	
$2l = 12$	
$l = 6$, no. ft. in length	

To solve a problem by an equation:

(1) **Let some letter stand for the number the problem asks you to find.** (For example, y for the number of yards, d for the number of dollars, etc.)

(2) **Using the information the problem gives you, form an equation using the letter.**

(3) **Solve the equation to find the value of the letter.**

(4) **Check.**

Explain how these four steps were used in the three solutions given above.

Solve these problems by equations. Check.

1. The sum of a certain number and 8 is 15. Find the number.

2. Four times $\_?\_$ less 2 equals 22.

3. I am thinking of a number. $\frac{2}{3}$ of it is 40. Find the number.

4. What number increased by 8 equals 27?

5. What number decreased by 9 equals 17?

6. If 8 is taken from a certain number, the result will be 35. What is the number?

7. 8% of how many dollars equals \$16?

8. Jean is paid $\frac{1}{5}$ of the amount of her sales of magazines. How many dollars' worth must she sell to earn \$4?

1. Translate into a statement in words: $\frac{1}{2}n + 3 = 8$.

2. Solve the equation in Ex. 1.

3. What is meant by *solving* an equation? What is meant by checking the solution of an equation?

4. State the laws, or principles, that are used in solving the equation $3x + 7 = 25$.

5. State the four laws that are used in solving equations.

6. Sam said that the ratio of a certain number to 8 is the same as the ratio of 5 to 4. He wrote this equation: $\frac{n}{8} = \frac{5}{4}$. Martha thought the equation should be written in this form: $\frac{5}{4} = \frac{n}{8}$. Who was correct? What was the "certain number" which Sam had in mind?

7. Can you solve and check these in 3 minutes?

(*a*) $\frac{x}{4} = \frac{5}{8}$	(*d*) $\frac{b}{5} = 7$	(*g*) $\frac{n}{2} = 5\frac{1}{2}$	(*j*) $\frac{x}{3} = \frac{1}{2}$
(*b*) $\frac{a}{3} = \frac{8}{5}$	(*e*) $\frac{c}{6} = 7$	(*h*) $\frac{e}{3} = 4\frac{1}{2}$	(*k*) $\frac{y}{5} = \frac{3}{2}$
(*c*) $\frac{y}{2} = \frac{7}{5}$	(*f*) $\frac{a}{4} = 10$	(*i*) $\frac{x}{5} = 3\frac{1}{4}$	(*l*) $\frac{p}{4} = \frac{7}{8}$

8. Solve and check the following equations:

(*a*) $\frac{3}{4}n = 180$	(*d*) $\frac{2}{3}p - 5 = 35$
(*b*) $\frac{2}{5}n + 1 = 25$	(*e*) $19 = \frac{3}{4}n + 1$
(*c*) $\frac{5}{6}y - 1 = 24$	(*f*) $\frac{5}{3}n + 2 = 47$

9. Make a list of the main topics in this chapter.

10. Solve by means of an equation: Three times a certain number increased by 4 equals 19. Find the number.

11. Make a graph that would be helpful to a garage man for finding what to charge for any number of gallons of gasoline up to 20 when the price is 19¢ a gallon.

Judith Ramsay kept house last summer while her mother took a week's vacation. Here are some of the arithmetic problems she met in her cooking.

By Ewing Galloway, N. Y.

1. She bought a $4\frac{1}{2}$-pound roast of beef. The cookbook said that if she wanted the meat well done she should allow 30 minutes' cooking for each pound. She wanted to serve it at noon. At what time should the roast be put in the oven to have it ready in time?

2. Judith baked muffins for dinner one day. The recipe said that the given amounts would make 18 muffins. She thought 12 muffins would be enough. How much milk should be used for the 12 muffins, if the recipe called for $1\frac{1}{2}$ cups?

3. The recipe (Ex. 2) also called for $1\frac{3}{4}$ cups of flour. How much flour should Judith use for the 12 muffins?

4. Judith decided to make some cherry jelly. This is the recipe she found: 3 cups cherry juice, $6\frac{1}{2}$ cups sugar, 1 cup Jelline (a preparation to make jelly congeal quickly).

Judith had only $2\frac{1}{2}$ cups of cherry juice. How much sugar and Jelline should she use?

5. The cherry jelly recipe (Ex. 4) said the yield would be 9 8-ounce glasses of jelly. Judith could find only 7 8-ounce glasses and 3 4-ounce glasses. As she washed them, she wondered whether she had enough to hold the jelly. Did she have enough?

A Vacation Trip*

The Glenns and their neighbors, the Allens, went on an automobile trip. The party consisted of Mr. Glenn, his wife, and his son Alfred, and Mr. and Mrs. Allen. They used Mr. Glenn's car and kept a record of all expenses so that an accurate settlement could be made at the end of the trip. Alfred had charge of the expense account. Here is a summary of his record:

Mileage: Distance traveled, 840 miles.
Gasoline: Car used 56 gallons of gasoline at 18¢ per gallon.
Oil: Car averaged 1000 miles per gallon of oil at $1.20 per gallon.
Wear on tires: Tires good for 16,000 miles; cost new $40 a set of four.
Repairs on car: $16.
Rooms at hotels: 5 nights at $1.00 a person per night.
Meals for 5 days: $7.25, $7.40, $5.95, $7.45, $6.75.
Incidentals: $10.

1. What did the meals and rooms cost?

2. How much was spent for gasoline?

3. What was the cost of the oil used on the trip?

4. What charge should be made for the wear on the tires?

5. What was the total cost of the vacation trip as found from Alfred's figures? What was the cost per person?

6. If the car decreased $750 in value during the first 25,000 miles, how much might well have been added to the cost of the trip for depreciation on the car?

7. Counting oil, gas, depreciation, repair, and tire wear, what did it cost per mile to run the car?

*NOTE. The starred pages in this book are optional. They may be used as the teacher wishes or they may be omitted without interfering with the sequence.

1. George joined a stamp club. The initiation fee was 25 cents, and the dues were 10 cents a month. Make a formula showing the relation between m (number of months) and c (number of cents spent for belonging).

2. Using the formula $m = 5p + 20$, copy and complete this table showing the relation between p and m:

If p is.....	1	2	4	10	12	15	20
then m is ..	25						

3. Using the following table, copy and complete this formula showing the relation between w and p: $p = 2w + (?)$

If w is.....	1	2	3	4	5	10	20
then p is...	3	5	7	9			

4. What numbers are missing from the table in Ex. 3?

5. Using the following table, copy and complete this formula showing the relation between s and p: $p = (?)\, s - 1$.

If s is	1	2	3	5	10	15	20
then p is ..	2	5	8	14			

6. What numbers are missing from the table in Ex. 5?

Solve and check the following equations:

7. $p + 20 = 80$
8. $x - 12 = 4$
9. $3b = 15$
10. $2x + 1 = 21$
11. $\frac{x}{2} = 9$
12. $40 = w + 4.5$
13. $\frac{2y}{3} = 8$
14. $2.5x = 50$
15. $y + \frac{1}{2} = 4$
16. $x - 1\frac{1}{2} = 5$
17. $2\frac{1}{2}m = 20$
18. $2x - 4 = 16$
19. $x + 2.5 = 6$
20. $.06x = 18$
21. $x + x = 28$
22. $12 = a - 5$
23. $2\frac{1}{2}y + 1 = 16$
24. $y - .15 = .25$

25. Three times a certain number plus 2 equals 17. Find the number.

1. A swimming-pool manager charges bathers 25¢ for the use of a bathhouse and 15¢ an hour for the use of the pool. Make a formula showing the relation between a (the amount due the manager) and h (the number of hours one swims).

2. Copy and complete this table, using the formula $g = .6\,p$ showing the relation between p and g.

If p is	1	2	4	6	8	10	12
then g is . . .	.6						

3. Using the following table, complete this formula showing the relation between a and b: $a = \frac{1}{2}\,b + (?)$

If b is	4	6	7	10	13	17	20
then a is	4	5	$5\frac{1}{2}$	7			

4. Copy and complete the table in Ex. 3.

5. What formula fits the following table?

If c equals . . .	1	3	4	7	8	10	12
then d equals	7	11	13				

6. Copy and complete the table in Ex. 5.

Solve and check the following equations:

7. $y + 3\frac{1}{2} = 10$
8. $s - 12\frac{3}{4} = 2\frac{1}{4}$
9. $3\frac{1}{2}\,y = 35$
10. $9\,y + 4 = 85$
11. $\frac{y}{3} = 2\frac{1}{2}$
12. $\frac{2\,y}{3} = 8.2$
13. $\frac{3\,x}{4} = 15$
14. $\frac{y}{6} = \frac{1}{4}$
15. $5\,y + 2\,y + 3 = 38$
16. $.08\,y = 12$
17. $2\frac{1}{2}\,x + 4 = 6$
18. $100 = y + 23.4$

Use equations in solving these problems:

19. Five times a certain number less 8 is 7. Find the number.

20. How much money must be placed on interest at 4% to earn an annual income of $100?

21. $\frac{3}{4}$ of what number equals 45?

1. In this table $c = 10m + 25$. Copy and complete it.

If m equals . .	2.5	3	3.5				10
then c equals .				30	35	45	

2. What formula fits the following table?

If x equals . . .	1	2	3	4		10	12
then y equals .	5	9	13		21		

3. Copy and complete the table in Ex. 2.

4. What formula did James have in mind when he made this table?

When m equals	1	2	3	4	5	6	7
then p equals	1	4	9	16	25	36	49

Solve and check the following equations:

5. $y + 17\frac{7}{8} = 80$
6. $33\frac{1}{3}y = 300$
7. $7x + 9 = 20$
8. $\frac{x}{12} = 6\frac{5}{8}$
9. $18 = 2y + 1 + y$
10. $\frac{2}{3}y = 12\frac{1}{2}$
11. $\frac{3}{4}n + 3 = 39$
12. $p - 12\frac{3}{10} = 2\frac{4}{5}$
13. $.05y = 1.2$
14. $25 = 2\frac{1}{2}a$
15. $3\frac{1}{4}x = 32\frac{1}{2}$
16. $27 = 5 + \frac{2}{3}x$
17. $62 = y + 15.4$
18. $\frac{y}{20} = 4\frac{1}{10}$
19. $\frac{2}{3} = \frac{3a}{4}$
20. $7\frac{3}{16} = x - 2\frac{7}{8}$
21. $\frac{7}{8}y - 1 = 13$
22. $.20y = 8\frac{1}{5}$
23. $22\frac{1}{2} = 12\frac{2}{3} + x$
24. $3.5x = 5.2$
25. $2\frac{1}{2}x + 4 = 8$
26. $s - 2.7 = 6.3$
27. $\frac{3y}{4} = 7.5$
28. $\frac{4}{3}n - 4 = 16$

Use equations in solving these problems:

29. Three tenths of my sales equals \$360. What is the amount of my sales?

30. I know that .80 × the selling price of an article = \$720. What is the selling price of the article?

31. If 1.20 × cost price = \$60, what is the cost price?

CHAPTER II

RECALLING THE FUNDAMENTALS OF ARITHMETIC

A School Picnic

By Ewing Galloway, N. Y.

The pupils in the eighth grade of the Tremont School decided to have a picnic for themselves and their families. They figured that 75 persons would come.

With the help of the domestic-science teacher the committee planned this menu: Meat Loaf Sandwich, Cheese Sandwich, Apple and Tomato, Graham Crackers, Milk, and Toasted Marshmallows.

The committee's next job was to estimate how much of each article of food should be bought. Before looking at the next page, make this estimate. (Allow each guest 1 meat loaf sandwich, 1 cheese sandwich, 1 tomato, 1 apple, $\frac{1}{2}$ pint of milk, 4 toasted marshmallows, and 4 graham crackers.)

This is the way the committee made its estimates. Tell how the blank spaces should be filled.

(*a*) Two rolls apiece for 75 persons for the sandwiches would be ___?___ rolls, or ___?___ dozen rolls. They decided to order 13 dozen rolls.

(*b*) To find out how much butter they would need, they experimented in the lunchroom one noon. They found they could spread 10 rolls with $\frac{1}{4}$ pound of butter. They figured a pound of butter would spread ___?___ rolls. To spread 156 rolls, they would need ___?___ pounds of butter. (They bought a whole number of pounds.)

(*c*) The butcher told them the meat loaf could be cut on the slicer to give them about 15 or 16 slices to the pound. For the 78 meat sandwiches ($\frac{1}{2}$ the 13 dozen rolls), they ordered ___?___ pounds of meat loaf.

(*d*) The cheese could be cut 20 or 21 slices to the pound. For the 78 cheese sandwiches they ordered ___?___ pounds of cheese.

(*e*) They found that there were about 4 tomatoes to the pound. They needed ___?___ pounds of tomatoes, but they bought 21 pounds.

(*f*) The apples ran about 3 to the pound. They bought ___?___ pounds of apples.

(*g*) They ordered ___?___ half-pint bottles of milk.

(*h*) They bought marshmallows in boxes containing 5 dozen each. They ordered ___?___ boxes.

(*i*) The graham crackers came in packages of 30. They bought ___?___ boxes.

After making the above estimates, the committee made a table in order to find out the cost of the food. Copy the first column of the table on the next page; then find the cost of each article and the total cost. (Do not write in your book.)

Article	Amount Needed	Price	Cost
Rolls	13 dozen	15¢ per dozen	
Butter	4 pounds	26¢ per pound	
Meat Loaf	5 pounds	35¢ per pound	
Cheese	4 pounds	40¢ per pound	
Tomatoes	21 pounds	3 lb. for 10¢	
Apples	25 pounds	5 lb. for 19¢	
Milk	75 bottles	5¢ per bottle	
Marshmallows	5 boxes	19¢ per box	
Graham Crackers	10 boxes	10¢ per box	
Drinking Straws	1 box (100 straws)	10¢ per box	
Paper Napkins	75	10¢ per package of 100	
Total Cost			

The committee then announced to the class that if the cost of the picnic was shared equally by the 24 pupils, each pupil should pay 60 cents. The committee found that the exact charge should be _?_ cents per person, but they decided that the 60-cent charge could be collected with less chance for error. They planned to put the extra money in the class treasury. How much would the class receive?

Do you ever have class picnics or parties? Have you ever been on a refreshment committee in school, church, or club? If so, will you discuss with your class some of the problems you met and how you solved them?

Finding Out What Help Is Needed

Miss Tyler had a new class of eighth-grade pupils. She wanted to find out what instruction, if any, they needed on the vocabulary and meanings of the fundamentals of arithmetic; so she gave them a typewritten test. Helen Kelly's test paper is shown on the next page. Read it carefully. Did Helen need help on any of the items?

Name: Helen Kelly Date: Sept. 7

548
223
670
1441

1. In this example 1441 is called the sum; 548 is one of the addends.

2. An addition example checks if you get the same answer by adding both ways.

1834
-972
862

3. In this example 1834 is called the minuend; 972 is the subtrahend; 862 is the difference, or remainder.

4. A subtraction example checks if the sum of the subtrahend and remainder equals the minuend.

106
25
530
212
2650

5. In this example 106 is the multiplicand; 25 is the multiplier; and 2650 is the product.

6. You can check a multiplication example by dividing the product by the multiplier. The result should equal the multiplicand.

20
23)473
46
13

7. In this example 473 is called the dividend; 23 is the divisor; 20 is the quotient; and 13 is the remainder.

8. In Ex. 7 the quotient could have been carried to 20.56. This answer expressed to the nearest tenth would be 20.6.

9. 25 exceeds 12 by 37.

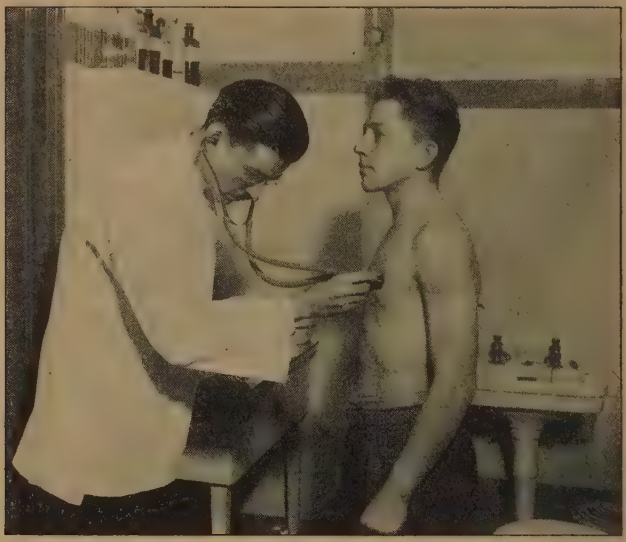
29)1862

10. In this example the first figure in the quotient is 6.

11. 144 × 2 gives the same product as 2 × 144.

12. Multiplying a whole number or a decimal by 100 moves the decimal point 2 places to the right.

13. Dividing a whole number or a decimal by 1000 moves the decimal point 3 places to the left.



This doctor specializes in *diagnosis*, in finding out what weaknesses or ailments people have. Have you ever had a complete physical diagnosis? The medical profession recommends that everyone have his physical condition — blood, digestion, vision, hearing, etc. — diagnosed from time to time even though his health is apparently good, so that any weakness may be found and remedied. "An ounce of prevention is worth a pound of cure."

Have you any weakness in arithmetic computation? No one can afford to take a chance on ailments of arithmetic. You should have your arithmetical ability diagnosed, and then obtain appropriate *remedial* help. The tests in this book called "Locating Difficulties in . . .," together with the remedial work which follows, will help you keep yourself "fit" arithmetically. The first one is on the next page.

Test 1. Locating Difficulties in Addition

Write the answers to these examples on folded paper. If you work incorrectly or do not complete the examples in any group, do the corresponding Practice Group on pages 40 and 41. After doing the required practice, come back and take this test again. Be sure to record your score each time you take a test (see page 42). See page 347 for answers. *Time, 11 minutes.*

Group I		Group II		Group III		Group IV	
(*a*)	(*b*)	(*a*)	(*b*)	(*a*)	(*b*)	(*a*)	(*b*)
		5	6				
		7	0			98	75
		6	5	$.28	$.98	86	86
3	4	8	4	.07	.67	43	67
8	7	3	3	.56	.35	59	58
2	4	4	9	.65	.82	68	70
7	6	2	8	.94	.70	47	81
6	8	9	7	.49	.59	85	33
5	0	8	6	.98	.98	96	80
9	9	7	5	.87	.47	99	29

Group V		Group VI		Group VII	
(*a*)	(*b*)	(*a*)	(*b*)	(*a*)	(*b*)
		5683	5373	$ 25.86	$ 8.67
		858	968	163.38	125.38
$3.75	$4.84	985	6736	27.94	69.56
9.06	7.75	5448	8485	568.57	77.89
8.74	8.86	867	579	3.06	6.75
6.68	2.43	6036	9647	75.73	138.64
7.37	9.69	7899	8374	994.69	74.98

Addition Practice

Practice Group I

(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)
5	8	5	5	2	5	4	3
4	4	6	7	4	3	6	9
7	9	7	6	7	8	7	8
9	2	8	8	8	9	8	7
3	6	9	4	5	6	4	4
6	8	3	8	9	7	8	5
8	3	5	3	3	8	2	6

Practice Group II

(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)
8	6	2	9	5	6	5	9
9	8	3	7	7	9	3	8
3	4	6	4	8	8	7	3
6	5	8	8	6	9	4	8
4	6	6	6	8	8	8	5
5	7	7	3	7	7	9	9
7	8	4	9	9	8	8	6
9	7	9	4	9	8	6	4
8	9	2	5	8	6	5	8
3	2	3	7	6	4	7	5

Practice Group III

(a)	(b)	(c)	(d)	(e)	(f)
$ .97	$ .77	$ .97	$ .58	$ .80	$ .59
.28	.88	.58	.70	.62	.83
.34	.53	.69	.45	.43	.01
.59	.59	.76	.06	.57	.60
.76	.74	.84	.78	.98	.72
.85	.65	.95	.80	.76	.46
.97	.86	.87	.07	.84	.85
.58	.97	.78	.96	.25	.38

Practice Group IV

(a)	(b)	(c)	(d)	(e)	(f)
57	44	73	63	94	58
20	56	68	79	75	79
38	78	53	84	26	22
62	69	34	35	87	73
36	72	67	67	33	36
84	23	88	98	58	67
75	36	96	56	99	44
47	64	35	73	30	98
98	58	69	25	64	76

Practice Group V

(a)	(b)	(c)	(d)	(e)
$6.39	$8.46	$7.56	$9.79	$5.84
8.02	7.77	6.08	8.68	7.73
7.76	5.84	4.49	3.45	8.06
3.98	6.08	3.83	8.07	6.64
9.47	3.39	2.36	5.63	4.97

Practice Group VI

(a)	(b)	(c)	(d)	(e)
5428	889	885	5689	569
979	3576	5946	5075	023
853	3705	5067	543	3117
8686	636	5599	8776	8898
6705	4404	683	8099	6285
467	4958	758	858	314
9358	863	9875	7367	7657

To the Teacher. If desired, the review work on fundamentals included in this chapter may be taken in short daily drill lessons to suit your convenience. The authors have found the Improvement Record an effective aid in learning, and recommend its use regardless of the time at which the tests are taken.

PRACTICE GROUP VII

(a)	(b)	(c)	(d)	(e)
$175.98	$155.89	$155.39	$ 45.00	$ 94.67
44.27	9.37	8.87	289.04	57.39
69.13	278.64	83.51	933.58	35.52
8.08	67.25	379.28	28.62	862.40
313.66	894.06	27.14	97.15	48.86
86.24	385.58	4.60	214.26	217.75
254.95	23.17	416.25	376.87	6.94

Each time James Brown takes a Locating Difficulties Test (as on page 39), he records his score. His record sheet, which he is keeping in a notebook, looks like this.

MY RECORD OF IMPROVEMENT IN COMPUTATION

Test		No. of examples in test	Number right		
			First trial	Second trial	Third trial
1	Addition	14			
2	Subtraction	18			
3	Multiplication	20			
4	Division	16			
5	Fractions	36			
6	Decimals	30			
7	Per Cents	34			

When James does all the examples in a test correctly, he puts a * after the number of examples in the table to show that he has made a perfect score. James found that keeping a record increases his interest in mathematics, and therefore his achievement.

Make a table like James's and keep a record of your improvement in computation.

Test 2. Locating Difficulties in Subtraction

Write the answers on folded paper. If you have difficulty with any example below, you will find the practice you need at the foot of this page and on the next page. If you made a mistake in Group I, do Practice Group I, etc. After practicing, take the test again. Record your score each time you take this test. See page 348 for answers. *Time, 8 minutes.*

Group I	Group II	Group III
(*a*) 823 (*b*) 513	(*a*) 632 (*b*) 663	(*a*) 8764 (*b*) 5286
497 286	429 357	347 839

Group IV	Group V	Group VI
(*a*) 509 (*b*) 605	(*a*) 8000 (*b*) 6000	(*a*) 84354 (*b*) 35633
278 432	2894 5489	74289 34983

Group VII

(*a*) \$5467.83 − 398.74

(*b*) \$6846.72 − 894.64

(*c*) \$7563.84 − 947.83

Group VIII

(*a*) \$7000.00 − 2899.74

(*b*) \$8000.00 − 2870.92

(*c*) \$6000.00 − 1989.05

Subtraction Practice

Practice Group I

(*a*) 543 − 286

(*b*) 624 − 589

(*c*) 773 − 487

(*d*) 622 − 448

(*e*) 513 − 186

Practice Group II

(*a*)	(*b*)	(*c*)	(*d*)	(*e*)
546	624	728	563	747
139	318	419	258	338

Practice Group III

(*a*)	(*b*)	(*c*)	(*d*)
8436	9654	8763	5683
987	586	974	783

Practice Group IV

(*a*)	(*b*)	(*c*)	(*d*)	(*e*)
509	207	707	406	508
376	189	186	305	289

Practice Group V

(*a*)	(*b*)	(*c*)	(*d*)
8000	6000	5000	4000
2765	2879	1864	2583

Practice Group VI

(*a*)	(*b*)	(*c*)	(*d*)
76345	52846	63201	74386
75983	52825	62984	73829

Practice Group VII

(*a*)	(*b*)	(*c*)	(*d*)
$6483.00	$5487.50	$6842.25	$6848.00
289.76	925.25	982.75	987.75

Practice Group VIII

(*a*)	(*b*)	(*c*)	(*d*)
$9000.00	$8000.00	$7000.00	$5000.00
3847.62	2876.75	5487.25	1554.72

By Ewing Galloway, N. Y.

Dorothy Bray's father, who is a salesman for a company that manufactures swimming suits, brought home for Dorothy 130 samples of last season's materials in various colors. Dorothy decided to use them to make a quilt for her bed.

She measured the blanket on her bed to find how large the quilt should be, and found that it was $56'' \times 80''$. Each sample was $5'' \times 7''$.

1. Dorothy's mother suggested that she fit her patches together on a muslin sheet, cut $56'' \times 80''$, and then stitch the patches to the sheet around all their edges. Dorothy did this. Draw a sketch of the quilt showing the best way to place the patches together to make a quilt of the desired size.

2. What is the smallest number of patches Dorothy could use to make the quilt the size she wished? Did she have enough samples? too many?

3. Then she needed to buy material to line the quilt—that is, cover the under side of it. She found she could get sateen 30 inches wide for 35¢ a yard. How much sateen did she need?

4. Dorothy bought $4\frac{1}{2}$ yards of sateen. Was that exactly the length she needed? How much did the sateen cost?

5. Dorothy had spent 75¢ for the sheet on which she sewed the patches and 15¢ for the heavy colored thread she used. How much did the quilt cost all together, not counting labor?

Test 3. Locating Difficulties in Multiplication

First do all the examples. Then check them. Use folded paper. If you have difficulty with any examples below, you will find the practice you need on the next two pages. After practicing, take the test again. Record your score each time you take this test. See page 349 for answers. *Time, 15 minutes.*

Group I		Group II		Group III	
(*a*)	(*b*)	(*a*)	(*b*)	(*a*)	(*b*)
75	86	286	895	5284	6428
39	25	347	286	397	439

Group IV		Group V		Group VI	
(*a*)	(*b*)	(*a*)	(*b*)	(*a*)	(*b*)
385	765	386	596	580	270
280	270	3800	3500	465	346

Group VII		Group VIII	
(*a*)	(*b*)	(*a*)	(*b*)
607	408	443	846
286	386	209	307

Group IX		Group X	
(*a*)	(*b*)	(*a*)	(*b*)
4009	7005	$39.86	$46.75
2008	4006	273	187

Practice Group I

(a)	(b)	(c)
38	75	39
87	46	84

Practice Group II

(a)	(b)	(c)
286	275	487
384	863	298

Practice Group III

2876	1684	5439
384	371	263

Practice Group IV

465	214	386
360	540	280

Practice Group V

(a)	(b)	(c)	(d)	(e)
347	684	573	546	879
2500	3700	2900	8300	9300

Practice Group VI

280	460	520	460	890
346	389	837	287	789

Practice Group VII

508	409	106	607	708
276	387	428	412	657

Practice Group VIII

538	214	325	463	678
209	408	109	501	905

Practice Group IX

3008	2001	6003	6008	7009
4009	4005	2004	5007	8006

PRACTICE GROUP X

(*a*)	(*b*)	(*c*)	(*d*)	(*e*)
\$64.25	\$58.27	\$74.69	\$75.75	\$86.97
386	132	321	486	789

Test 4. Locating Difficulties in Division

If you have any difficulty with the examples in this test, you can find the practice you need on the next page. Record your score each time you take this test. See page 350 for answers. Express answers to the nearest tenth. Do all the examples. Then go over your work to check. *Time, 15 minutes.* (Copy the examples before counting time.)

GROUP I

(*a*) $7\overline{)6153}$ (*b*) $9\overline{)6786}$

GROUP II

(*a*) $35\overline{)735}$ (*b*) $24\overline{)528}$

GROUP III

$64\overline{)5248}$ $55\overline{)2475}$

GROUP IV

$24\overline{)876}$ $35\overline{)906}$

GROUP V

$86\overline{)77493}$ $93\overline{)65914}$

GROUP VI

$84\overline{)5889}$ $93\overline{)5671}$

GROUP VII

$83\overline{)8106}$ $57\overline{)5104}$

GROUP VIII

$76\overline{)684608}$ $69\overline{)621552}$

Practice Group I

(a)	(b)	(c)	(d)
8)7008	9)8865	7)5528	6)5207

Practice Group II

34)750	23)992	33)697	22)755

Practice Group III

82)5836	71)5751	62)6173	92)8200

Practice Group IV

27)918	36)926	47)2930	36)1932

Practice Group V

87)69687	99)89199	76)68803	85)76723

Practice Group VI

86)7803	75)5294	68)5475	94)6596

Practice Group VII

78)7296	69)6734	49)4121	19)1173

Practice Group VIII

86)688774	95)855665	73)584657	69)552483

Miss Tyler asked her eighth-grade pupils: "What is the meaning of a fraction?" She chose these three answers as best representing the meanings of a fraction:

(*a*) *Joe Withers:* A fraction tells you into how many equal parts something has been divided and how many of the equal parts have been considered.

For example, if AE is 1 inch and if it is divided into 4 equal parts as shown, then $AB = \frac{1}{4}$ inch; $AC = \frac{2}{4}$ or $\frac{1}{2}$ inch; $AD = \frac{3}{4}$ inch.

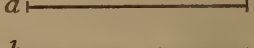
(*b*) *Mary Ames:* A fraction makes you think of division. Thus, the number of gallons in seven quarts is $7 \div 4$, or $\frac{7}{4}$.

(*c*) *Jane Allen:* We have just learned in cooking, that when 3 parts of sugar are mixed with 4 parts of flour, the ratio of the sugar to the flour is 3 to 4, or $\frac{3}{4}$. It seems to me that a fraction represents the *ratio* of two numbers.

1. Give another illustration of each of the three meanings of fractions expressed above.

2. Which of the following examples illustrate the fact that a fraction is a ratio? a division of one number by another number? Which example suggests that something has been divided into a certain number of equal parts?

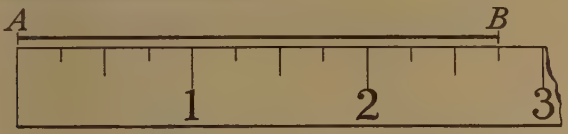
(*a*) Mother cut a pie into 6 equal pieces and put 5 of them into a picnic box ($\frac{5}{6}$).

(*b*) When we compare the lengths of these lines, we say a is $\frac{5}{6}$ of b, or $\frac{a}{b} = \frac{5}{6}$. 

(*c*) If 6 pairs of stockings cost \$5, the number of dollars one pair costs is $\frac{5}{6}$.

(*d*) Tom solved 5 examples correctly while May was solving 6 that were equally difficult. Tom was $\frac{5}{6}$ as efficient as May.

3. Would you say that AB is $\frac{11}{4}$ inches long? Would you say it is $2\frac{3}{4}$ inches long?



4. With your ruler draw a line which shows that $1\frac{3}{4}$ inches equals $\frac{7}{4}$ inches.

5. Draw two pies, dividing each into 6 equal parts. Now suppose that $\frac{1}{6}$ of one of the pies is removed. How many sixths are left in the two pies?

Numbers like $2\frac{1}{4}$, $1\frac{3}{4}$, and $6\frac{1}{2}$ are called *mixed numbers*.
Numbers like $\frac{3}{2}$, $\frac{5}{4}$, $\frac{4}{3}$, and $\frac{5}{5}$ are called *improper fractions*.

6. The above drawing shows that the mixed number $2\frac{3}{4}$ equals the improper fraction $\frac{11}{4}$.

7. Express each of the following mixed numbers as an improper fraction: $3\frac{1}{2}$, $4\frac{1}{2}$, $2\frac{1}{4}$, $3\frac{1}{4}$, $6\frac{1}{2}$, $8\frac{3}{4}$, $7\frac{3}{8}$, $9\frac{2}{3}$, $8\frac{1}{4}$, $7\frac{5}{8}$.

8. Complete: To express $5\frac{3}{4}$ as fourths, I multiply __?__ by __?__, add __?__, and write the result over 4.

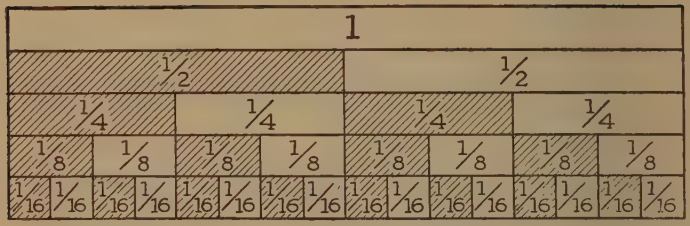
9. State the rule for writing *any* mixed number as an improper fraction.

10. Make a drawing to show that $\frac{9}{4}$ equals $2\frac{1}{4}$.

11. Express each of the following improper fractions as a mixed number: $\frac{7}{2}$, $\frac{9}{4}$, $\frac{10}{3}$, $\frac{27}{4}$, $\frac{35}{8}$, $\frac{57}{7}$, $\frac{16}{5}$, $\frac{17}{8}$, $\frac{25}{6}$, $\frac{33}{4}$.

12. What kind of fraction is $\frac{a}{b}$, if a is larger than b? if a is smaller than b? if a is equal to b?

13. The *denominator* of a fraction tells the number of parts into which something (a unit) has been divided; the *numerator* tells how many of these parts are being considered. Thus, in the fraction $\frac{4}{7}$, the 7 shows . . . and the 4 shows . . .



1. What does the above diagram show about $\frac{1}{2}$, $\frac{2}{4}$, $\frac{4}{8}$, and $\frac{8}{16}$? Would you call them equivalent fractions? (*"Equivalent" means equal in value.*)

2. Use your ruler to find how many sixteenths of an inch there are in $\frac{5}{8}$ of an inch. How many sixteenths are equivalent to $\frac{3}{8}$? $\frac{5}{8}$? $\frac{1}{4}$? $\frac{3}{4}$? $\frac{1}{2}$?

A fraction may be changed to an equivalent fraction by multiplying or dividing both its numerator and denominator by the same number.

4. Think of a proper fraction, such as $\frac{2}{3}$. Add some number to both the numerator and the denominator. Is the result equal to the fraction with which you started? What conclusion do you draw from this?

5. Write any proper fraction. Now subtract the same number from the numerator and the denominator. Have you changed the value of the original fraction? What conclusion do you draw from this?

6. Does $\frac{2}{3}$ equal $\frac{5 \times 2}{5 \times 3}$?

7. Does $\frac{1}{2}$ equal $\frac{1+1}{2+1}$?

8. Does $\frac{5}{10}$ equal $\frac{5-1}{10-1}$?

9. Does $\frac{n}{d}$ equal $\frac{n+1}{d+1}$?

10. Does $\frac{4}{12}$ equal $\frac{4 \div 4}{12 \div 4}$?

11. Does $\frac{n}{2}$ equal $\frac{2n}{4}$?

12. Does $\frac{n}{d}$ equal $\frac{3n}{3d}$?

13. Does $\frac{n}{d}$ equal $\frac{\frac{1}{2}n}{\frac{1}{2}d}$?

(1) Six boys wished to share equally the cost of 12 gallons of gasoline at 18 cents a gallon. How much should each pay?

(2) Which of these solutions of the problem do you prefer?

$$(a)\ \frac{12 \times 18}{6} = \frac{216}{6} = 36 \qquad (b)\ \frac{\overset{2}{\cancel{12}} \times 18}{\underset{1}{\cancel{6}}} = 36 \qquad (c)\ \frac{\overset{2}{\cancel{12}} \times \overset{3}{\cancel{18}}}{\underset{1}{\cancel{6}}} = 6$$

(3) As you have already learned, expressions such as $12 \times \$.18$, 8×24, $3x$, and $10p$ are called *products*. In the product $12 \times \$.18$, the 12 and the \$.18 are the *factors*. What are the factors in each of the other products?

(4) To solve the problem in Ex. (1), you had to "divide a product by a number." To divide the product 12×18 by 6, you divide only the 12 by 6 or only the 18 by 6.

To divide a product of two or more factors by a number, you divide only one of the factors by the number.

Perform these divisions:

1. $\frac{12 \times 8}{4}$
2. $\frac{6 \times 4 \times 8}{2}$
3. $\frac{6 \times 9}{3}$
4. $\frac{4xy}{y}$
5. $\frac{16x}{16}$
6. $\frac{12lw}{6}$
7. $\frac{3n}{3}$
8. $\frac{\pi D}{D}$
9. $\frac{4 \times 20}{4}$
10. $\frac{1wh}{w}$

11. In finding $\frac{6 \times 8}{24}$, you might write: $\frac{\overset{1}{\cancel{6}} \times \overset{2}{\cancel{8}}}{\underset{4}{\cancel{24}}} = \frac{1 \times 2}{1} = 2.$

We call that ***cancellation***. Does it make use of the principle that "to divide a product by a number, divide only one of the factors by the number"?

12. Can you find four errors in the following? Tell what the correct results are.

$(a)\ \frac{6 \times 15}{3} = 6 \times 5$ $\qquad (b)\ \frac{8y}{2y} = 4y$ $\qquad (c)\ \frac{8 \times 6}{2} = 4 \times 6$

$(d)\ \frac{12x}{4} = 3$ $\qquad (e)\ \frac{4}{8 \times 3} = 6$ $\qquad (f)\ \frac{6}{12 \times 9} = \frac{1}{18}$

$(g)\ \frac{5y}{5} = y$ $\qquad (h)\ \frac{12x}{3x} = 4$ $\qquad (i)\ \frac{.06x}{.06} = 1$

Test 5. Locating Difficulties with Fractions

If you have any difficulty with the examples in this test, you can find the practice you need on the next two pages. Record your score each time you take this test. See page 350 for answers. (Copy the examples before timing the test.)

Part I (*Time, 10 minutes*)

Group I	Group II	Group III
(*a*) $\frac{1}{8}+\frac{1}{4}+\frac{1}{2}$	(*a*) $\frac{2}{5}+\frac{1}{2}+\frac{7}{10}$	(*a*) $3\frac{1}{10}+4\frac{1}{5}+7\frac{1}{2}$
(*b*) $\frac{5}{12}+\frac{1}{3}+\frac{1}{6}$	(*b*) $\frac{7}{8}+\frac{1}{3}+\frac{1}{2}$	(*b*) $4\frac{1}{6}+5\frac{1}{4}+\frac{1}{3}$
Group IV	**Group V**	**Group VI**
(*a*) $4\frac{5}{12}+2\frac{2}{3}+1\frac{5}{6}$	(*a*) $\frac{7}{8}-\frac{1}{3}$	(*a*) $6\frac{3}{8}-2\frac{1}{3}$
(*b*) $3\frac{2}{3}+5\frac{5}{8}+2\frac{1}{2}$	(*b*) $\frac{3}{4}-\frac{1}{5}$	(*b*) $7\frac{5}{6}-4\frac{1}{2}$
Group VII	**Group VIII**	**Group IX**
(*a*) $6\frac{1}{3}-2\frac{5}{12}$	(*a*) $17-\frac{5}{6}$	(*a*) $16-3\frac{5}{8}$
(*b*) $12\frac{1}{2}-3\frac{7}{8}$	(*b*) $4-\frac{2}{3}$	(*b*) $4\frac{5}{8}-2$

Part II (*Time, 8 minutes*)

Group X	Group XI	Group XII
(*a*) $\frac{2}{5}\times\frac{2}{3}$	(*a*) $3\frac{1}{4}\times 8$	(*a*) $2\frac{1}{3}\times\frac{3}{4}$
(*b*) $\frac{1}{3}\times\frac{5}{6}$	(*b*) $9\times 2\frac{1}{3}$	(*b*) $\frac{5}{6}\times 1\frac{2}{3}$
Group XIII	**Group XIV**	**Group XV**
(*a*) $2\frac{1}{2}\times 3\frac{2}{5}$	(*a*) $\frac{1}{2}\div\frac{3}{4}$	(*a*) $6\div\frac{2}{3}$
(*b*) $4\frac{1}{2}\times 1\frac{1}{3}$	(*b*) $\frac{1}{8}\div\frac{5}{8}$	(*b*) $\frac{3}{4}\div 9$
Group XVI	**Group XVII**	**Group XVIII**
(*a*) $\frac{3}{4}\div 1\frac{1}{2}$	(*a*) $1\frac{3}{5}\div 5$	(*a*) $2\frac{1}{2}\div 3\frac{2}{3}$
(*b*) $1\frac{1}{4}\div\frac{1}{7}$	(*b*) $10\div 2\frac{1}{2}$	(*b*) $3\frac{1}{2}\div 2\frac{3}{4}$

Part I

Practice Group I

(a) $\frac{1}{12} + \frac{1}{4} + \frac{1}{2}$

(b) $\frac{1}{4} + \frac{1}{3} + \frac{1}{6}$

(c) $\frac{1}{8} + \frac{1}{2} + \frac{1}{4}$

(d) $\frac{1}{5} + \frac{1}{4} + \frac{1}{10}$

(e) $\frac{1}{5} + \frac{1}{10} + \frac{1}{20}$

Practice Group II

(a) $\frac{3}{4} + \frac{1}{2} + \frac{7}{8}$

(b) $\frac{3}{5} + \frac{3}{4} + \frac{1}{2}$

(c) $\frac{2}{3} + \frac{5}{6} + \frac{1}{2}$

(d) $\frac{2}{3} + \frac{3}{4} + \frac{5}{6}$

(e) $\frac{5}{8} + \frac{1}{3} + \frac{1}{2}$

Practice Group III

(a) $4\frac{1}{5} + 5\frac{1}{10} + 7$

(b) $7\frac{1}{10} + 6\frac{1}{4} + 3\frac{1}{5}$

(c) $3\frac{3}{8} + 7 + 2\frac{1}{4}$

(d) $5\frac{1}{4} + \frac{1}{3} + 7\frac{1}{6}$

(e) $4\frac{5}{12} + 2\frac{1}{4} + 9$

Practice Group IV

(a) $4\frac{5}{8} + 2\frac{1}{3} + 4\frac{1}{2}$

(b) $6\frac{2}{3} + 4\frac{3}{4} + \frac{5}{6}$

(c) $3\frac{1}{2} + 5\frac{5}{6} + 1\frac{2}{3}$

(d) $7\frac{3}{5} + 4\frac{3}{4} + 8$

(e) $5\frac{3}{4} + \frac{1}{2} + 7\frac{7}{8}$

Practice Group V

(a) $\frac{5}{8} - \frac{1}{2}$

(b) $\frac{3}{4} - \frac{2}{5}$

(c) $\frac{5}{6} - \frac{1}{4}$

(d) $\frac{2}{3} - \frac{3}{8}$

(e) $\frac{4}{5} - \frac{1}{2}$

Practice Group VI

(a) $6\frac{4}{5} - 2\frac{1}{3}$

(b) $8\frac{2}{3} - 4\frac{5}{8}$

(c) $9\frac{5}{6} - 2\frac{3}{4}$

(d) $8\frac{3}{4} - 1\frac{3}{5}$

(e) $3\frac{7}{8} - 2\frac{1}{3}$

Practice Group VII

(a) $6\frac{1}{2} - 2\frac{5}{6}$

(b) $3\frac{1}{3} - 2\frac{3}{8}$

(c) $6\frac{1}{2} - 5\frac{4}{5}$

(d) $8\frac{2}{5} - 6\frac{3}{4}$

(e) $9\frac{5}{8} - 4\frac{3}{4}$

Practice Group VIII

(a) $18 - \frac{3}{4}$

(b) $20 - \frac{7}{8}$

(c) $16 - \frac{2}{3}$

(d) $4 - \frac{5}{6}$

(e) $8 - \frac{3}{8}$

Practice Group IX

(a) $17 - 3\frac{7}{8}$

(b) $19 - 2\frac{3}{4}$

(c) $20 - 4\frac{1}{8}$

(d) $5\frac{2}{3} - 2$

(e) $6\frac{3}{5} - 1$

Part II

Practice Group X

(a) $\frac{3}{4} \times \frac{5}{6}$

(b) $\frac{2}{3} \times \frac{3}{4}$

(c) $\frac{7}{8} \times \frac{4}{5}$

(d) $\frac{3}{7} \times \frac{2}{3}$

(e) $\frac{5}{9} \times \frac{3}{4}$

Practice Group XI

(a) $4\frac{1}{2} \times 6$

(b) $7 \times 2\frac{1}{3}$

(c) $4\frac{1}{2} \times 9$

(d) $8 \times 1\frac{1}{2}$

(e) $3\frac{1}{2} \times 8$

Practice Group XII

(a) $3\frac{1}{3} \times \frac{3}{4}$

(b) $5\frac{1}{2} \times \frac{4}{5}$

(c) $\frac{3}{4} \times 2\frac{1}{3}$

(d) $\frac{7}{8} \times 4\frac{1}{3}$

(e) $6\frac{1}{3} \times \frac{6}{7}$

Practice Group XIII

(a) $2\frac{1}{3} \times 1\frac{1}{4}$

(b) $5\frac{1}{2} \times 1\frac{2}{3}$

(c) $6\frac{1}{4} \times 1\frac{1}{3}$

(d) $3\frac{1}{2} \times 2\frac{1}{3}$

(e) $1\frac{3}{5} \times 1\frac{1}{4}$

Practice Group XIV

(a) $\frac{3}{4} \div \frac{1}{2}$

(b) $\frac{7}{8} \div \frac{3}{4}$

(c) $\frac{5}{6} \div \frac{2}{3}$

(d) $\frac{3}{8} \div \frac{6}{7}$

(e) $\frac{2}{3} \div \frac{5}{6}$

Practice Group XV

(a) $7 \div \frac{1}{2}$

(b) $8 \div \frac{2}{3}$

(c) $\frac{4}{5} \div 16$

(d) $\frac{7}{8} \div 2$

(e) $9 \div \frac{3}{4}$

Practice Group XVI

(a) $\frac{7}{8} \div 3\frac{1}{2}$

(b) $\frac{5}{6} \div 2\frac{1}{2}$

(c) $4\frac{1}{2} \div \frac{2}{3}$

(d) $3\frac{2}{3} \div \frac{1}{3}$

(e) $\frac{1}{5} \div 1\frac{1}{5}$

Practice Group XVII

(a) $2\frac{1}{3} \div 7$

(b) $3\frac{1}{3} \div 5$

(c) $8 \div 1\frac{1}{3}$

(d) $9 \div 1\frac{1}{2}$

(e) $3\frac{2}{3} \div 2$

Practice Group XVIII

(a) $4\frac{1}{2} \div 1\frac{1}{2}$

(b) $3\frac{1}{3} \div 2\frac{1}{2}$

(c) $1\frac{1}{2} \div 3\frac{3}{4}$

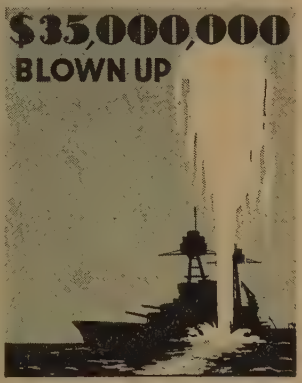
(d) $2\frac{1}{3} \div 3\frac{1}{2}$

(e) $4\frac{1}{2} \div 2\frac{1}{2}$

Those who read newspapers or magazines must be able to read and understand large numbers. Notice, for instance, the figures in the following quotation from a newspaper:

The World War cost the United States $51,000,000,000. As recently as 1885, $51,000,000,000 would have bought the whole United States, with everybody and everything in it, lock, stock, and barrel.

Today $51,000,000,000 would buy sixteen states like Alabama, or three enormously rich Californias, with a couple of Colorados thrown in for good measure. It would buy the entire state of New York—including the metropolis, wonder city of the world—and there still would be left over enough change to acquire four states like Maryland.



World Peaceways, Inc.

If you had started throwing away dollars the day Christ was born, and kept it up ever since, at the rate of a dollar a minute without pausing either to eat or to sleep, you would only now be starting on your second billion, with 95,000 years more to go.

At 5 per cent, $51,000,000,000 would provide an annual income of $2,550,000,000, or enough to pension 2,125,000 old and broken-down workers at $100 a month, virtually abolishing poverty in this country.

In how many ways does the writer of the above article help you to appreciate the enormous sum of $51,000,000,000? Can you check his statements in the third and fourth paragraphs?

The extract above is a sample of the way large numbers are used. Spend a week making a collection of clippings containing large numbers for class discussion. Try particularly to find articles in which the writer helps you to understand the numbers used by comparisons, graphs, diagrams, or in other ways.

1. According to the last census, the population of the United States was 122,775,046. For most practical purposes we say that the population was 123,000,000. Why do we say 123 rather than 122 million?

2. Do you know the diameter of the earth? You might find any one of these figures for it: (*a*) 7918.57 miles; (*b*) 7900 miles; (*c*) 8000 miles. Each number is inaccurate to some extent. Which result is most accurate? least accurate?

3. In saying that the diameter of the earth is 8000 miles, we mean that the diameter is 8000 miles *to the nearest thousand*. What does it mean to say that the diameter is 7900 miles?

4. When the population of the United States is given as 123,000,000, we say it is stated *in round numbers*. In what round numbers is the diameter of the earth sometimes given?

5. We say that 7918.57 miles "rounded off" to *thousands* is 8 thousand miles; rounded off to hundreds it is 79 hundred miles. How is 122,775,046 expressed when rounded off to millions?

6. Mr. Jones threshed 702 bushels of wheat. Rounded off to hundreds, that was __?__ bushels.

7. The number 31.576 rounded off to the nearest hundredth is 31.58; to the nearest tenth it is 31.6; to the nearest unit it is 32. What is 7918.17, rounded off to the nearest tenth? to the nearest unit? to hundreds? to thousands?

31.576
31.58
31.6
32.

8. According to the last census, the population of New York City was 6,930,446, and the population of Philadelphia was 1,950,961. New York was about how many times as large as Philadelphia? (Round off before dividing.)

9. A great scientist found that the speed of light is 186,284 miles per second. If you remember the result in thousands, that will be accurate enough for most purposes.

Miss Simms was preparing the eighth-grade honor roll. Roy Peterson worked out the averages and gave her the list shown here.

Ethel Winters	$94\frac{5}{6}$
Harry Fast	$94\frac{7}{8}$
James Garner	$94\frac{2}{3}$
Robert Fuller	$92\frac{2}{3}$
Jane Millman	$92\frac{4}{7}$
Portia Heller	$90\frac{3}{4}$

Miss Simms: Thank you, Roy. I wish, however, you had written the fractional remainders as decimals, expressed to the nearest tenth. Do you see why?

Roy: I believe I do. Ethel, Harry, and James happen to be so close that you might not know at a glance which one is really ahead by looking at those common fractions.

Miss Simms: That's it exactly. Decimal fractions are much more convenient for use when you wish to compare the sizes of fractions. Also, if you need to add unlike common fractions, it is often easier to change them to decimals and then add.

Roy revised the list like this. Check his work. Do you consider this list easier to read than Roy's first one?

Harry Fast	94.9
Ethel Winters	94.8
James Garner	94.7
Robert Fuller	92.7
Jane Millman	92.6
Portia Heller	90.8

Any fraction whose denominator is 10, 100, 1000, etc., may be written as a decimal.

1. $\frac{5}{6} = 6\overline{)5.00}$ $.83$

State the rule for changing common fractions with denominators other than 10, 100, 1000, etc., to decimal fractions.

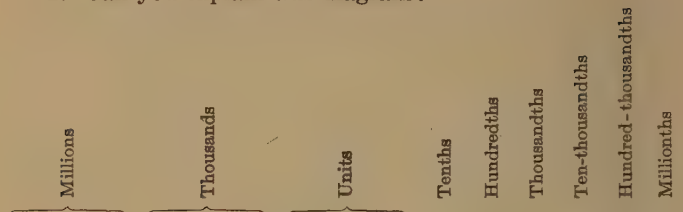
2. Express as decimals to the nearest hundredth and arrange in order of size beginning with the smallest:

$\frac{1}{4}$, $\frac{3}{4}$, $\frac{2}{5}$, $\frac{4}{5}$, $\frac{1}{6}$, $\frac{1}{7}$, $\frac{3}{7}$, $\frac{5}{7}$, $\frac{1}{8}$, $\frac{3}{8}$.

3. Arrange these mixed numbers in order of size. (First express each as a decimal to the nearest hundredth.)

$94\frac{5}{8}$, $94\frac{3}{5}$, $94\frac{1}{3}$, $93\frac{3}{3}$, $93\frac{4}{7}$, $94\frac{8}{9}$, $94\frac{7}{12}$.

1. Can you explain this diagram?



2. The number to the left of the decimal point is a whole number. In reading a number composed of a whole number and a decimal, when we come to the decimal point we say "and." 304.4 is read "three hundred four *and* four tenths."

Read these decimals:

	(*a*)	(*b*)	(*c*)	(*d*)	(*e*)
3.	27.20	.08	.3094	.95876	.74385
4.	.8	.009	.89750	8.6202	.598247
5.	.82	27.643	2.04632	.87054	.046273

6. Perhaps your teacher will dictate some decimals to see whether you can write them correctly in figures.

7. Arrange these numbers in order of size, beginning with the largest: 2.22 2.2220 2.202 2.2202 2.2002

8. With the help of the examples below state the rule for changing a decimal fraction to a common fraction.

$$.25 = \frac{25}{100} = \frac{1}{4} \qquad .375 = \frac{375}{1000} = \frac{75}{200} = \frac{3}{8}$$

Change these decimals to common fractions:

	(*a*)	(*b*)	(*c*)	(*d*)	(*e*)	(*f*)
9.	.75	.62½	.15	.345	.35	.37
10.	.50	.375	.45	.66⅔	.87½	.33⅓

Herbert measured the length of a line AB. He had a very accurately constructed scale which he could read to tenths of an inch. He placed the zero of the scale on point A and noted that the length of the line was very close to 2.6 inches. He recorded its length as 2.6 inches.

Herbert might have *misread* the scale and recorded the measurement as 2.4 instead of 2.6 inches. In the exercises that follow it is assumed that no errors of this kind were made.

1. Herbert said that his result, 2.6 inches, was only *approximately* correct. What did he mean?

2. If the length of Herbert's line were between 2.5 inches and 2.6 inches, but a little closer to the 2.6-inch mark on the ruler than to the 2.5 mark, Herbert would call it _?_ inches. If the length of the line were between 2.6 inches and 2.7 inches, but a little closer to the 2.6-inch mark on the ruler than to the 2.7 mark, Herbert would call it _?_ inches.

Herbert said that his result was *correct to tenths* of an inch. He meant that the true length of the line was closer to 2.6 inches than to 2.5 inches or to 2.7 inches.

3. Herbert assumed that he could tell, with his eye, whether the end of the line was closer to the 2.6-inch mark than to the 2.7-inch mark. If Herbert says that a line is 2.6 inches long, he means that its true length is between 2.55 inches and 2.65 inches. Illustrate with a large drawing.

4. If Herbert has measured a line and given the length as 3.8 inches and has stated that the measurement is correct to tenths of an inch, he means that the true length of the line is between ___?___ and ___?___ inches.

5. If the true length of the line in Ex. 4 is between 3.75 in. and 3.85 in., then the difference between the stated length and the true length cannot be greater than ___?___ in. This difference we may call the *inaccuracy* of the stated measure. Thus, if a true distance of 3.824 in. is stated as 3.8 in., the difference (.024) is the inaccuracy of the stated measurement (3.8).

The *inaccuracy* of a stated measurement (assuming the scale has not been misread) is the amount by which the stated measurement differs from the true length.

6. Fred said that a line was 4.4 inches long. If he did not misread the scale, the true measure is between ___?___ inches and ___?___ inches. The inaccuracy of his statement was not over ___?___ inch.

7. Thomas and Marie measured the length of a line. Thomas said the line was 6.20 inches long, and Marie said it was 6.2 inches. Which pupil *claimed* to have measured more precisely? What is the possible inaccuracy of Marie's measure? of Thomas's?

8. What is the possible inaccuracy of a measurement (*a*) with your metric ruler? (*b*) with your inch ruler?

9. Express the following numbers to the nearest tenth:

(*a*)	(*b*)	(*c*)	(*d*)
.79	.80	9.864	.875
.84	.94	72.607	.209
.77	1.89	2.540	.793

10. Express the numbers in Ex. 9 (*c*) and (*d*) to the nearest hundredth.

Test 6. Locating Difficulties with Decimals

If you have any difficulty with the examples in this test, you can find the practice you need on the following pages. Record your score. See page 352 for answers.

PART I (*Time, 5 minutes*)

Addition and Subtraction

GROUP I

(*a*) .73 (*b*) .49
.24 .33
.98 .86
.74 .03
.63 .77
.29 .28

GROUP II

(*a*) 2.9 (*b*) 6.49
4.83 4.76
8.36 9.33
.4 .2
9.65 8.48
7.5 .9

GROUP III

(*a*) .84 + 3.25 + 6.4
(*b*) 25 + 2.8 + .9

GROUP IV

(*a*) 3.48 − 2
(*b*) 75 − 68.5

GROUP V

(*a*) .63 (*b*) .75
−.29 −.43

GROUP VI

(*a*) 2.23 (*b*) 23.5
−1.845 −7.24

PART II (*Time, 8 minutes*)

Multiplication and Division

Copy the answers in these examples and place the decimal point correctly in each:

GROUP VII

(*a*) .75 × .63 = 4725
(*b*) .84 × .35 = 2940

GROUP VIII

(*a*) 4.3 × 7.5 = 3225
(*b*) 8.2 × .64 = 5248

(*Go right on to the next page.*)

Diagnostic Test (Continued)

GROUP IX

(*a*) $.07 \times .25 = 175$

(*b*) $.05 \times .75 = 375$

GROUP X

(*a*) $.08 \times .07 = 56$

(*b*) $.11 \times .06 = 66$

GROUP XI

(*a*) $8\overline{)2584}$ $\quad 323$

(*b*) $7\overline{)45.633}$ $\quad 6519$

GROUP XII

(*a*) $7.5\overline{)4.725}$ $\quad 63$

(*b*) $.63\overline{)4.725}$ $\quad 75$

GROUP XIII

(*a*) $24.6\overline{).1968}$ $\quad 8$

(*b*) $2.74\overline{).1644}$ $\quad 6$

Express the answers to the nearest hundredth:

GROUP XIV

Divide: (*a*) 29.5 by .84

(*b*) 295 by .35

GROUP XV

Divide: (*a*) 15 by 17

(*b*) 8 by 9

Practice with Decimals

PRACTICE GROUP I

(*a*)	(*b*)	(*c*)	(*d*)	(*e*)
.42	.93	.47	.31	.94
.43	.06	.76	.27	.87
.76	.82	.08	.68	.68
.95	.40	.85	.19	.55
.87	.77	.39	.40	.03
.63	.08	.92	.84	.87

PRACTICE GROUP II

(*a*)	(*b*)	(*c*)	(*d*)
3.6	.51	4.3	8.59
.92	3.0	.66	7.0
4.21	6.29	8.8	.28
.35	8.18	.97	4.63
7.7	.67	7.34	.7
5.89	1.4	1.55	6.45

Practice Group III

(*a*) .56 + 2.75 + 8.6
(*b*) 9.4 + 7 + 8.75
(*c*) 2.3 + .54 + 9
(*d*) 38 + 24.5 + .90

Practice Group IV

(*a*) 9 − 8.3
(*b*) 2.43 − .86
(*c*) 12 − 5.6
(*d*) 3.42 − .8
(*e*) 9.76 − 4
(*f*) 3 − 2.75

Practice Group V

(*a*)	(*b*)	(*c*)	(*d*)	(*e*)
.73	.82	.40	.61	.41
− .24	− .56	− .39	− .54	− .33

Practice Group VI

(*a*)	(*b*)	(*c*)	(*d*)
3.46	8.35	23.8	10.00
− 2.856	− .786	− 7.94	− 9.84

Copy the answers to these examples and place the decimal point correctly in each:

Practice Group VII

(*a*) .85 × .63 = 5355
(*b*) .28 × .96 = 2688
(*c*) .5 × .62 = 310
(*d*) .8 × .7 = 56

Practice Group VIII

(*a*) 2.2 × 6.4 = 1408
(*b*) 7.5 × .38 = 2850
(*c*) .55 × 8.4 = 4620
(*d*) 3.6 × 7.8 = 2808

Practice Group IX

(*a*) .5 × .12 = 60
(*b*) .06 × .25 = 150
(*c*) .4 × .22 = 88
(*d*) .35 × .07 = 245

Practice Group X

(*a*) .09 × .06 = 54
(*b*) .11 × .07 = 77
(*c*) .08 × .05 = 40
(*d*) .004 × .7 = 28

Practice Group XI

(*a*) $6\overline{)37.2}$ = 62

(*b*) $7\overline{)48.58}$ = 694

(*c*) $5\overline{)2.5325}$ = 5065

(*d*) $9\overline{)2701.8}$ = 3002

Practice Group XII

(*a*) $2.8\overline{)2.688}$ = 96

(*b*) $.63\overline{)5.355}$ = 85

(*c*) $.22\overline{)14.08}$ = 64

(*d*) $.75\overline{)2.850}$ = 38

Practice Group XIII

(*a*) $6.4\overline{).1408}$ = 22

(*b*) $38\overline{)2.850}$ = 75

(*c*) $9.6\overline{).2688}$ = 28

(*d*) $27.5\overline{).1650}$ = 6

Practice Group XIV

Express the answers to the nearest hundredth:

(*a*) 26.4 ÷ 3.85

(*b*) 27 ÷ 3.5

(*c*) 28 ÷ .75

(*d*) 4.3 ÷ 2.15

(*e*) 5.8 ÷ .31

(*f*) 2.46 ÷ 1.8

(*g*) 35.6 ÷ 24

(*h*) 75 ÷ .28

Practice Group XV

Express the answers to the nearest hundredth:

(*a*) 8 ÷ 11

(*b*) 7 ÷ 9

(*c*) 23 ÷ 37

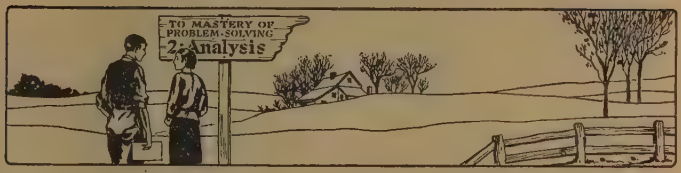
(*d*) 18 ÷ 23

(*e*) 16 ÷ 27

(*f*) 14 ÷ 56

(*g*) 75 ÷ 126

(*h*) 84 ÷ 109



Help No. II. Analyzing Problems

In solving problems you should—

(1) Read the problem carefully. (Be sure that you know the meaning of all the words.)

(2) Decide what you need to find.

(3) Note all the facts stated in the problem, and look up any other facts you may need to know.

(4) Decide what to do — whether to add or to divide, etc.

(5) Perform accurately the processes decided upon and check. (No computation is satisfactory until it has been checked.)

(6) Estimate the result, if possible, and see whether the answer looks reasonable.

Use the above steps in solving the following problems:

1. A general-science class used 20 cu. ft. of gas in an experiment. Find the cost of the gas at \$1 per 1000 cu. ft.

2. Mr. Smith bought 100 acres of land at \$70 an acre and 40 acres at \$65 an acre. He sold all the land at \$68 an acre. Did he gain or lose? How much?

3. A storekeeper buys soap at \$4.32 a gross (144 cakes) and sells it at 5¢ a cake. What is the margin on each gross? (The margin is the difference between selling price and cost price.)

4. A grocer paid \$4.50 for a barrel of apples ($2\frac{3}{4}$ bu.). One peck of the apples spoiled. He sold the rest at 80 cents a peck. How much was his margin?

5. A committee was appointed to buy class colors for the eighth grade. Each pupil was to have two ribbons, of different colors, each 8 inches long. The ribbon sold for $.24 a yard. There were 10 boys and 8 girls in the class. How much did the committee have to collect from the class to pay the bill?

6. John read that light, traveling 186,000 miles per second, required 1.2 *seconds* to reach us from the moon. With that information how could he compute the distance between the moon and the earth? Make the computation.

7. John also read that light requires 8.2 *minutes* to come from the sun. What is the distance from the earth to the sun?

8. If Will Rogers is to speak on the radio in California at 4:15 Pacific Standard Time, at what time (Eastern Standard Time) should Marie Miller, who lives in Carlisle, Pennsylvania, tune in so that she can hear his program?

9. James read: "Is not the person who jaywalks (crosses diagonally at a street intersection) foolish to risk 17,000,000 minutes of his life just to gain one minute or less?" He figured out that the person who risks 17,000,000 minutes of his life must expect to live about __?__ years longer.

10. Between 9 A.M. and 12:30 P.M. a man drove 105 miles. At this rate how far would he travel between 2 P.M. and 5 P.M.?

11. Jane bought maple syrup at $1.50 a gallon and sold it for $.25 a pint. Her expenses were, on the average, $.20 a gallon. How much did she gain? The gain was what fractional part of the cost price?

12. A cabinetmaker charges $1.20 an hour for repair work. He spent two 7-hour days refinishing a chest for Mrs. Jones. What was the amount of her bill?

13. A gasoline station sold gasoline at 16.3¢ a gallon. How much change would you get from a 5-dollar bill after buying 8 gallons of gasoline?

14. A certain kind of silk sells regularly for \$3 a yard. A remnant 27 in. long sold for \$1. This was a reduction of what fractional part of the regular price?

15. Three boys, A, B, and C, agreed to do a piece of work for which they were to receive \$2.10. A worked $1\frac{1}{2}$ hours, B worked 2 hours, and C worked $3\frac{1}{2}$ hours. How much should each receive?

16. A certain kind of automobile, under ordinary care, depreciates $\frac{1}{4}$ of the purchase price the first year, $\frac{1}{5}$ of the purchase price the second year, and $\frac{1}{6}$ the third year. By taking excellent care of his car, which he bought new for \$1200, Mr. Brown was able to sell it for \$600 at the end of 3 years. His excellent care had saved him __?__ dollars.

17. At 1:30 P.M. a man started to drive west from Columbus, Ohio. At 5:10 P.M. he was 121 miles west of Columbus. What was his average speed?

18. If telephone poles are placed 100 ft. apart, how many poles will you pass in a minute if you are traveling at the rate of 40 miles an hour?

19. A grocer bought oranges for \$3.15 a crate of 10 dozen. He sold them at 6 for 25¢. Allowing for the fact that 6 of the oranges spoiled in shipment, what was his margin?

20. Thomas has a sheet of paper $8\frac{1}{2}$ inches wide. He needs to make on it one column $2\frac{3}{4}$ inches wide and four other equal columns taking up the remaining space. How wide will each of his four columns be?

21. Our car travels an average of 16 miles to the gallon of gasoline. At 19¢ a gallon, how much will the gasoline for a 500-mile trip cost?

22. If .3 inch on a map represents a mile, how long a line should you draw to represent 25 miles?

The persons who buy goods or service are sometimes dissatisfied with the goods or service they receive. In such cases they usually are justified in making complaints to the business firm involved. As a rule, that firm is only too glad to do what it can to remedy the situation. Such complaints should always be made in a dignified manner, after the person has made a careful study of the *facts* and *figures* in the case. Consider, for example, the complaint of Mrs. Jackson, who was being annoyed by "wrong number" telephone calls. One morning in June on a particularly busy day she had several such calls. She was irritated. She might have gone to the telephone and in angry tones denounced the telephone company for the nuisance; but she didn't. Instead she decided to keep a list of all the "wrong number" calls she had over a period of time. Then she wrote the telephone company the following letter:

June 29, 1936

Gentlemen:

From June 5 to June 22 inclusive, I had eight "wrong number" telephone calls. I have kept a record of the date, hour, and number wanted in each case. Now, I figure that if I have 8 such calls in 18 days, I have 13 of them per month. If each of your 5000 subscribers is being annoyed in a similar way, it means there are 65,000 "wrong number" calls being put through per month. When you consider the fact that 2 parties are always inconvenienced by each call, it means you irritate 130,000 persons per month in this manner.

There is another aspect to the case. If 65,000 "wrong number" calls are made per month, they mount up to 780,000 per year. At 5 cents per call your company is therefore receiving the tidy sum of $39,000 per year, in return for which you are giving not service but annoyance.

I feel that you should try to eliminate this public nuisance, the "wrong number" call.

Yours truly,

(Mrs. G. W.) Ethel Jackson

By Ewing Galloway, N. Y.

1. How did Mrs. Jackson (page 70) figure that she has 13 wrong-number telephone calls per month?

2. Check all the computations Mrs. Jackson made.

3. Within a few days Mrs. Jackson received a courteous letter from the telephone company, stating that they would investigate the causes of the wrong-number calls and eliminate as many as possible in the future. During nineteen days immediately following the receipt of that letter, Mrs. Jackson had only two wrong-number calls. At that rate, she would receive only ___?___ wrong-number calls in a year.

4. Did the telephone company immediately reduce the number of wrong-number calls about $\frac{1}{4}$, $\frac{1}{2}$, or $\frac{3}{4}$?

5. Does your school board supply you with writing pads? If so, perhaps you could write a letter to the pupils of your town urging them not to waste paper and showing the yearly needless cost to the school board if each pupil in the town wastes two sheets of paper a day.

Do you agree that facts and figures concerning a need are much more convincing than vague and uncertain statements? A well-educated person reads carefully the figures he meets in newspapers and magazines and learns to use figures accurately and carefully in both conversation and writing.

1. John is y years and 7 months old. Write a formula which expresses his age a in months.

2. Upon what formula is the following table based?

If a equals...	1	2	3	6	7	8	10
then c equals.	5	9	13	25			

3. What numbers are missing from the table in Ex. 2?

Use an equation to solve each of the following problems:

4. Harry has a strip of silver picture molding 24 inches long. With it he wishes to make a picture frame that is twice as long as it is wide. What dimensions will it have?

5. A farmer and his helper cleared $84.20 on a potato crop. They had agreed that the farmer should take 3 times as much of the profit as the helper. How much should each take?

6. *Puzzle.* Ann's grandmother is 5 times as old as Ann. The sum of their ages is 78 years. How old is Ann?

7. A carpenter and his helper together received $13.50 for doing a piece of work. They decided that the carpenter, because of greater skill, should receive twice as much as his helper. How should they divide the $13.50?

8. Robert decided that the taxicabs in his city use the formula $c = 5f + 15$, in which c stands for the cost in cents and f the distance in fourths of a mile. What would it cost to ride 1 fourth of a mile? 3 fourths? $1\frac{1}{2}$? Make a graph of the formula. (HINT. Make a table first.)

9. .25 of what number equals 100?

10. An art student is to draw a triangle, two sides of which are equal. The equal sides are each to be twice as long as the third side, and the perimeter is to be 30 inches. How long should she make each of the sides of the triangle?

1. In the equation $y = 6x + 7$, what is y if $x = 4$?

2. Ruth is selling soap. She is allowed to keep $\frac{2}{5}$ of the amount of her sales. She wishes to earn \$20. How much soap must she sell to earn what she wishes? (Use an equation.)

Solve and check each of these equations:

3. $2x - 3 = 1$

4. $\frac{x}{5} = 12$

5. $.06x = 72$

6. $5x + 1 = 36$

7. Express 2.64 to the nearest tenth.

8. A bank said in one of its advertisements: "Our 3896 depositors have deposited with us \$408,365.47." Express the number of depositors to the nearest thousand.

9. Express the amount of deposits (Ex. 8) to the nearest hundred thousand dollars.

10. Using your answers to Ex. 8 and Ex. 9, would you estimate the amount of the average account in that bank to be approximately \$10, \$100, or \$1000?

11. When Mr. Kline started on an automobile trip, his odometer registered 3246.8 miles. (An *odometer* is an instrument for measuring distance traveled.) At the end of 16 hours' driving it read 3716.0 miles. What was his average rate of driving in whole miles per hour?

12. It is the custom at the Merivale School to consider that at the end of a year's use a textbook is worth $\frac{2}{3}$ of what it was worth at the beginning of the year, to the nearest nickel. Marie bought a new mathematics book for \$1. She used it a year and her brother used it the next year. Now they want to sell it. How much should they ask for it?

Standards: *A, 10.* *B, 8.* *C, 6.*

CHAPTER III

PER CENTS

Fractional Parts

Miss Green's eighth-grade class is entering a county arithmetic contest. They have received a list of the types of problems upon which they will be tested. Here is a part of the list:

FRACTIONAL PARTS

TYPE I. Finding a fractional part of a number. ($\frac{7}{8}$ of 16 = ?)

TYPE II. Finding what part one number is of another. (8 is what fractional part of 12?)

TYPE III. Finding a number when a fractional part of it is given. ($\frac{5}{8}$ of what number equals 15?)

1. Miss Green asked James to give a problem of the first type. What would you have given if you had been in James's place?

James said: "If a pound of cookies costs 20¢, $\frac{3}{4}$ pound will cost $\frac{3}{4}$ of 20¢, or 15¢."

2. Helen gave this example of the second type: "Out of a list of 20 questions, I have answered 16 correctly. I have answered $\frac{16}{20}$, or $\frac{?}{}$, of them correctly."

3. Leo gave this as an illustration of the third type: "After I had poured 8 gallons of water into an empty tank, the gauge showed it to be $\frac{2}{3}$ full. I wondered how many gallons the tank would hold."

This is the way Leo found out:

$\frac{2}{3}$ of the contents of the tank = 8 gal.
$\frac{1}{3}$ of the contents = $\frac{1}{2}$ of 8 gal. = 4 gal.
$\frac{3}{3}$ of the contents (whole volume) = 3×4 gal., or 12 gal.

4. Solve these examples, which Miss Green gave her pupils to do in preparation for the test on fractional parts:

(*a*) 40 is what part of 75?
(*b*) 42 is $\frac{6}{7}$ of what?
(*c*) 28 is what part of 35?
(*d*) $\frac{5}{6}$ of 108 equals what?
(*e*) 25 is $\frac{5}{8}$ of what?
(*f*) $\frac{15}{16}$ of 20 equals what?
(*g*) 48 is what part of 64?
(*h*) 28 is $\frac{7}{9}$ of what?
(*i*) 20 is $\frac{4}{5}$ of what?
(*j*) 12 is what part of 40?

These are the problems on fractional parts which were included in the county contest. Can you solve them?

1. A basketball team won 7 of the 12 games which it played and had a tie score in 2 games. What part of the games played did the team win? What part of the games played did the team lose? What part were tied?

2. Harry is preparing 1000 advertisements for mailing. He has finished 72 in $\frac{3}{4}$ hour. At that rate how many advertisements can he do an hour? How long should he allow himself to prepare all the advertisements?

3. The list price of a victrola was $56. It was offered to a school, however, for $\frac{3}{8}$ off the list price. If the school buys the victrola, how much will it cost?

The circular announcing the county contest also listed the following *types of per cent problems* upon which the pupils would be tested:

PER CENTS

TYPE I. Finding a per cent of a number. (6% of 90 = ?)

TYPE II. Finding what per cent one number is of another. (10 is what per cent of 40?)

TYPE III. Finding a number when a per cent of it is known. (12 is 60% of what number?)

NOTE. Pupils will need to know the *fractional equivalent* of many common per cents to solve problems of the above types.

Miss Green planned the following review of per cents for her class.

(1) Every per cent can be written in fractional form. It is often easier to compute with a fraction than with a decimal.

(2) $37\frac{1}{2}\% = .37\frac{1}{2} = \frac{3}{8}$. Is it easier to find $37\frac{1}{2}\%$ of 56 as $\frac{3}{8}$ of 56 or as $.37\frac{1}{2} \times 56$?

(3) Your work in per cents will be made much easier if you know these scales of *fractional equivalents* of decimals.

0	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{3}{8}$	$\frac{1}{2}$	$\frac{5}{8}$	$\frac{3}{4}$	$\frac{7}{8}$	1
0	$.12\frac{1}{2}$	.25	$.37\frac{1}{2}$	.50	$.62\frac{1}{2}$	.75	$.87\frac{1}{2}$	1.00

0	$\frac{1}{10}$	$\frac{1}{5}$	$\frac{3}{10}$	$\frac{2}{5}$	$\frac{1}{2}$	$\frac{3}{5}$	$\frac{7}{10}$	$\frac{4}{5}$	$\frac{9}{10}$	1
0	.10	.20	.30	.40	.50	.60	.70	.80	.90	1.00

0	$\frac{1}{12}$	$\frac{1}{6}$	$\frac{1}{4}$	$\frac{1}{3}$	$\frac{5}{12}$	$\frac{1}{2}$	$\frac{7}{12}$	$\frac{2}{3}$	$\frac{3}{4}$	$\frac{5}{6}$	$\frac{11}{12}$	1
0	$.8\frac{1}{3}$	$.16\frac{2}{3}$	.25	$.33\frac{1}{3}$	$.41\frac{2}{3}$	.50	$.58\frac{1}{3}$	$.66\frac{2}{3}$	.75	$.83\frac{1}{3}$	$.91\frac{2}{3}$	1.00

(4) Make a scale like the one given above, without the help of the book. Include in your scale 0, $\frac{1}{16}$, $\frac{1}{8}$, $\frac{3}{16}$, $\cdots$ 1 and the equivalent decimal fractions.

To express a per cent as a decimal, omit the per cent sign and move the decimal point two places to the left. (For example, $5\% = .05$; $2.5\% = .025$.)

1. Express these per cents as decimals:

14%	$4\frac{1}{2}\%$	$62\frac{1}{2}\%$	300%	3.4%
270%	4%	.3%	$\frac{1}{8}\%$	.5%

2. Write each of the following in three forms — as a common fraction, as a decimal fraction, and as a per cent.

(*a*) $\frac{1}{10}$	(*f*) 1.25	(*k*) $37\frac{1}{2}\%$	(*p*) $\frac{2}{50}$	(*u*) 0.0625
(*b*) $12\frac{1}{2}\%$	(*g*) $\frac{2}{5}$	(*l*) $\frac{1}{4}\%$	(*q*) 70%	(*v*) $137\frac{1}{2}\%$
(*c*) 0.8	(*h*) $62\frac{1}{2}\%$	(*m*) $\frac{3}{4}$	(*r*) $2\frac{1}{4}\%$	(*w*) 0.035
(*d*) $\frac{7}{8}$	(*i*) 0.2	(*n*) 60%	(*s*) $\frac{9}{25}$	(*x*) 110%
(*e*) 2%	(*j*) $2\frac{5}{8}$	(*o*) 0.9	(*t*) 0.5	(*y*) 0.16

A coat sold regularly for $20. At a sale it was offered for 75% of the regular price. Find the sale price of the coat.

We can get the answer in two ways:

SOLUTION I

$75\% = .75$

$.75 \times \$20 = \$15$

SOLUTION II

$75\% = .75 = \frac{3}{4}$

$\frac{3}{\cancel{4}_{1}} \times \cancel{\$20}^{5} = \$15$

Per cent of = "hundredths times."

By Ewing Galloway, N. Y.

Solve these examples in the two ways mentioned above:

1. 60% of 15 =
2. 50% of 96 =
3. 25% of 40 =
4. 10% of 60 =
5. $87\frac{1}{2}\%$ of 200 =
6. 75% of 36 =
7. 40% of 80 =
8. 30% of 50 =
9. 5% of 100 =

Solve these by the way that seems easier to you:

10. $33\frac{1}{3}\%$ of 24 =
11. $87\frac{1}{2}\%$ of 120 =
12. 16% of 96 =
13. 75% of 100 =
14. 26% of 20 =
15. 25% of 28 =
16. $66\frac{2}{3}\%$ of 18 =
17. $83\frac{1}{3}\%$ of 48 =
18. 8% of 54 =

19. Lester sold eighteen dollars' worth of magazines. He is to keep out as his pay 20% of the amount of the sales. How much should he send to the company?

20. Mr. Kline borrowed $200. He has to pay 6% of $200 each year for the use of the money. If he returns the money at the end of one year, how much in addition to the $200 must he pay?

Here are some examples in finding a per cent of a number that sometimes cause trouble. Study these solutions until you are sure you understand them. Check each answer.

Copy Exs. (1) to (5) and see whether you can do them without looking at the book. Then look to see whether your answers are right.

Troublesome Per Cent Examples of Type I

(1) Find 250% of 64.

SOLUTION. 250% = 2.50
2.50 × 64 = 160

(2) Find .4% of 28.

SOLUTION. .4% = .004
.004 × 28 = .112

(3) Find 3.5% of 18.

SOLUTION. 3.5% = .035
.035 × 18 = .63

(4) Find $4\frac{1}{3}\%$ of 81.

SOLUTION. $4\frac{1}{3}\% = .04\frac{1}{3}$
$.04\frac{1}{3} \times 81 = 3.51$

(5) Find $\frac{1}{2}\%$ of 90.

SOLUTION. $\frac{1}{2}\%$ means $\frac{1}{2}$ of 1%
1% of 90 = .01 × 90 = .90
$\frac{1}{2}\% = \frac{1}{2}$ of .90 = .45

Work these examples; then look at the answers on page 84 and correct any mistakes you made.

1. 300% of 82 =
2. 2.4% of 25 =
3. $\frac{1}{2}\%$ of 28 =
4. $3\frac{1}{3}\%$ of 15 =
5. .6% of 72 =
6. $6\frac{1}{2}\%$ of 20 =
7. 4% of 72 =
8. .5% of 84 =
9. 3.5% of 64 =
10. 110% of 18 =
11. 16% of 94 =
12. $2\frac{1}{2}\%$ of 100 =
13. 270% of 18 =
14. $\frac{1}{3}\%$ of 72 =
15. .8% of 39 =
16. 350% of 24 =
17. $3\frac{1}{4}\%$ of 200 =
18. 4.3% of 72 =
19. $87\frac{1}{2}\%$ of 120 =
20. $\frac{1}{4}\%$ of 18 =
21. .3% of 400 =
22. 150% of 900 =
23. 1.5% of 800 =
24. $\frac{1}{2}\%$ of 1000 =
25. 320% of 100 =
26. 2.6% of 5400 =
27. $\frac{3}{4}\%$ of 760 =

Jean answered 20 out of 25 history questions correctly. What per cent did she answer correctly?

First we must remember that 20 out of 25 questions is $\frac{20}{25}$ of the questions, and that per cent means hundredths. So we must think $\frac{20}{25} = \frac{?}{100}$. We can solve problems of this kind in two ways.

Solution I	Solution II
$\frac{20}{25} = \frac{4}{5} = \frac{80}{100} = 80\%$	$\frac{20}{25}$ means $20 \div 25$
Check	$25\overline{)20.00}$ (.80; 20.0; 0)
$.80 \times 25 = 20$	$.80 = 80\%$

Solve these in the two ways shown above:

1. 6 is ?% of 30
2. 48 is ?% of 60
3. 72 is ?% of 108
4. 12 is ?% of 48
5. 16 is ?% of 20
6. 24 is ?% of 48
7. 12 is ?% of 20
8. 24 is ?% of 60
9. 36 is ?% of 60

Solve these by the method that seems easier to you. Express the answers to the nearest whole per cent. (For example, 14 is ? % of 19. $14 \div 19 = .736^{+} = 73.6\%$ or 74%.)

10. 12 is ?% of 18
11. 9 is ?% of 11
12. 7 is ?% of 15
13. 19 is ?% of 27
14. 15 is ?% of 21
15. 70 is ?% of 120
16. ?% of 200 is 50
17. ?% of 160 is 80
18. ?% of 113 is 5

Think carefully as you solve these:

19. 5% of 60 = ?
5 is ?% of 60
20. 20% of 80 = ?
20 is ?% of 80
21. ?% of 40 is 25
25% of 40 = ?
22. 16 is ?% of 28
16 % of 28 = ?
23. 25% of 70 = ?
25 is ?% of 70
24. $12\frac{1}{2}$% of 40 = ?
$12\frac{1}{2}$ is ?% of 40

Here are some of the "trouble makers" in finding what per cent one number is of another. Study them carefully.

Copy Exs. (1) to (5) and see whether you can solve them; then look at the book to see whether your answers are right.

Troublesome Per Cent Examples of Type II

(1) 12 is what per cent of 13?
SOLUTION. $12 \div 13 = 12.00 \div 13 = .923 = 92.3\%$

(2) 40 is what per cent of 10?
SOLUTION. $40 \div 10 = 40.00 \div 10 = 4.00 = 400\%$

(3) What per cent of 12 is 15?
SOLUTION. $15 \div 12 = 15.00 \div 12 = 1.25 = 125\%$

(4) What per cent of 240 is 1.2?
SOLUTION. $1.2 \div 240 = 1.200 \div 240 = .005 = .5\%$

(5) 1.25 is what per cent of 50?
SOLUTION. $1.25 \div 50 = 1.250 \div 50 = .025 = 2.5\%$

Work these examples. Check every example. Then look at the answers on page 84 and correct any mistakes you made.

1. 50 is ?% of 10.
2. 10 is ?% of 50.
3. ?% of 25 is 30.
4. 60 is ?% of 15.
5. 15 is ?% of 60.
6. 1 is ?% of 500.
7. ?% of 800 is 4.
8. 3.2 is ?% of 400.
9. ?% of 40 is 2.6.
10. ?% of 250 is 375.
11. 400 is ?% of 200.
12. ?% of 84 is 2.856.
13. ?% of 45 is 50.
14. 2.1 is ?% of 350.

15. James bought some popcorn for 10¢. He popped it and sold five bags for 10¢ each. His gain was _?_ ¢. His gain was what per cent of the cost of the popcorn?

(1) Fred receives for selling brushes 30% of the amount of his sales. He wants to earn $18 to buy a bicycle. How many dollars' worth of brushes must he sell?

Here are two ways to solve the problem.

SOLUTION I	SOLUTION II
30% of his sales = $18	Let s represent amount of sales
1% of sales = $\frac{1}{30}$ of $18 = $.60	Then $.30\,s = 18$
100% of sales = 100 × $.60 = $60	and $s = \frac{18}{.30} = 60$ (*Ans.*)
(100% of sales = all of sales)	CHECK. $.30 \times 60 = 18$
CHECK. .30 × $60 = $18	

Which solution do you prefer? Why?

(2) A newspaper has announced that the population of Woodlawn has increased 50% during the last year and that the present population is 1500. Can you tell what the population was a year ago? Here are two ways to solve the problem.

SOLUTION I	SOLUTION II
100% of last yr. pop. + 50% of last yr. pop. = 1500	Let p = last yr. population
150% last yr. pop. = 1500	Then $p + .50\,p = 1500$
1% last yr. pop. = 10	or $1.50\,p = 1500$
100% last yr. pop. = 1000	and $p = \frac{1500}{1.50} = 1000$
CHECK. 1000 + 500 = 1500	CHECK. 1000 + 500 = 1500

Which solution do you prefer? Why?

1. The school paper announced that 250 new pupils had entered this year, which was an increase of 25% over the number of new pupils entering last year. Can you tell from the information given in the paper how many pupils entered last year?

1. Grace earned money by taking orders for toilet articles. She was allowed to keep 40% of the amount of her sales as commission. One week she wished to earn \$8. How many dollars' worth of merchandise would she have to sell?

2. How many dollars' worth of goods would Grace (Ex. 1) have to sell in a month to earn \$30?

3. 25% of ___?___ is 10.
4. 75% of ___?___ is 18.
5. $12\frac{1}{2}$% of ___?___ is 4.
6. 100% of ___?___ is 60.
7. 150% of ___?___ is 15.
8. 20 is 20% of what?
9. 30 is 25% of what?
10. 2 is 50% of what?
11. $\frac{1}{2}$ is 10% of what?
12. $2\frac{1}{2}$ is 5% of what?

Solve and check each of these equations:

13. $.10\,p = 60.$
14. $.05\,p = 80.$
15. $2.5\,p = 60.$
16. 10% of $n = 25.$
17. 5% of $n = 30.$
18. $2\frac{1}{2}$% of $n = 15.$
19. $1.5\,x = 45.$
20. $1.06\,x = 53.$
21. $1.04\,n = 26.$

22. Which of the equations below could be used to find what number increased by 20% of itself equals 72?

(*a*) $n + .20\,n = 72$
(*b*) $.20\,n = 72$
(*c*) $1.20\,n = 72$
(*d*) 120% of $n = 72$

23. My brother said his salary was reduced \$3 a week, which was a 15% reduction. What was his salary before the reduction? after the reduction?

24. A piece of dust blew into Harry's eye and the eye became infected. The school nurse told him to wash it out with a 10% solution of argyrol. When the druggist made that solution, he used 1 oz. of argyrol crystals and ___?___ oz. of water. (How many ounces of *mixture* did the druggist make? Check.)

25. If the nurse had ordered a 20% solution of argyrol, the druggist would have used ___?___ oz. of water to each ounce of argyrol crystals.

Study these solutions; then copy the examples and see whether you can do them without help. Check each one. Then look at the book to see whether your answers are right.

Troublesome Per Cent Examples of Type III

(1) 90 is 150% of what number?

150% of a number = 90
1% of the no. = $\frac{90}{150}$ = .6
100% of the no. = 100 × .6 = 60

$1.50\,x = 90$
$x = \frac{90}{1.50}$
$x = 60$

CHECK. 60 + (.50 × 60) = 60 + 30 = 90

---

(2) What number increased by 12% of itself equals 336?

100% of the no. + 12% of the no. = 336
112% of the no. = 336
1% of the no. = $\frac{336}{112}$ = 3
100% of the no. = 300

$x + .12\,x = 336$
$1.12\,x = 336$
$x = \frac{336}{1.12}$
$x = 300$

CHECK. 300 + (.12 × 300) = 300 + 36 = 336

---

(3) What number decreased by 6% of itself equals 470?

100% of the no. − 6% of the no. = 470
94% of the no. = 470
1% of the no. = $\frac{470}{94}$ = 5
100% of the no. = 500

$x - .06\,x = 470$
$.94\,x = 470$
$x = \frac{470}{.94}$
$x = 500$

CHECK. 500 − (.06 × 500) = 500 − 30 = 470

Work these examples; then look at the answers on page 84 and correct any mistakes you made. Check every example.

1. 90 is 3% of what?
2. 260 is 130% of what?
3. 360 is 90% of what?
4. 90 is 18% of what?
5. 1000 is 200% of what?
6. 5% of what number is 25?
7. 125% of what number is 25?
8. 250% of what number is 25?
9. 240% of what number is 120?
10. 300% of what number is 1800?
11. What number increased by 20% equals 600?
12. What number decreased by 8% equals 1104?
13. What number increased by 25% equals 1000?

1. Jane bought a pair of shoes at a sale for 20% below the marked price of $4.00. How much did she save?

2. Ruth is selling soap. She receives 25% commission. How much soap will she need to sell in order to earn enough to buy a 75-cent class pin?

3. A grocer bought 50 doz. oranges at 20¢ a dozen. He estimates that he will lose 5% of the oranges through spoilage. If he sells the oranges at 30¢ a dozen, how much more will he receive for the oranges than he paid?

4. I saw a coat that I liked. It was marked $28.00. The salesman said I could have it for $25.00 if I paid cash. I was allowed a reduction of $\frac{?}{}$ dollars. The reduction was what per cent of the marked price? (Express the answer to the nearest whole per cent.)

5. If you know what per cent of the pupils in a class are boys, how can you find what per cent are girls?

Answers to page 78

1. 246	**6.** 1.3	**11.** 15.04	**16.** 84	**21.** 1.2	**26.** 140.4
2. .6	**7.** 2.88	**12.** 2.5	**17.** 6.5	**22.** 1350	**27.** 5.7
3. .14	**8.** .42	**13.** 48.6	**18.** 3.096	**23.** 12	
4. .5	**9.** 2.24	**14.** .24	**19.** 105	**24.** 5	
5. .432	**10.** 19.8	**15.** .312	**20.** .045	**25.** 320	

Answers to page 80

1. 500%	**4.** 400%	**7.** .5%	**10.** 150%	**13.** $111\frac{1}{9}$%
2. 20%	**5.** 25%	**8.** .8%	**11.** 200%	**14.** .6%
3. 120%	**6.** .2%	**9.** 6.5%	**12.** 3.4%	**15.** 40; 400%

Answers to page 83

1. 3000	**4.** 500	**7.** 20	**10.** 600	**13.** 800
2. 200	**5.** 500	**8.** 10	**11.** 500	
3. 400	**6.** 500	**9.** 50	**12.** 1200	

1. At the close of the winter Mr. Ball announced that he would reduce the selling price of all skates 40%. Leslie bought a pair for $2.40. Do you know what the selling price of the skates was before the sale? Here are two ways to begin the computation to find out:

100% represents the regular selling price.	Let p = regular selling price.
40% is the discount allowed.	.40 p = discount
Therefore the skates sold for 100% − 40%, or 60%, of the regular selling price.	p − .40 p = 2.40
60% of the regular selling price equals $2.40.	.60 p = 2.40

Complete the solution and check your answer.

2. What was the regular selling price of a pair of skates that sold at the sale (Ex. 1) for $1.20? for $1.80? for $2.10? $2.40? $2.70? $3.00? $3.60? Check each answer.

3. Alice bought a dress for $8. That was at a 20% discount. The regular selling price of the dress must have been ___?___. Check your answer.

4. Alfred bought a fountain pen at a discount of 12½%. He paid $3.50 for it. Before the sale the pen had been marked ___?___. Check your answer.

5. Find the regular selling price of the following articles for which the per cent of discount and sales price are given. Check your answers.

Article	Per Cent of Discount	Sale Price
(*a*) Chair	12½%	$21
(*b*) Table	10%	$18
(*c*) Footstool	15%	$ 9
(*d*) Desk	20%	$24

6. Make a graph to show the relation between regular price and sale price of goods being sold at a discount of 20%; of 30%. (Put both lines on the same graph and label them as in the graph on page 6.)

1. The enrollment of the Jones School was 900 when the school was new. Now the enrollment is 1200. What is the per cent of increase in enrollment?

$$\textit{Per cent of increase} = \frac{\textit{increase}}{\textit{original}} \textit{ of } 100\%.$$

The per cent of increase in enrollment in the Jones School was therefore $\frac{1200-900}{900}$ of 100%, or $\frac{300}{900}$ of 100%, or ___?___ %.

2. When Mary entered Hawthorne School, the enrollment was 400. Now it is 600. Find the per cent of increase.

3. Mrs. Smith said: "Eggs are certainly expensive. They are 50 cents a dozen now. Only a short time ago they were 30 cents a dozen. That's an increase of ___?___ per cent!"

4. When Grace was 12 years old, she weighed 80 lb. A year later she weighed 85 lb. Find the per cent of increase in her weight during the year.

5. Tom's wages were $15 a week; now they are $12 a week. Find the per cent of decrease in his wages.

$$\textit{Per cent of decrease} = \frac{\textit{decrease}}{\textit{original}} \textit{ of } 100\%.$$

The per cent of decrease in Tom's wages was therefore $\frac{15-12}{15}$ of 100%, or $\frac{3}{15}$ of 100%, or ___?___ %.

6. A knife that was marked to sell for 50 cents was sold for 35 cents. What was the per cent of decrease in the selling price?

7. The population of Smithville in 1900 was 1240. Thirty years later the population was 800. Find the per cent of decrease in the population.

8. Mr. Adams could have sold his hogs in December for 6 cents a pound. He sold them the next month at $5\frac{1}{2}$ cents a pound. Find the per cent of decrease in the price.

1. If the population of a town is now 4200 and you know it has increased 40 % during the last year, what was the population a year ago?

2. Mr. Green has $15,000 invested in his business. He says that is an increase of 200 % over his original investment. His original investment must have been ___?___.

3. My father received a tax bill the other day for $78. It was stamped: "Increased 4 % because not paid within 60 days of issue." Do you know what the bill was before it was raised?

4. The Allied Drug Company is reducing the salaries of its employees. There will be a 10 per cent reduction on the amount by which each person's weekly salary exceeds $10. Harry Gordon has a present weekly salary of $25. His salary exceeds $10 by ___?___. 10 % of $15 is ___?___. His reduced salary will be ___?___.

5. Find the reduced salaries of the following employees of the Allied Drug Company (see Ex. 4):

PART OF PRESENT WEEKLY PAYROLL

J. R. Arnold.....	$18	L. G. Drake.....	$30
S. L. Black.....	$20	M. N. Fisher.....	$12
P. C. Downes.....	$45	L. P. Good.....	$10

6. When a business concern is forced to reduce salaries, why does it seem fairer to reduce each employee's salary as in Ex. 4 than to take a flat sum (such as $3, $4, $5, or the like) from each employee's salary?

7. The Chase Novelty Company employs 80 persons. The company recently issued a statement that it would increase its list of employed 15 % this month. How many new people does the company plan to employ? (Can you show two methods of solving this problem?)

8. Ten years ago the enrollment in our school was 264. Now the enrollment is 935. What is the per cent of increase in enrollment?

9. Is it possible for a person driving an automobile at 20 miles an hour to increase the speed less than 100%? 100%? more than 100%? Illustrate.

10. Is it possible for a person driving an automobile at 20 miles an hour to decrease the speed less than 100%? 100%? more than 100%? Illustrate.

11. When John was 10 years old, he was 50 inches tall and weighed 64 pounds. Now he is 12 years old, is 54 inches tall, and weighs 74 pounds. His age has increased __?__ %, his height has increased __?__ %, and his weight __?__ per cent.

12. All prices in the Retail Clothing Shop are to be cut 12%. What will be the new price of a suit now marked $18? (Can you show two methods of solving this problem?)

13. The Welfare Guild was helping 46 families last month. This month it had to help only 31 families. The need for help decreased __?__ % during the month.

14. Jean Alward said that the company for which she works is going to decrease the salaries of its employees 8% and that it will mean a cut of $1.20 in her weekly salary. Her present salary must be __?__.

15. Mr. Jones owns a house worth $3500. He says it has increased $16\frac{2}{3}$% in value since he bought it 10 years ago. How much was the house worth when he bought it?

16. Mr. Barker gets $20 a week and 25% of his week's sales. What will he earn during a week in which he sells $40 worth of goods? $80 worth? $120 worth? $160 worth?

17. Make a graph to show the relation between Mr. Barker's weekly earnings and sales (Ex. 16).

1. Mrs. Yates wants to buy a desk that is marked $36. If she waits until the August furniture sale, she can get it at a discount of $12\frac{1}{2}\%$. How much will she save by waiting?

2. A radio company advertised: OUR REGULAR 45-DOLLAR RADIOS NOW ON SALE FOR $35. What *rate of discount* was being offered on the radios?

The amount of a bill after the discount has been deducted is called the *proceeds*.

3. A discount of 5% was allowed on a bill of goods for $16.60. Find the proceeds.

MARIE'S SOLUTION. $.05 \times \$16.60 = \$.8300$ (*Discount*)
$\$16.60 - \$.83 = \_\_?\_\_$ (*Proceeds*)

TOM'S SOLUTION. $.95 \times \$16.60 = \_\_?\_\_$ (*Proceeds*)

Which of these solutions do you prefer?

4. A power and light company gives a discount of 2% on all bills that are paid within ten days. If the following bills are paid within ten days, how much will they amount to? Vary your method in solving these problems.

(*a*) $1.98	(*c*) $5.35	(*e*) $12.45
(*b*) $2.65	(*d*) $7.84	(*f*) $21.85

5. Jane said: "I saved 25¢ by buying this hat at a 20% discount sale." How much had the hat been marked before the sale? How much did Jane pay for it?

6. Mr. Allen said: "I had to sell my house for 10% less than my asking price. That means $500 out of my pocket." What was Mr. Allen's asking price? How much did he get?

7. Advertisement: THESE TWO-DOLLAR BOOKS NOW ON SALE FOR $1.50. What rate of discount is being offered on those books?

8. Tom paid $4 for a tennis racket at a 20% discount sale. What would the racket have cost Tom before the sale?

1. Mr. Kaufman works in a store which sells radios. He gets a salary of \$15 a week and a commission of 3% on all his sales. This week he has sold 450 dollars' worth of goods. What should his pay check amount to?

2. How much does Mr. Kaufman (Ex. 1) earn during a week in which he sells 300 dollars' worth of goods? 500 dollars' worth? 650 dollars' worth?

3. James is going to sell magazines. He is to receive 30% of the amount of his sales. He wants to earn \$15 before Christmas. How many dollars' worth of magazines will he have to sell?

4. If James (Ex. 3) earns \$18, how many dollars' worth of magazines will he sell? How much money will he turn over to the company?

5. Martha sold her baby sister's carriage for \$10. Her mother said she could keep \$2. Martha said: "My rate of commission for making the sale is __?__ per cent."

6. Tom was offered a position at \$12 a week and 4% of the amount of his sales. How much will his sales have to amount to if he wishes to earn \$25 a week?

7. An agent is allowed to keep 20% of the amount of his sales as commission. He has to send the *proceeds* of his sales (the amount of his sales less his commission) to the company at the end of each week. This week he sent the company \$12. Can you tell what his sales amounted to this week?

8. A company allows its agents 25% of their sales. Last week the proceeds turned in by all the agents amounted to \$1275. The manager said: "Our commissions for the week amounted to __?__."

9. Make a graph such as the manager (Ex. 8) might use to tell the commission on sales up to \$1600.

Courtesy of Port of New York Authority

1. The Holland Tunnel, between New York City and Jersey City, is the longest vehicular tunnel in the world. It consists of twin tubes, each 9250 ft. or __?__ miles long.

2. Each tube accommodates two lanes of traffic. More than 1900 motor vehicles an hour can pass through each tube. In a recent year 11,403,863 vehicles paid revenue for using the tunnel. That was an average of __?__ vehicles daily.

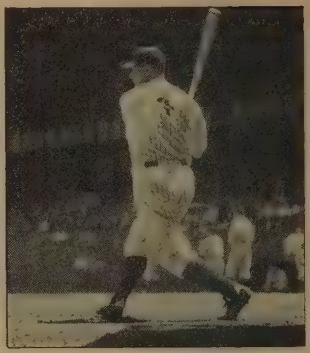
3. The cost of the Holland Tunnel was approximately $50,589,000. At 4% interest the annual charge for interest is __?__ dollars.

4. During a recent year the gross receipts were $5,999,-187.41. The net income that year was $2,440,987.15. The net income was approximately __?__ % of the gross receipts.

5. It is said that the Holland Tunnel is *self-liquidating*. (Receipts from its operation will pay for its construction and maintenance.) Do the data (facts) given above show that it is self-liquidating?

MAJOR LEAGUE STANDING

American	Won	Lost	Percentage	National	Won	Lost	Percentage
N. Y.	79	35	.693	Chic'go	63	50	.558
Ath's	70	47	.598	Pitts.	61	53	.535
Cleve.	68	47	.591	Bklyn.	63	56	.529
Wash.	63	51	.553	Phillies	61	58	.513
Detroit	58	54	.518	Boston	58	61	.487
St. L.	53	62	.461	St. L.	56	59	.478
Chicago	36	75	.324	N. Y.	54	60	.474
Boston	30	86	.259	Cincin.	51	70	.421



By Ewing Galloway, N. Y.

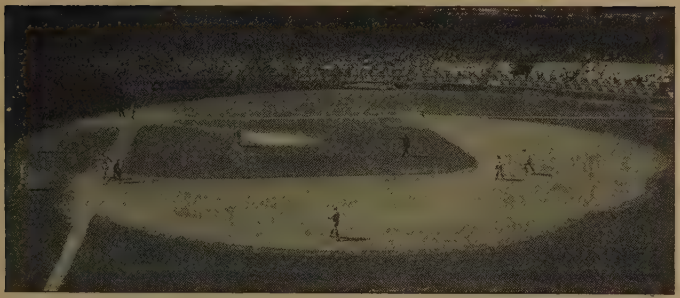
1. This clipping shows the standing of the Major League baseball teams near the close of a recent baseball season. It shows that in the American League the New York team (the Yankees) won 79 games and lost 35. Therefore they played ___?___ games all together. Since they won 79 of the 114 games played, they won $\frac{79}{114}$ of their games. Newspapers always express this fraction as a decimal correct to the nearest thousandth. In baseball this decimal is called the team's *percentage*.

2. Earl said: "I wouldn't call .693 a percentage. I think that it would be clearer to say that New York's percentage is 69.3." Do you agree with Earl? Explain.

3. There is an error in the percentages of the teams as given in the clipping. Can you find it?

4. In the National League the Pittsburgh team won ___?___ games and the Brooklyn team won ___?___ games. How do you account for the fact that Pittsburgh's percentage is higher than Brooklyn's percentage although the number of games won is smaller?

5. Find the "percentage" of a team which won 44 games and lost 68 games.



By Ewing Galloway, N. Y.

6. This clipping shows the batting averages of some of the players in the National and American Leagues. (*G* means number of *games* played. *AB* means number of times *at bat*. *R* means number of *runs*; *H* number of *hits*; *P. C., percentage*.)

Let someone in the class tell the exact meaning of the terms *at bat, hit, run,* etc.

Leading Batters

NATIONAL LEAGUE

	G.	A.B.	R.	H.	P.C.
Medwick, St. Louis..	52	216	47	81	.375
Terry, New York....	55	215	43	78	.363
Leslie, Brooklyn.....	55	213	29	77	.362
Urbanski, Boston....	52	226	46	81	.359
Vaughan, Pittsburgh.	50	179	54	63	.352

AMERICAN LEAGUE

Gehringer, Detroit...	53	204	50	83	.407
Manush, Washington.	54	226	43	91	.403
Travis, Washington..	39	147	23	56	.381
Vosmik, Cleveland...	44	172	29	63	.366
Higgins, Philadelphia.	52	190	34	69	.363

Medwick made 81 hits in the 216 times he was at bat. That is, he was successful $\frac{81}{216}$ of the time. This fraction expressed as a decimal correct to the nearest thousandth is ___?___. The .375 is called Medwick's percentage or batting average.

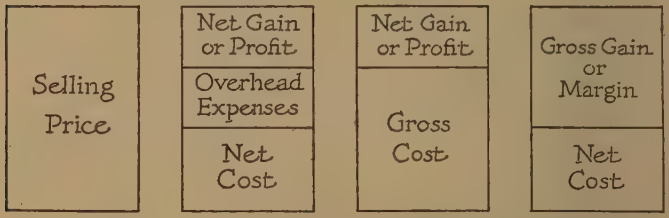
7. Check the batting averages of the other players as given.

8. How do you account for the fact that Urbanski, with the same number of hits as Medwick, has a lower batting average?

(1) Harry paid \$60 for some Christmas trees the week before Christmas. He sold them all for \$70. His overhead expense (cost of handling the trees) amounted to \$2. The gain is what per cent of the cost?

The ***net cost*** of the trees was \$60.
The ***gross cost*** was \$60 + \$2 = \$62.
The ***selling price*** was \$70.
The ***net gain*** or ***profit*** was \$70 − \$62, or \$8.
The ***gross gain*** or ***margin*** was \$70 − \$60, or \$10.
Cost in this book is used to mean ***net cost.***
Gain in this book is used to mean ***net gain.***
Loss in this book is used to mean ***net loss.*** (*Net loss = gross cost − selling price.*)

(2) Can you explain these diagrams?



1. A grocer bought a case containing 2 doz. cans of corn for \$2.40. He estimates that his overhead expenses equal 20% of the cost of goods handled. He wants to sell the case at a profit of 10% of the cost. What must he get for the corn?

2. At what figure must the grocer set the selling price of a can of corn to gain the per cent he wishes? (Ex. 1.)

3. A dealer bought a dozen umbrellas for \$18. He estimates that his overhead expenses are equal to 15% of the cost of goods handled. What must he get for the umbrellas so that he may have a profit of 10% of the cost? At what figure must the dealer set the selling price of each umbrella?

1. At the end of his first year in the grocery business Mr. Mills found that his bills for merchandise bought had amounted to $12,645 and that his overhead expenses totaled $1895. He said: "That shows that my overhead expenses are about ___?___ % of the cost of goods handled."

2. The following year Mr. Mills decided to market his goods by adding 25 % to the cost. He said: "That allows me about a 10 % profit." Was his statement correct?

3. Mr. Mills bought a crate containing 12 dozen eggs for $3.00. How much must he receive for the eggs to cover cost and 15 % of the cost for overhead expenses and make a profit of 10 % on the cost?

4. For how much a dozen should Mr. Mills sell the eggs?

5. When I saw that Mr. Mills had oranges marked 30¢ a dozen, I thought: "Those oranges probably cost him ___?___ cents a dozen."

6. How much must Mr. Mills get for the following articles so that the selling price will cover cost and overhead expenses and allow a profit of 10 % on the cost?

	Cost		Cost
(*a*) Carton of crackers	$2.40	(*e*) Basket of spinach	$ .80
(*b*) Crate of oranges	$3.00	(*f*) Tub of butter	$4.20
(*c*) Case of tomatoes	$1.92	(*g*) Box of bread	$3.20
(*d*) Crate of eggs	$2.16	(*h*) Case of corn	$4.32

7. Make a graph that Mr. Mills could use to find the selling price of an article when he knew the cost. Measure cost on the horizontal scale.

8. Mr. Mills paid $3 for some merchandise. He sold it for $4. If his overhead expenses are 15 % of the cost, his profit on that merchandise was what per cent of the cost?

9. What is Mr. Mills's per cent of profit or loss on goods bought at $4 and sold for $5? bought at $4 and sold for $4? bought at $4 and sold for $4.25?

(1) At the end of the second year of business Mr. Mills studied his accounts carefully. This is what he found:

Receipts (total sales)		$17,543
Spent for merchandise (net cost)	$13,956	
Overhead expenses	2,195	
Total expense (gross cost)		16,151
Profit		$ 1,392

(2) Mr. Mills thought: "I'd like to use this information to help me figure out about what per cent of my receipts I am entitled to withdraw from the business each month during the next year, to use for the living expenses of my family."

Can you suggest a way for him to find out what per cent of his receipts (selling price of his goods) is profit?

(3) Mr. Mills said: "$\frac{1392}{17,543}$, or $\frac{?}{}$%, of this year's receipts was profit. Therefore, if during this next year I withdraw each month about 8% of my receipts for the month, I'll be 'living within my means.' "

(4) During January Mr. Mills's receipts totaled $1286. He withdrew that month $110 for family expenses. Judging by last year's business, was his withdrawal a fair one?

(5) State a fair withdrawal for Mr. Mills to make at the end of a month in which his receipts are $1472; $1386; $1117; $1528.

(6) Although Mr. Mills's accounts show how much he spends for merchandise during any one month, does he actually know the cost of goods sold during that month? Explain your answer.

(7) Why is it much easier for Mr. Mills to find his monthly profit as a per cent of the selling price, rather than as a per cent of the cost of goods sold?

(8) If the gross cost of an article is $16 and the selling price is $18, the profit is what per cent of the selling price?

Test 7. Locating Difficulties in Per Cents

If you have any difficulty with the examples in this test, you can find the practice you need on the following pages. Record your score each time you take this test.

Part I (*Time, 6 minutes*)

Finding a Per Cent of a Number

I. (*a*) 26% of 72 (*b*) 16% of 32

II. (*a*) 5% of 38 (*b*) 8% of 38

III. (*a*) 200% of 28 (*b*) 300% of 52

IV. (*a*) 130% of 14 (*b*) 260% of 48

V. (*a*) $37\frac{1}{2}$% of 40 (*b*) $16\frac{2}{3}$% of 72

VI. (*a*) $\frac{1}{4}$% of 36 (*b*) $\frac{1}{5}$% of 22

VII. (*a*) $4\frac{1}{4}$% of 30 (*b*) $5\frac{1}{2}$% of 46

VIII. (*a*) .4% of 38 (*b*) .5% of 29

IX. (*a*) 2.5% of 84 (*b*) 3.4% of 72

Part II (*Time, 6 minutes*)

Finding What Per Cent One Number Is of Another

I. (*a*) 21 is _?_% of 35 (*b*) _?_% of 11 is 4?

II. (*a*) _?_% of 10 is 12? (*b*) 15 is _?_% of 12?

III. (*a*) 30 is _?_% of 10? (*b*) _?_% of 30 is 120?

IV. (*a*) _?_% of 120 is .6? (*b*) 1.32 is _?_% of 165?

Part III (*Time, 6 minutes*)

Finding a Number When a Per Cent of It Is Known

I. (*a*) 60 is 50% of what? (*b*) 30 is $37\frac{1}{2}$% of what?

II. (*a*) 120 is 125% of what? (*b*) 80 is 200% of what?

III. (*a*) What number increased by 25% of itself equals 55?
(*b*) What number increased by 20% of itself equals 36?

IV. (*a*) What number decreased by 30% of itself equals 28?
(*b*) What number decreased by 15% of itself equals 340?

Part I—Practice

Finding a Per Cent of a Number

I. (*a*) 24 % of 73
(*b*) 18 % of 24
(*c*) 56 % of 38
(*d*) 47 % of 56
(*e*) 12 % of 39

II. (*a*) 6 % of 480
(*b*) 5 % of 164
(*c*) 1 % of 29
(*d*) 4 % of 63
(*e*) 8 % of 28

III. (*a*) 400 % of 28
(*b*) 200 % of 36
(*c*) 300 % of 39
(*d*) 500 % of 84
(*e*) 600 % of 63

IV. (*a*) 120 % of 62
(*b*) 240 % of 34
(*c*) 350 % of 89
(*d*) 165 % of 75
(*e*) 125 % of 63

V. (*a*) $37\frac{1}{2}$ % of 96
(*b*) $16\frac{2}{3}$ % of 108
(*c*) $87\frac{1}{2}$ % of 72
(*d*) 25 % of 64
(*e*) $41\frac{2}{3}$ % of 240

VI. (*a*) $\frac{1}{2}$ % of 280
(*b*) $\frac{1}{4}$ % of 32
(*c*) $\frac{1}{3}$ % of 276
(*d*) $\frac{1}{10}$ % of 900
(*e*) $\frac{1}{5}$ % of 23

VII. (*a*) $5\frac{1}{4}$ % of 35
(*b*) $6\frac{1}{2}$ % of 82
(*c*) $2\frac{1}{3}$ % of 63
(*d*) $6\frac{1}{4}$ % of 95
(*e*) $4\frac{1}{2}$ % of 84

VIII. (*a*) .5 % of 78
(*b*) .6 % of 64
(*c*) .4 % of 39
(*d*) .1 % of 83
(*e*) .2 % of 25

IX. (*a*) 2.5 % of 28
(*b*) 3.4 % of 64
(*c*) 4.8 % of 39
(*d*) 12.5 % of 87
(*e*) 15.3 % of 25

Part II—Practice

Finding What Per Cent One Number Is of Another

I. (*a*) 8 is what per cent of 10?
(*b*) What per cent of 12 is 7?
(*c*) 6 is what per cent of 13?
(*d*) What per cent of 15 is 5?
(*e*) 4 is what per cent of 9?

II. (*a*) 10 is ___?___ % of 8.
(*b*) ___?___ % of 15 is 18.
(*c*) 11 is ___?___ % of 6.
(*d*) ___?___ % of 7 is 9.
(*e*) 15 is ___?___ % of 13.

III. (*a*) What per cent of 10 is 40?
(*b*) 120 is what per cent of 60?
(*c*) What per cent of 15 is 45?
(*d*) 80 is what per cent of 10?
(*e*) What per cent of 8 is 80?

IV. (*a*) ___?___ % of 130 is .65.
(*b*) .96 is ___?___ % of 240.
(*c*) ___?___ % of 320 is 1.92.
(*d*) 1.75 is ___?___ % of 250.
(*e*) ___?___ % of 245 is .49.

Part III—Practice

Finding a Number When a Per Cent of It Is Known

Practice Group I

(*a*) 120 is 25% of what number?
(*b*) 120 is 50% of what number?
(*c*) 120 is 20% of what number?
(*d*) 90 is 60% of what number?
(*e*) 60 is 30% of what number?
(*f*) 40 is 80% of what number?

Practice Group II

(*a*) 60 is 200% of what number?
(*b*) 60 is 300% of what number?
(*c*) 60 is 400% of what number?
(*d*) 120 is 125% of what number?
(*e*) 120 is 150% of what number?
(*f*) 100 is 250% of what number?

Practice Group III

(*a*) What number increased by 25% of itself equals 150?
(*b*) What number increased by 20% of itself equals 480?
(*c*) What number increased by 15% of itself equals 460?
(*d*) What number increased by 12% of itself equals 224?
(*e*) What number increased by $87\frac{1}{2}$% of itself equals 150?

Practice Group IV

(*a*) What number decreased by 20% of itself equals 720?
(*b*) What number decreased by 30% of itself equals 630?
(*c*) What number decreased by 40% of itself equals 480?
(*d*) What number decreased by 25% of itself equals 300?
(*e*) What number decreased by 75% of itself equals 300?

Give a close *estimate* for each of the following:

1. 49.75×30.2. Think: "$50 \times 30 = 1500$"
2. 69×61
3. 19.27×20.5
4. $70\frac{1}{2} \times 29¢$
5. $399 \div 39.8$
6. $989 \div 39.6$
7. 11×79.8
8. 6.90×7.02
9. $21\frac{1}{3} \times \$0.99$
10. The cost of 36 dozen articles at 24 cents a dozen.
11. The cost of 48 campaign badges at 13 cents each.
12. What you would earn in $8\frac{1}{2}$ hours at 32 cents an hour.
13. The area of a rectangle 26 in. by 36 in.
14. The sum of 497, 98, and 104.
15. The cost of 54 articles at 34 cents each.
16. Seventy-five per cent of \$478.50.
17. Four fifths of \$98.86.
18. $37\frac{1}{2}\%$ of \$24.10.
19. $6\frac{1}{4}\%$ of \$63.75.
20. 67% of \$25.
21. A suit of clothes was marked \$35. After Christmas the price was reduced 15%. Estimate the price after the reduction was made.
22. Estimate the cost of $9\frac{1}{2}$ yd. of silk at \$1.75 a yard.
23. *Father:* What mark did you make in spelling?
Maud: I spelled 15 out of 19 words, or about __?__ %.
24. 20 is __?__ % of 23.
25. 4 is __?__ % of 41.
26. 78 is 24% of __?__.
27. 87 is 51% of __?__.
28. 19 is $9\frac{1}{2}\%$ of __?__.
29. 12 is __?__ % of 118.
30. 60 is $74\frac{1}{2}\%$ of __?__.
31. 63 is __?__ % of 65.
32. Which player has the better batting average, one who makes 80 hits in 201 times at bat or one who makes 51 hits in 150 times at bat?

1. Helen wants a pair of shoes marked \$4.50 in Lane's Department Store. Her father is a clerk at Lane's. All the employees are allowed an 8% discount on their purchases. How much will the shoes Helen wants cost her father?

2. Harry receives a 15% commission for selling office supplies. How many dollars' worth of supplies must he sell to earn \$15?

3. James bought a turkey for \$3 and sold it for \$3.50. He spent 25¢ for feed while he owned the turkey. His gain was what per cent of the cost? what per cent of selling price?

4. The enrollment in our school is 750 pupils. The principal says the enrollment has increased 25% in the last year. That means the enrollment a year ago must have been __?__.

5. A sweater at a 20% discount sale was marked \$2. Before the sale the selling price of the sweater must have been __?__.

6. The per cent of pupils present in a schoolroom plus the per cent of pupils absent equals __?__ per cent.

7. $2x + 5 = 13$

8. $2n + n = 9$

9. $\frac{n}{2} = 6$

10. $10 = 2x + 4$

11. $3x - 9 = 6$

12. $\frac{1}{4}y = 3$

13. $11x + 4 = 70$

14. $\frac{2n}{3} = 4$

15. $5n - 2 = 8$

16. A number plus three times itself equals 36. Find the number. (Use an equation.)

17. Find the cost of $3\frac{7}{8}$ pounds of meat at 28¢ a pound.

18. If gasoline sells for 15.7¢ a gallon, how much change will you get from a 10-dollar bill after buying 7 gallons?

19. Jane has a paper $8\frac{1}{4}$ in. wide. She wants to divide it into 6 equal columns. How wide must she make each column?

1. Mr. Mason estimates that his overhead expenses average $33\frac{1}{3}\%$ of the cost of goods handled. For what should he sell a table which cost him \$30 in order to make a net profit of \$5? His profit is what per cent of the selling price?

2. Which would be the better offer for selling magazines, a salary of \$6 a week or a commission of 25%, if you estimate that you can sell at least \$30 worth of magazines each week?

3. If you know what per cent of the voters in the United States are men, how can you find out what per cent of the voters are women?

4. The Lincoln School wants to raise \$50 for the Red Cross. So far only \$36 has been raised. The eighth-grade class president said: "We are only __?__ % of the way toward our goal."

5. Yesterday I read in the paper that food prices have dropped 15% in the last year. Our grocery bill this month was \$49.50. Do you know about how much the same amount of food would have cost a year ago?

6. What is 200% of 20?

7. Express .048 to the nearest whole per cent.

8. $5n+2n-1=27$

10. $2.5y+4=34$

12. $\frac{2n}{3}+1=11$

9. $2\frac{1}{2}b=20$

11. $15=\frac{y}{4}-1$

13. $\frac{1}{2}y+y-1=8$

14. A triangle is to have two sides equal, a perimeter of 27 inches, and a base of 11 inches. How long should each of the equal sides be? (Use an equation.)

15. If 10 is subtracted from $\frac{1}{4}$ a certain number, the result is 5. Find the number. (Use an equation.)

16. Round off each of these to hundredths:

2.784 1.698 .808 3.1416

1. In sponging 4 yd. of broadcloth for a customer, Mr. Miller at the Emporium Department Store found the material shrank to $3\frac{7}{8}$ yd. How much did the material shrink? Mr. Miller said: "I must remember that this material shrinks $\frac{1}{?}$ of its length in sponging."

2. When another customer told Mr. Miller (Ex. 1) that he needed exactly $7\frac{3}{4}$ yards of the broadcloth, Mr. Miller said: "Then you should buy ___? yd. to allow for shrinkage in sponging." Check your answer.

3. At \$3.95 a yard, what will the broadcloth needed by the customer in Ex. 2 cost?

4. A man whose business is being closed out can pay only $83\frac{1}{3}\%$ of his debts. How much can he pay Mr. Trainer, to whom he owes \$675.50?

5. Express $.00\frac{3}{4}$ as a per cent.

6. Tom bought 240 boxes of berries at 10¢ a box. Later he examined 18 boxes carefully and found they contained only enough perfect berries to fill 12 boxes. If the rest of the boxes are in a similar condition, how much must he get for each box of good berries to make a margin of \$6?

7. $1\frac{1}{2}a + 2\frac{1}{2} = 5\frac{1}{2}$

8. $2.5x + x - 2 = 19$

9. $\frac{m}{3} + 5 = 8$

10. $.05x = .25$

11. $\frac{5x}{9} - 7 = 8$

12. $10 = \frac{1}{2}d + 8$

13. $5 + \frac{3x}{2} = 20$

14. $.05p = 100$

15. $20 = \frac{4t}{5} - 4$

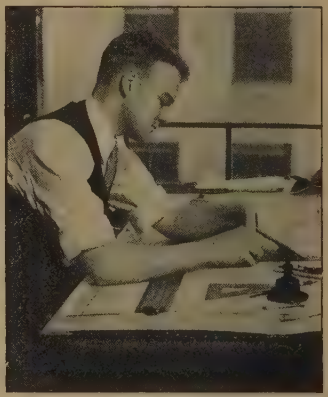
16. *Puzzle.* A horse trader sold two horses for \$100 each. He says he gained 20% on one of them and lost 20% on the other. What did the two horses together cost him?

17. A merchant paid \$2 for a watch. He wished to mark it to sell at a profit of 10% of the selling price. He figured that his overhead averaged 10% of his sales. What should he mark the watch to sell for?

CHAPTER IV

DIRECT MEASUREMENT

Using an Inch Scale



By Ewing Galloway, N. Y.

1. Some pupils measured the width of the picture on page 91. They used a scale graduated to sixteenths of an inch. Among the results obtained were $3\frac{7}{8}$ in., $3\frac{13}{16}$ in., $3\frac{14}{16}$ in., and $3\frac{27}{32}$ in. What result do you get when you measure it?

2. Arrange the numbers in Ex. 1 in order of size.

3. Measure the height of the pictures on this page. Did your classmates obtain differing results? If so, why? List some of the causes of errors that you are likely to make in measuring length.

4. Arrange the measures of Ex. 3 in order of size, as in Ex. 2. Find the average of these measurements. Would the average of the results probably be more accurate than the first measurement in the list?

1. With your inch ruler draw a line segment 4 in. long. Now measure the line with the metric scale of your ruler. Complete: 4 in. = __?__ cm. Compute the number of centimeters in 1 inch. (Record the result.)

2. Find the average of the results obtained by several of your classmates. If you were very careful, you found that

1 inch = 2.54 cm.

3. How many centimeters are there in 1 foot? in 1 yard?

4. The smallest unit on your metric ruler is a millimeter. What part of a centimeter is it?

5. Change each of these to centimeters: (*a*) 15 mm., (*b*) 22 mm., (*c*) 50 mm., (*d*) 100 mm., (*e*) 110 mm., (*f*) 1000 mm., (*g*) 10 mm., (*h*) 6 mm., (*i*) 5 mm.

6. Change each of these to millimeters: (*a*) 2 cm., (*b*) 25 cm., (*c*) 10 cm., (*d*) .5 cm., (*e*) .2 cm., (*f*) .05 cm., (*g*) .15 cm., (*h*) 1.1 cm.

7. With your metric scale find the width of the picture on page 136. Find the average of the results of several of your classmates. Compare this average with that of Ex. 4 on the preceding page. Are they equivalent?

8. Do you know some other metric units of length? It is important to remember the length of a meter (100 cm.) in inches.

1 meter = 39.37 inches (approximately)

9. Albert said that if 2.54 centimeters = 1 inch, then 1 centimeter = $\frac{1}{2.54}$ inches. Was he correct?

10. Does $\frac{1}{2.54}$ inch equal .3937 inch? (Change $\frac{1}{2.54}$ to a decimal.)

11. If 1 centimeter = .3937 inch, how many inches would 100 centimeters (or 1 meter) equal?

(Look at the table of metric units on page 364.)

1. This scale is graduated to eighths of an inch. How long would you say segment AB is?

FIG. 1

2. Here the scale is graduated to sixteenths of an inch. How long would you say segment AB is?

FIG. 2

3. With a scale graduated to thirty-seconds of an inch, what is your reading for the length of segment AB?

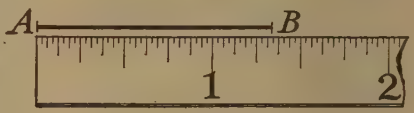


FIG. 3

4. How could you measure the length of AB still more accurately?

5. Using a scale graduated to eighths of an inch, Grace read the length of a segment as $2\frac{5}{8}$ inches. Irene thought: "If Grace read the scale correctly, the *true length* cannot be *more* than $2\frac{5}{8} + \frac{1}{16}$, or __?__ inches. It cannot be *less* than $2\frac{5}{8} - \frac{1}{16}$, or __?__ inches." Is that correct?

6. Using a scale graduated to eighths of an inch, Mary read the length of a segment as $3\frac{3}{8}$ inches. The true length was not more than what? not less than what?

7. Using a scale graduated to sixteenths of an inch, Henry read the length of a segment as $4\frac{11}{16}$ inches. Its true length was not more than what? not less than what?

8. Assuming that the scale is not misread, what is the largest inaccuracy there might be in the reading of a scale graduated to eighths of an inch? sixteenths of an inch? thirty-seconds of an inch?

The largest possible inaccuracy in the reading of a scale (if not misread) is one half of the smallest unit of the scale.

1. Joseph said that he lived 12 blocks from school. Do you know how many feet that was? Why not? Was Joseph using a unit of measure that has a definite fixed length?

2. William said that it was about 30 paces from his home to his garage. Do you know how many feet that was? Why not? Does one person's pace always equal every other person's pace?

A unit such as a foot, meter, or mile is called a *standard* unit. A unit such as a block or pace is not standard.

A standard unit is one that is defined and accepted.

3. Would you call a yard a standard unit?

4. Think of some place that is about a mile from your school. (1 mile = 5280 feet.) About how many minutes do you think it would take you to walk a mile?

5. How many yards are there in a mile? How many rods? (1 rd. = $16\frac{1}{2}$ ft.)

6. Mary's friend, who was spending a year in France, said in her last letter: "This morning we walked over to the village of Bohn, a distance of about 5 kilometers . . ." Mary turned to a metric table, saw that 1 kilometer = 1000 meters, and then figured that 1 kilometer = 1000×39.37 inches = $\frac{1000 \times 39.37}{12}$ feet = $\frac{1000 \times 39.27}{12 \times 5280}$ miles = .62 mile. Was her work correct?

7. How far did Mary's friend walk? (Ex. 6.)

Remember: **1 kilometer = $\frac{5}{8}$ mile (approximately).**

8. About how long would it take you to walk a kilometer?

9. Your classmates would enjoy hearing a report on the history of some of our standard units of length. Consult the dictionary or a good encyclopedia, or some other source.

1. The unit for measuring angles is the ***degree***. A degree is a very small angle. It is defined as $\frac{1}{360}$ of a complete rotation. How many degrees are there in one half of a rotation? in one quarter of a rotation?

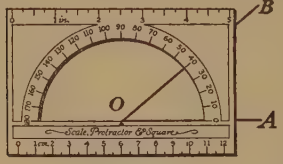


FIG. 1

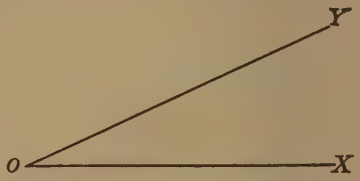


FIG. 2

2. An angle of 90 degrees (often written as 90°) is called a ***right angle***. Would you say that the angle formed at the corner of a sheet of paper is a right angle? Draw a right angle, by estimate, with a ruler. Test it with the corner of a sheet of paper.

3. The sides of a 90-degree angle are ***perpendicular*** to each other. If two lines are perpendicular to each other, they form an angle or angles of ___?___ degrees.

4. An ***acute angle*** is an angle of less than 90°. An ***obtuse angle*** is an angle of more than 90° and less than 180°. What are the dictionary definitions of *acute* and *obtuse?*

5. Draw an acute angle. After drawing the angle, extend the sides. Have you made the angle larger? Why?

6. Draw a figure like this on the blackboard. How many angles has it? Is angle A obtuse or acute? angle B? angle C? angle D? Name the sides of angle A; of angle B; angle C; angle D.

7. How many degrees are there in angle AOB of Fig. 1?

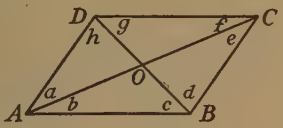
8. Measure angle XOY in Fig. 2.

9. If the sides of an angle are not long enough to reach the scale on the protractor, they must be ___?___ before the angle may be measured.

10. Draw by estimate an angle of 40°. Measure it and find your error. (Your error should be less than 5°.) Do the same for angles of 30°; 60°; 10°; 80°.

11. Draw any triangle and measure each of its angles. What is the sum of the three angles? Compare your sum with the answers of your classmates.

12. Draw a figure like this on the blackboard. How many angles have A for a vertex? If angle a equaled 30° and angle b equaled 25°, how many degrees would there be in $\angle DAB$? ($\angle$ is the symbol for "angle.")



13. Read all the angles of the above figure, tracing each angle with a pointer as you read it. Can you read $\angle g$ in two different ways?

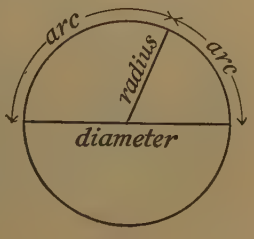
14. Measure angles e and f. What is their sum? Check your answer by measuring $\angle BCD$.

15. How many angles have O for a vertex? Name the sides of these angles. Tell without using your protractor what the sum of the angles with vertex O is. Check with your protractor.

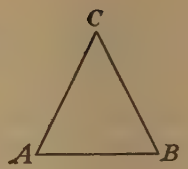
16. Refer to the figure and complete these sentences:

(a) A portion of a circle is called an ___?___.

(b) A line from the center of a circle to any point on the circle is called a ___?___ of the circle.



1. An ***isosceles triangle*** is a triangle having two equal sides. In this isosceles triangle, AC and BC are the equal sides. Can you construct an isosceles triangle?



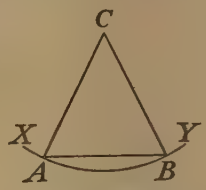
2. Ben constructed an isosceles triangle with compass and ruler as follows:

(*a*) With any point C as the center, and with any convenient radius, draw an arc XY.

(*b*) Mark any two points A and B on the arc.

(*c*) Draw AB, AC, and BC.

Why is triangle ABC isosceles?



The angle formed by the equal sides of an isosceles triangle is called the *vertex angle*. AB is called the *base*.

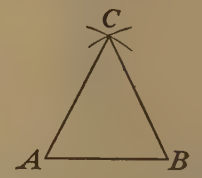
3. Construct an isosceles triangle ABC, as shown in Ex. 2.

4. Grace used the following method of constructing an isosceles triangle:

(*a*) Draw any convenient segment AB.

(*b*) With A as a center and with any convenient radius, draw an arc.

(*c*) With B as a center and with the same radius as in (*b*), draw an arc cutting the first arc.



(*d*) Call the point of intersection C, and draw AC and BC.

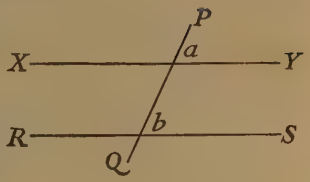
Why is triangle ABC isosceles?

5. Construct an isosceles triangle ABC, as shown in Ex. 4.

6. Do you think that an isosceles triangle always contains two equal angles? How could you test your answer?

7. Construct a triangle ABC in which AB is any convenient length, angle A is 40°, and angle B is 40°. Do you think that the triangle is isosceles? Test your answer.

1. Do you think that XY and RS in this figure are *parallel lines?*

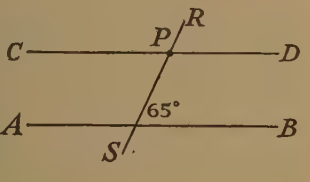


Angle a and angle b in the figure are called *corresponding angles*. PQ is the *transversal*.

Two lines are parallel if they make equal corresponding angles with a transversal.

2. Complete: If angle a = angle b, then XY and RS are ___?___.

3. How would you draw a line CD through a given point P and parallel to a given line AB? (How large will angle DPR have to be made in order to make CD parallel to AB?)



4. Draw a line parallel to a given line, using the principle stated above.

5. Construct a *parallelogram* (a four-sided figure whose opposite sides are parallel).

6. With ruler and protractor construct a *rectangle* (a parallelogram whose angles are right angles).

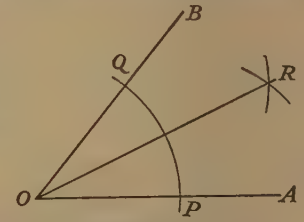
7. Construct a *square* (an equal-sided rectangle).

8. Construct a *trapezoid* (a four-sided figure with only one pair of parallel sides).

9. Which of the following geometric figures contain parallel lines? right angles? perpendicular lines? equal lines?

Square Rectangle Parallelogram Trapezoid

Do you recall how to ***bisect an angle?*** (To *bisect* means to "divide into two equal parts.") Here is a convenient method you may know:



(*a*) With the vertex O of the angle as center, and with any convenient radius, draw an arc cutting the sides of the angle at P and Q.

(*b*) With P and Q as centers, and with the same radius, draw arcs intersecting at R.

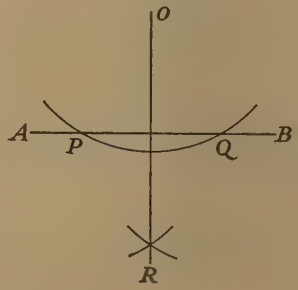
(*c*) Connect O and R.

1. Draw an angle AOB and use the above method for bisecting it. How could you check your construction? Is it necessary to use the same radius for the last two arcs as for the first arc?

2. In the construction for bisecting angle O, show that you made OP equal to OQ and PR equal to QR.

3. If you draw PQ, PR, and QR in the figure for Ex. 1, show that you would have two isosceles triangles, with a common side PQ between them.

Can you ***drop a perpendicular*** from a given point O to a given line AB? Here is one method:



(*a*) With O as center and with a suitable radius, draw an arc cutting AB at P and Q.

(*b*) With P and Q as centers, and with the same radius, draw arcs intersecting at R.

(*c*) Connect O and R.

4. Use this method to drop a perpendicular from a point to a line, and test your construction with your protractor. Could you use a shorter radius for locating R? How short?

1. Construct an isosceles triangle in which the vertex angle is 50° and the equal sides are each 3.5 centimeters.

2. Construct an isosceles triangle in which the base is $1\frac{7}{8}$ inches and the equal sides are $2\frac{3}{4}$ inches. Find the perimeter of the triangle.

3. Construct an isosceles triangle with a vertex angle of 46° and equal sides 3.3 cm. How many degrees are there in each of the three angles of the triangle? What is the sum of all three angles of the triangle? Bisect the vertex angle and extend the bisector until it meets the base. How many degrees are there in each part of the bisected angle? Check your answer with your protractor.

4. Draw any triangle. Bisect the three angles. Do the three bisectors appear to meet in one point?

5. Draw an angle ABC of 73°. Bisect the angle and call the bisector BD. Place a point P on BD $2\frac{7}{16}$ inches from B. At P construct a line perpendicular to BD. Extend the perpendicular until it meets the sides of the angle. Call the points E and F. How long is BE? BF? Are they equal?

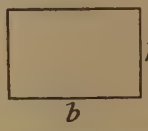
6. Draw a line AB $2\frac{1}{2}$ inches long. Construct the perpendicular bisector of AB. (With A and B as centers and the same radius, draw arcs intersecting at P and Q above and below AB. A line through P and Q is the ***perpendicular bisector*** of AB.) Place a point C on the perpendicular bisector $3\frac{1}{4}$ inches from AB. Connect C with A and B. Is triangle ABC isosceles? Using your protractor, show that $\angle ACB$ is bisected.

7. Using your protractor, make a parallelogram in which $\angle A$ is 60°, AB and DC are 2 inches each, and AD and BC are 1 inch each. Draw the diagonal AC. From D construct a line perpendicular to AC. From B construct a line perpendicular to AC. How long is each of these perpendiculars? Are they equal?

1. Harry accidentally broke the slate blackboard in his classroom. His teacher said: "That's too bad, Harry. Slate black costs thirty cents a square foot . . . "

What would you have to know to find the cost of a new blackboard?

2. The blackboard was rectangular in shape, 4 ft. by 6 ft. How many square feet were there in the blackboard? How much would it cost?



(**Area of a rectangle**) ——> $A = bh$

3. Use the formula $A = bh$ to find the area of rectangles in which—

(*a*) $b = 4\frac{1}{2}$, $h = 6\frac{1}{4}$
(*b*) $b = 5.5$, $h = 2$
(*c*) $b = 8$ in., $h = 2$ ft.
(*d*) $b = 2\frac{1}{4}$ yd., $h = 4$ ft.
(*e*) $b = 6$ cm., $h = 4$ cm.
(*f*) $b = 10$ ft., $h = \frac{1}{2}$ ft.

4. What is the area of a rectangular floor whose dimensions are 16 ft. and 20 ft.?

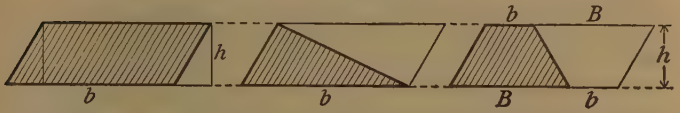
5. What is the area of a floor whose dimensions are 50% greater than those of Ex. 4? (Is the area just 50% greater?)

6. Sketch on your blackboard each of these units for measuring area: (1) square inch; (2) square foot; (3) square yard. Complete: A square foot = _?_ square inches; a square yard = _?_ square feet.

7. Find the area of your classroom floor (1) in square feet, (2) in square yards, (3) in square meters.

8. Which of these formulas would you use to find the area of a square? (1) $A = bh$ (2) $A = lw$ (3) $A = s \times s$ (4) $A = s^2$ (Remember: s^2 is read "s square.")

9. Using $A = s^2$, find A when $s = 4$; 8; 16; 32; 64.



1. Using the first figure above, can you show that the formula for the area of a parallelogram is as follows?

(**Area of a parallelogram**) $\longrightarrow A = bh$

2. Find the area of a parallelogram in which —

(*a*) $b = 6, h = 1$
(*b*) $b = 3.4, h = 8$
(*c*) $b = 12\frac{3}{4}, h = 16$
(*d*) $b = 2\frac{7}{8}, h = \frac{3}{4}$
(*e*) $b = 6, h = 6\frac{1}{8}$
(*f*) $b = 3\frac{3}{8}, h = 4$
(*g*) $b = 12\frac{1}{2}, h = 8\frac{1}{4}$
(*h*) $b = .5, h = 4.2$

3. Find the area of the triangle on page 158.

4. How does the second figure suggest that the formula for the area of a triangle is this?

(**Area of a triangle**) $\longrightarrow A = \frac{bh}{2}$

5. Find the missing numbers for these triangles:

A	*b*	*h*	*A*	*b*	*h*
?	16 in.	1 ft.	1 sq. in.	2 in.	?
?	2 ft.	16 in.	1 sq. ft.	1 ft.	?
?	2 yd.	2 ft.	1 sq. yd.	?	2 yd.
?	$1\frac{1}{2}$ ft.	2 yd.	10	10	?
?	1 ft.	$1\frac{1}{3}$ ft.	20	5	?

6. How does the third figure suggest the formula for the area of a trapezoid?

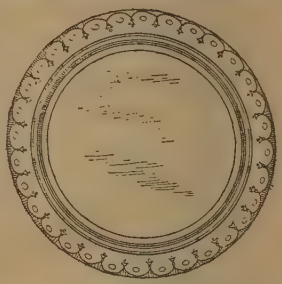
(**Area of a trapezoid**) $\longrightarrow A = h \times \frac{(B + b)}{2}$

7. Find the areas of these trapezoids:

B	*b*	*h*	*B*	*b*	*h*
6	4	3	20	16	12
12	10	8	6.4	3.6	5.4

8. If $A = \frac{bh}{2}$, does $bh = 2A$? does $h = \frac{2A}{b}$? does $b = \frac{2A}{h}$?

1. Margaret wished to make enough lace to sew on the edge of a circular tray cover. She knew that the diameter of the tray cover was 18 inches. Do you know how to find how much lace she needed? Can you use the following formula?



(Circumference of circle) $\longrightarrow c = \pi d$

(π = approx. 3.14 or approx. $3\frac{1}{7}$.)

2. Margaret made $1\frac{3}{4}$ yards of lace. Was that more than she needed?

3. John had a piece of wire 50 in. long. He planned to make a circular hoop from it. How large a diameter could the hoop have, assuming no loss in joining the ends?

4. Find the circumference of this circle.

5. If $c = \pi d$, does $d = \frac{c}{\pi}$? does $\pi = \frac{c}{d}$? does $d = \pi c$?

6. If d is doubled, will c also be doubled? If d is halved, will c also be halved? Would you say that c varies as d varies?

7. Sometimes this formula for finding the circumference of a circle is used:

(Circumference of a circle) $\longrightarrow c = 2\pi r$

(r stands for the number of units in the length of the radius of the circle.)

(*a*) Use $c = 2\pi r$ to find the circumference of a circle whose radius is 10.

(*b*) Use $c = \pi d$ to find the circumference of the circle in (*a*) above.

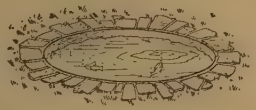
8. Can you show that $c = 2\pi r$ may be changed to $c = \pi d$?

9. Find the circumferences of circles in which —

(*a*) $d = 10$	(*d*) $r = 3$	(*g*) $d = 100$
(*b*) $r = 8$	(*e*) $d = .6$	(*h*) $r = 100$
(*c*) $d = 4\frac{1}{2}$	(*f*) $r = 8.5$	(*i*) $d = 28$

1. George was going to help his father lay a cement bottom in their circular fish pond. They were wondering what the cost would be. What would they have to know in order to find out?

(*a*) George said they would have to know the cost per square foot and the number of square feet. Was that correct?



(*b*) They found by measurement that the diameter of the fishpond was 10 ft. George then used the following formula:

$$\text{(Area of a circle)} \longrightarrow A = \frac{\pi d^2}{4}$$

He wrote: $A = \frac{3.14 \times 10 \times 10}{4} = 78.5$ sq. ft. Do you think that was correct?

(*c*) His father used the formula:

$$\text{(Area of a circle)} \longrightarrow A = \pi r^2$$

He wrote: $A = 3.14 \times 5 \times 5 = 78.5$ sq. ft. (Do you see how he got the 5?) The results agreed; so they concluded that the area of the pond was 78.5 sq. ft. (How would they find the total cost of the cement bottom for the pond?)

2. You remember that r^2 means r times r and is read "r square." The 2 in r^2 is called an ***exponent***.

An ***exponent*** *tells how many times the number above which it is written is to be used as a factor.*

3. What is 5^2? 8^2? 12^2? 20^2? $\left(\frac{2}{3}\right)^2$? $\left(\frac{5}{2}\right)^2$? 1.5^2?

4. Use either of the above formulas to find the area of circles in which —

(*a*) $r = 4$	(*c*) $r = 6$	(*e*) $r = 10$	(*g*) $r = 2\frac{1}{2}$
(*b*) $d = 4$	(*d*) $d = 6$	(*f*) $d = 10$	(*h*) $d = 2\frac{1}{2}$

1. Do you think that the area of this circle is about $\frac{1}{2}$ of the area of the square? $\frac{3}{4}$? $\frac{7}{8}$?

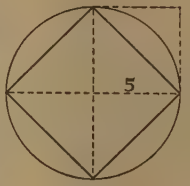
(*a*) If each side of the square is 10, what is the diameter of the circle? What is the area of the circle?

(*b*) Do you now see that the area of the circle is $\frac{78.5}{100}$ or 78.5% of the area of the square? Do you think that would be true if the square were larger or smaller? Try it for two or three other squares.

(*c*) In this figure the circle is said to be *inscribed* in the square.

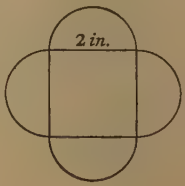
2. A tinsmith cut the largest possible circle from a 10-inch square piece of tin. What per cent of the tin was wasted in the cutting?

3. Do you think that the area of this circle is about 2 times the area of the inscribed square? $1\frac{1}{2}$ times? $1\frac{1}{4}$ times?



(*a*) If the diameter of the circle is 10, what is its area? What is the area of the square? (Note that the area of the large square is 2 times the area of the small square whose side is 5.)

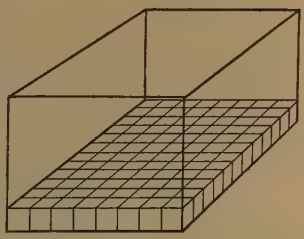
(*b*) Complete: The area of the circle is $\frac{78.5}{50}$ of the area of the square, or ___?___ times the area of the square. Do you think this would be true if the diameter were larger or smaller? Try it for two or three other circles.



4. Find the perimeter of the figure pictured here, in which the diameters of the semicircles are 2 inches.

5. What is the area of the figure?

1. Henry made some building blocks for his small brother. The blocks were 1 inch on each side. He has a box 12 inches long, 8 inches wide, and 6 inches high (inside dimensions). How can he find how many of the building blocks (inch cubes) he can get into the box?



(*a*) How many cubes can Henry place in the bottom layer? How many layers of cubes can he place in the box? How many blocks can he place in the box all together?

(*b*) Using V to represent the number of inch cubes in the box, and l, w, and h the number of inches in the length, width, and height or depth (*altitude*), then $V = l \times w \times h$ or $V = lwh$.

A solid, all of whose sides are rectangles, is called a ***rectangular prism.***

(**Volume of a rectangular prism**) ——→ $V = lwh$

Letting s represent the side of a cube, and remembering that $s \times s \times s = s^3$ (read "s cube"), then the formula for the *volume of a cube* is $V = s^3$.

2. Sometimes the formula for the volume of a prism is written $V = Bh$. In this case B stands for the area of the rectangular *base* of the prism. How would you find B?

3. In finding the volume of a prism in which $l = 6$ inches, $w = 4$ inches, and $h = 2$ inches, do you get the same answer when you use the formula $V = Bh$ as when you use $V = lwh$?

4. When the formula $V = lwh$ is used, must l, w, and h be expressed in the same kind of units?

5. Find the volume, V, of a rectangular prism in which —

(*a*) $l = 4$, $w = 4$, $h = 4$ (*c*) $l = 1$ ft., $w = 8$ in., $h = 6$ in.

(*b*) $l = 6$, $w = 4\frac{1}{2}$, $h = 5$ (*d*) $l = 18$ in., $w = 1$ ft., $h = \frac{1}{2}$ ft.

In this picture you have a very old example of a geometric form. It is a *pyramid.* All the sides of a pyramid are tri-

THE PYRAMIDS OF EGYPT

angles, and all of them meet in a common point, the vertex. (See the illustrations below.)

Pyramids are named according to the number of sides in their bases, as shown below. The *altitude* (or height) of a pyramid is the height of its vertex above the level of its base.

TRIANGULAR PYRAMID

SQUARE PYRAMID

HEXAGONAL PYRAMID

The story of the building of the great Egyptian pyramids contains interesting material for special reading.

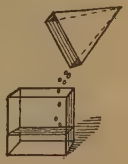
1. Construct from cardboard a square pyramid, as suggested by this figure. The flaps on the edges of the paper should be pinned or pasted on to the sides.

An experiment to find how to compute the volume of a pyramid

(1) Make a pyramid and a prism with equal bases and equal altitudes. Fill the pyramid with rice, water, or sand, and pour the contents into the prism. Repeat the process until the prism is full. What does this experiment show about the relative size of a pyramid and a prism with the same base and the same altitude?

(2) If performed accurately, the experiment shows that *the volume of a pyramid is one third the volume of a prism which has the same base and the same altitude.* Show that the formula for the volume of a pyramid is:

$$V = \tfrac{1}{3}\,(\text{Area of base} \times \text{altitude})$$



(3) If B represents the area of the base, and h the altitude, the formula is as follows:

$$\text{(Volume of a pyramid)} \longrightarrow V = \frac{Bh}{3}$$

1. A certain pyramid has a square base, 30 in. on a side. Its altitude is also 30 in. What is its volume? What would its volume be if each side of the base were 60 in.?

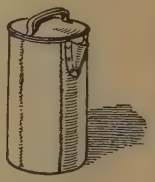
2. The area of the base of a triangular pyramid is 42 square inches; its altitude is $18\frac{1}{2}$ inches. What is its volume?

3. Which is larger, a piece of cake in the form of a rectangular prism 2″ by 1″ by 1.5″, or one in the form of a rectangular pyramid whose base is 2″ by 3″ and whose height is 1.5″?

4. The area of the base of a hexagonal pyramid is 64.8 sq. in., and its altitude is 18 in. Find the volume.

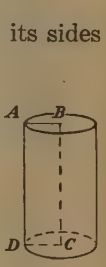
5. The Pyramid of Cheops in Egypt has a base 764 feet square and an altitude of 480 feet. Does it cover more or less than an acre of ground? (An acre = 43,560 sq. ft.) About how many cubic yards of stone would it contain, if solid?

Mr. Keal, the tinsmith, took an order to make a gasoline can which would hold exactly 3 gallons. He wanted the radius of the base to be 3.5 inches. In order to find how high to make the can, he had to know how to find the *volume of a cylinder*. If you understand how to find the volume of a cylinder, you may omit the first seven exercises.



1. If a rectangle $ABCD$ is rotated about one of its sides BC as an axis, the other sides trace a cylinder.

2. The *radius of the base* of this cylinder is 5 inches. How many square inches are there in the area of the base?



3. If it were possible to fit cubic inches (inch cubes) into the bottom of this cylinder, how many could you get into the bottom layer?

4. If the *height* of the cylinder is 18 inches, how many layers of inch cubes could be placed in the cylinder?

5. What is the volume of this cylinder; that is, how many cubic inches could be placed in the cylinder?

6. If the radius of the base of a cylinder is r inches, show that πr^2 cubic inches could be placed in the bottom layer.

7. If the height of the cylinder is h inches, show that the volume is $\pi r^2 h$ cubic inches.

(**Volume of a cylinder**) $\longrightarrow V = \pi r^2 h$

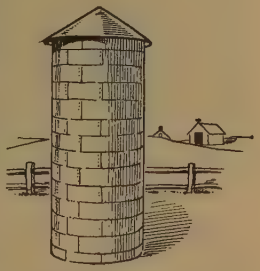
8. Find V if $r = 8$ inches and $h = 10$ inches.

9. Find V if $r = 6$ inches and $h = 1$ foot.

10. How many cubic inches are there in a cylindrical bucket, the radius of whose base is 4 in. and whose height is 7 in.? (Use $\pi = 3\frac{1}{7}$.)

11. Many objects are made in the form of a cylinder. Water pipes, oil tanks, boilers, cisterns, and silos are usually cylindrical in shape. See if you can bring to class a list of five objects seen around home, in the shop, on the farm, or in the store, that are good examples of cylinders.

12. A farmer has a cylindrical silo which is 10 ft. in diameter inside and 25 ft. high. How many cubic feet will it hold?



13. If the silo had a 5-foot inside diameter and were 25 feet high, how much would it hold?

14. The radius of a cylinder is 4 inches; its volume is 301 cubic inches. Find its height.

15. Mr. Keal's gasoline can (see preceding page) was to hold 3 gallons, or 693 cubic inches. The radius of its base was 3.5 inches. Can you now find how high it had to be made?

16. What do you have to know to find the volume of a cylinder? of a rectangular prism?

17. William wanted to make a cylindrical cup which would hold a quart. He knew that its volume must be 57.75 cubic inches. He made the diameter 4 inches and the height 4.6 inches. Did it hold a quart?

18. Find the volumes of cylinders in which—

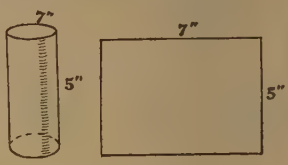
(*a*) $r = 4.8$ in.; $h = 4.8$ in. (Use $\pi = 3.14$.)
(*b*) $r = \frac{1}{2}$ ft.; $h = 1$ ft.
(*c*) $r = 1$ in.; $h = 1$ ft.
(*d*) d (diameter) $= 1$ yd.; $h = 1$ yd.
(*e*) $d = 2.6$ ft.; $h = 3$ ft.

In making the gasoline can, Mr. Keal, the tinsmith, had to know how much tin to use. He had to find the total *surface* of the can. Mr. Keal had to find the area of the top and bottom and the curved surface. How many separate parts are there in the total surface of a cylinder?

Since you have already learned how to find the area of the circular parts, we need to make a special study only of the *curved surface* of the can.

1. Cut out a piece of paper so that it will just cover the curved surface of a cylinder. What is the shape of the paper when you lay it out flat? How can you find its area?

2. Look at the cylindrical piece of tin shown in this figure. Its circumference is 7 in. When it has been split open and flattened out, what shape does it take? What are its dimensions? What is its area?



3. What are the dimensions of a piece of paper that will exactly cover the curved surface of a cylinder which has a circumference of 20 in. and a height of 12 in.? What is the area of the paper? What is the area of the curved surface?

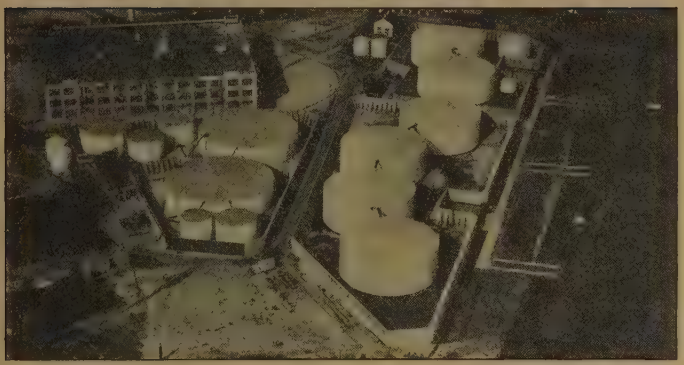
4. A tin cup is $3\frac{1}{2}$ in. high and 8 in. around. How many square inches of tin were used for the curved surface?

5. What is the area of the curved surface of a cylinder $3\frac{1}{2}$ in. high and 2 in. in diameter?

6. (*a*) How much tin would Mr. Keal need for the curved surface of the 3-gallon gasoline can? That is, what is the area of the curved surface of the cylinder whose height is 18 in. and whose circumference is 22 in.?

(*b*) How much tin is needed for the top of the can, not counting the handle? for the bottom?

(*c*) What is the total amount of tin needed?



By Ewing Galloway, N. Y.

AN AIRPLANE VIEW OF PETROLEUM TANKS

7. A group of pupils measured the dimensions of a large cylindrical tank, built by an oil company, for storing gasoline. The circumference was 78 ft. and the height was 20 ft. What problems do you think the pupils wanted to solve? State two problems that you could solve by using these facts.

8. A painter wishes to find the cost of painting a cylindrical water tower whose diameter is 16 ft. and whose height is 80 ft. What would the painting cost at 15 cents a square yard? (He paints the side only.)

9. How many square feet of sheet iron would you need in order to make a stovepipe 3 ft. long and 8 in. in diameter, allowing 1 in. for overlapping?

10. Mr. Farmer offered his son Henry 10 cents a square yard to paint the surface of their silo. It was 20 ft. high and 12 ft. in diameter. Henry figured that he could complete the task in 4 days, thereby earning $\_\_?\_\_$ dollars a day.

Find the entire surface of cylinders having these dimensions:

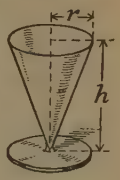
11. $r = 4, h = 6$.

12. $r = 8.4, h = 4.2$.

13. $r = 1$ ft., $h = 6$ in.

14. $r = 8$ in., $h = \frac{3}{4}$ ft.

Mary Ellen has some cone-shaped drinking glasses. Each glass is 4 inches deep, and the diameter of the top is 3 inches. She wonders how many of these glasses she can fill with a gallon of sweet cider (231 cubic inches). How can she find out without measuring the liquid? These exercises will show you how to find the *volume of a cone*.

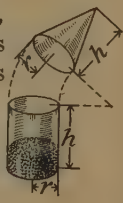


An experiment

1. We may think of a cone as being formed by rotating a right triangle about one of its sides. Show how the cone represented here is formed.

2. What is the radius of the cone shown here? What is the altitude? (See *BO*.)

3. Try to get a cylinder and a cone with the same radius and the same altitude. See the next figure. Fill the cone with rice, water, or sand, and pour it into the cylinder. Repeat the process until the cylinder is full. The cylinder holds how many times as much as the cone?



4. This experiment, if performed accurately, shows that *the volume of a cone is one third of the volume of a cylinder with the same base and the same altitude*. Show that the formula is $V = \frac{3\frac{1}{7} \times r^2 \times h}{3}$.

$$\textbf{(Volume of a cone)} \longrightarrow V = \frac{\pi r^2 h}{3}$$

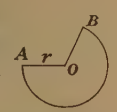
5. Find the volume of a cone in which $r = 8$ inches and $h = 12$ inches. Make a drawing of the cone.

6. Now can you find the volume (in cubic inches) of one of the drinking glasses referred to at the top of the page? How many cubic inches are there in its volume? How many glasses could be filled with a gallon of cider?

7. If the cider costs 50¢ a gallon, what is the cost of each glass of cider?

8. Would the formula $V = \frac{Bh}{3}$ apply to a cone as well as to a pyramid?

9. Construct a cone by cutting out of heavy paper a sector (part) of a circle, as suggested by this figure. Place OA along OB. Find the volume of the cone you have constructed.



10. How many loads of sand (27 cubic feet to the load) are there in a conical-shaped pile of sand, 18 feet across its base and 8 feet high?

11. Find the number of bushels of wheat in a conical pile, 12 feet across the base and $5\frac{1}{4}$ feet high. (Use 1.24 cubic feet for 1 bushel.)

12. John bought grape juice at \$1.50 a gallon and sold it in paper conical cups at 15¢ a cup. The cups were filled with juice to an average depth of 4 inches, and the diameter of the cone of juice was the same as the depth. The cups cost $\frac{1}{2}$¢ each. How much did he make on a gallon?

13. Find the volume of a cone in which r is $\frac{1}{2}$ ft. and h is 8 in.

Find the volumes of cones whose dimensions are:

14. $r = 10$ in., $h = 10$ in.

15. $r = 5$ in., $h = 10$ in.

16. $r = 5$ in., $h = 5$ in.

17. $r = 1$, $h = 3$.

18. $r = 1$, $h = 6$.

19. $B = 12$, $h = 6$.

20. The volume of a cone is 376.8 cu. in. The radius of the cone is 6 in. Find its altitude. (Use $\pi = 3.14$.)

21. The volume of a cone is 308 cu. in. Its altitude is 6 in. Find the area of its base.

1. The boys in a gymnasium class decided to put a new leather cover on their "mass" ball. They knew that it was a sphere 30 inches in diameter. Their coach said: "Find the *surface* of the ball, and then add 10% for waste in cutting and sewing." Can you compute the amount of leather the boys had to buy? If so, begin with Ex. 2.

An experiment to find how to compute the surface of a sphere

(1) Cut a spherical object into two equal parts (hemispheres). An old croquet ball will serve your purpose.

(2) Wrap a string on the flat surface, as indicated in the figure. (You need a heavy cord.)

(3) Wrap a string on the curved surface of one of the hemispheres, as shown here.

(4) Compare the amount of string used in (2) and (3).

(5) Show that the flat surface contains πr^2 square units.

(6) If your work was accurately done, you used twice as much string on the curved surface as on the flat surface. Show that the surface of the hemisphere, therefore, is $2 \times \pi r^2$.

(7) Show that the *total* surface of the sphere is $4 \times \pi r^2$ square units.

(Surface of a sphere) $\longrightarrow S = 4\pi r^2$

2. Find the surface of the mass ball referred to above. How much leather is needed to make it?

3. How much leather will be needed to make a volley ball 10 inches in diameter? Allow 10% for waste and seams.

4. The radius of one sphere is 4 inches; the diameter of another sphere is 4 inches. Compare their surfaces.

5. Find the surfaces of spheres in which —

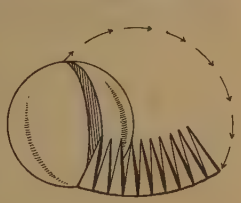
(*a*) $r = 5$ in. (*c*) $r = 20$ in. (*e*) $r = 1$ in.
(*b*) $d = 2$ in. (*d*) $d = 8$ in. (*f*) $d = 1$ in.

1. Martha, who did the family buying, had a chance to buy oranges, 3 inches in diameter, for \$.25 a dozen. The grocer urged her to buy oranges of the same kind 4 inches in diameter at \$.50 a dozen. She refused to buy the latter. Do you think Martha was a good buyer? Why? (Disregard thickness of the orange skin.)

To be certain about a problem like Martha's, you should know how to find the *volume of a sphere.*

An experiment to find how to compute the volume of a sphere

(1) Cut a spherical object into pyramids as suggested by this figure. (An apple or a potato will do.)



(2) What is the height of each pyramid?

(3) How would you find the volume of each pyramid?

The volume of each pyramid equals its base times what?

(4) What is the sum of the areas of all the bases? Show that this is $4\ \pi r^2$.

(5) Show that the sum of the volumes of all pyramids equals the surface of the bases ($4\ \pi r^2$) multiplied by one third of the height.

(6) Remembering that $r \times r \times r = r^3$, show that the volume of a sphere equals $\frac{1}{3}$ of r times $4\ \pi r^2$ or that $V = \frac{4}{3}\ \pi r^3$.

(**Volume of a sphere**) $\longrightarrow$ $V = \frac{4}{3}\ \pi r^3$

2. What is the volume of an orange whose radius is $1\frac{1}{2}$ inches? an orange whose radius is 2 inches?

3. The volume of an orange of 2-inch radius is how many times as large as the volume of an orange of $1\frac{1}{2}$-inch radius? Was Martha a good buyer?

4. A glass bowl in the form of a hemisphere 10 inches in diameter will hold __?__ gallons.

1. The Hortonsphere, used by the Detroit Edison Company for the storage of gas at 6 lb. pressure, is 44 ft. in diameter. Find (*a*) its surface in square feet, and (*b*) its volume in cubic feet.

Underwood & Underwood

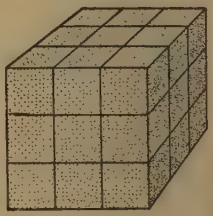
THE HORTONSPHERE

2. Compute the volume of a cone with a 2-inch radius and a 3-inch altitude.

3. How many cylindrical ink bottles $1\frac{1}{2}$ inches in diameter and $1\frac{1}{2}$ inches deep can be filled with a pint of ink? (1 gal. = 231 cu. in.)

4. George wants to know how long it will take to pump his swimming pool full of water. The pool is rectangular, 20 ft. by 40 ft., and when filled has an average depth of $4\frac{1}{2}$ ft. The pump delivers 300 gallons an hour. George thought it would take about 3 days. Was his estimate about right?

5. This 3-inch cube is painted gray. If it is cut into cubic inches, how many of the cubes will be gray on 6 sides ? on 5 sides? on 4 sides? 3 sides? 2 sides? 1 side? no sides? There are __?__ cubes.



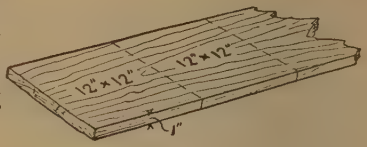
6. How deep should a tinsmith make a cylindrical pail 6 inches in diameter, to hold a gallon?

7. If a spherical metal ball having a 2-inch radius is lowered into a 4-inch cubical box filled with water, what per cent of the water will be displaced?

Find in Column II the formula that matches each expression in Column I.

Column I	Column II
1. Area of parallelogram	(*a*) $A = s^2$
2. Area of triangle	(*b*) $V = s^3$
3. Area of square	(*c*) $A = bh$
4. Area of circle	(*d*) $V = lwh$
5. Area of trapezoid	(*e*) $A = \pi r^2$
6. Surface of sphere	(*f*) $V = \pi r^2 h$
7. Volume of cube	(*g*) $V = \frac{1}{3}\pi r^2 h$
8. Volume of rectangular prism	(*h*) $S = 4\pi r^2$
9. Volume of pyramid	(*i*) $V = \frac{4}{3}\pi r^3$
10. Volume of cylinder	(*j*) $V = \frac{Bh}{3}$
11. Volume of cone	(*k*) $A = \frac{bh}{2}$
12. Volume of sphere	(*l*) $A = h \times \frac{B + b}{2}$

Lumber is measured by the *board foot*. A ***board foot*** is the amount of wood in a board 1 foot square and 1 inch thick. (See illustration.)



Boards less than an inch thick are considered as if they were 1 inch thick.

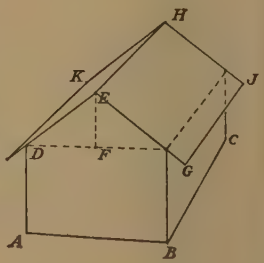
1. Harry bought a board 8 feet long, 12 inches wide, and 1 inch thick, to use in making his tool shelf. How many board feet did it contain? At \$3 per hundred board feet, how much did it cost?

2. If Harry's board had been less than 1 inch thick, would it be considered as having the same number of board feet?

3. Lumber is usually sold by the 1000 board feet. At \$60 per M, what is the cost of 1 board foot? (M = 1000.)

4. How many board feet of $\frac{3}{4}''$ flooring are needed for covering a floor 16 feet by 30 feet? How much would the flooring cost at \$50 per M? Allow 10% additional for waste.

5. How much lumber is needed to cover the surface of the garage shown here, if $AB = 10$ ft., $BC = 16$ ft., $AD = 6$ ft., $EF = 3$ ft., and $EG = 7$ ft.? The garage has a concrete floor. Allow 10% additional for waste. (Boards $\frac{7}{8}''$ thick.)



6. What are *rafters? joists? sills? studding?*

To find the number of board feet in any piece of lumber an inch or more thick, multiply the number of square feet on one face by the number of inches in the thickness.

By Ewing Galloway, N. Y.

Find the number of board feet in the following:

7. 12 planks 2 in. thick, 12 in. wide, and 16 ft. long.

8. 4 beams 4 in. thick, 6 in. wide, and 20 ft. long.

9. 10 joists 2 in. thick, 8 in. wide, and 18 ft. long.

10. 60 boards $\frac{3}{4}$ in. thick, 8 in. wide, and 16 ft. long.

11. 30 rafters 2 in. thick, 4 in. wide, and 16 ft. long.

12. Find the cost of an oak board 16 in. wide, 2 in. thick, and 20 ft. long, at 20¢ a board foot.

13. A *section* of land is a square one mile on each side. How many board feet of lumber would be needed to fence in a section of land, if the boards are 1 in. thick, 4 in. wide, and $16\frac{1}{2}$ ft. long, and if the fence is 4 boards high? How many posts would be needed if they were placed $8\frac{1}{4}$ ft. apart? What would the boards cost at $22.50 per M feet? the posts at 16¢ each? the lumber for the whole fence?

1. A hidden treasure was known to have been buried 20 ft. from a given pine tree. Just where would you dig if you were searching for it?

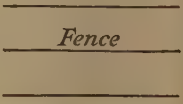
This problem suggests the important geometric principle:

In a flat surface, all points a given distance from a given point lie on a circle having the given point as its center and the given distance as its radius.

2. The hidden treasure (Ex. 1) was also known to be northeast of the given pine tree. Could you now find the position of the hidden treasure? How? (The pine tree might be called a *reference point*.)

3. A buried treasure was believed to be 30 ft. from a given oak tree and 50 ft. from a given maple tree. The distance between the trees was 60 ft. Where would you dig, if you were searching for the treasure?

4. A squirrel buried some hickory nuts 10 ft. from a given fence. Is this fact sufficient to enable you to locate the nuts?



This problem suggests the important geometric principle:

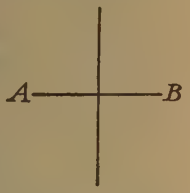
All points a given distance from a given line lie on lines parallel to the given line and the given distance from it.

5. Make a scale drawing to show the position of a hidden treasure that is 12 ft. north of an east-west fence and 16 ft. east of a north-south intersecting fence. (The fences might be called *reference lines*.)

6. Make a scale drawing to help you decide whether the position of an object can be determined if it is known to be 40 ft. from a given road and 60 ft. from a given spring.

7. Is the position of an object determined if it is known to be northwest of a given tree and east of a given post?

8. A point P is known to be equally distant from each of two given points, A and B. Where might P lie? This problem suggests the geometric principle:



Any point that is equally distant from the end points of a line segment lies on the perpendicular bisector of the segment.

9. Make a scale drawing to show the position of an object that is equidistant from two given trees A and B and west of a given tree C.

10. An object is equally distant from two intersecting lines. Would you say that the object lies on the bisector of the angle formed by the lines? Draw an angle, bisect it, mark some point on the bisector, and find the lengths of the lines from the point perpendicular to the sides of the angle.

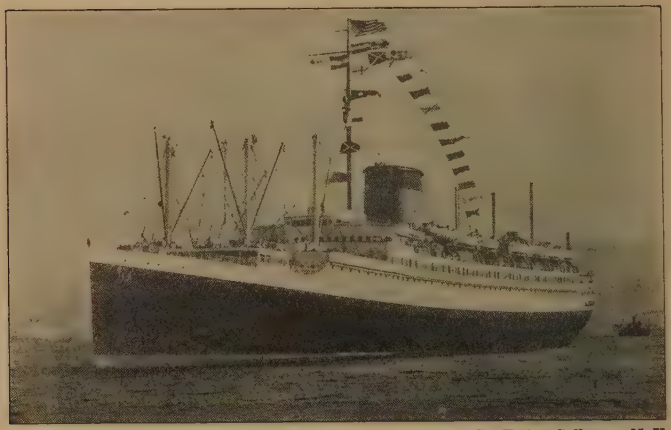
Ex. 10 suggests this important geometric principle:

Any point that is equally distant from two intersecting lines lies on the bisector of the angle formed by the lines.

11. Albert read that a hidden treasure was 100 yards from the point of intersection of two roads, AB and CD, and equally distant from them. Make a sketch to show the possible positions of the treasure.

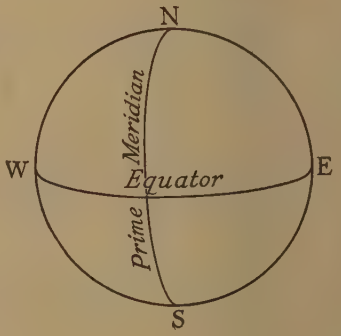
12. We usually determine the position of an object by (*a*) finding its distance and direction from a particular reference point; or (*b*) finding its distance east or west of one fixed reference line and its distance north or south of another reference line. (The reference lines are usually perpendicular.)

Which of the problems above make use of (*a*)? of (*b*)?



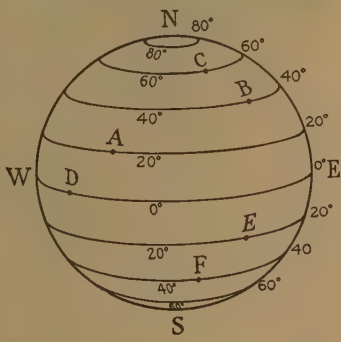
By Ewing Galloway, N. Y.

1. The ship on which Robert was crossing the ocean received a wireless signal from a sister ship in distress. It went immediately to the rescue. Robert wondered how the position of a ship at sea is located. Do you know?



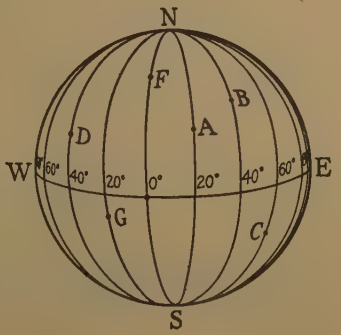
2. The earth rotates about an *axis*, an imaginary line through its center. The ends of this axis are called the *poles*—one the north pole, the other the south pole. An imaginary circle on the earth, halfway between the poles, is called the *equator*. An imaginary circle through the poles and through Greenwich, England, is called the *prime meridian*. Which of these circles suggests a north-south reference line? an east-west reference line?

3 Distances north or south from the equator are measured in degrees. In that case a degree is $\frac{1}{360}$ of a circle that passes through the poles. In this figure, point *A* is 20° north of the equator. What is the location of point *B*? *C*? *D*? *E*? *F*?



4. The ***latitude*** of a place is its distance north or south of the equator, measured in degrees. Where might a ship be if its latitude is 30° north of the equator? Would you call its position fully determined? Why not? What place on the earth's surface is 90° north of the equator (90° N.)? 90° south (90° S.)?

5. Distances east or west of the prime meridian are measured in degrees. In this case a degree is $\frac{1}{360}$ of the equator or of any circle on the earth parallel to it. In this figure *A* is 20° east of the prime meridian. What is the location of point *B*? *C*? *D*? *F*? *G*?

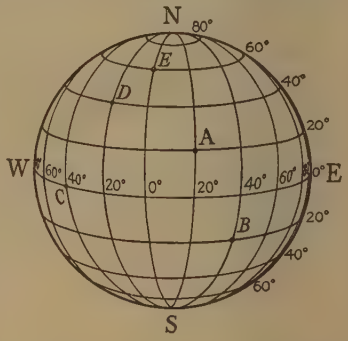


6. The ***longitude*** of a place is its distance east or west of the prime meridian, measured in degrees. What is the longitude of *A*? *B*? *C*? *D*? *F*? *G*?

7. Where might a ship be if its longitude is 20° E.? 40° W.? 120° E.? 179° W.? Is a place fully determined when its longitude is known?

8. On a sketch of the earth, show the location of a place that is 30° E. of the prime meridian and 40° N. of the equator. Is its position fully determined?

9. To locate a place on the earth's surface, it is customary to give its latitude and longitude. In this figure the location of *A* is 20° E., 20° N. What is the location of *B*? *C*? *D*? *E*?



10. What is the location of Chicago? Los Angeles? Boston? London? (Look at a globe or a map.)

11. The earth makes one complete rotation (360°) in 24 hours. How many degrees does it turn in one hour?

12. At noon, where you live, what time is it at places 15° east of you? 15° west of you? 30° east of you? 90° east? 30° west? 90° west? 60° west?

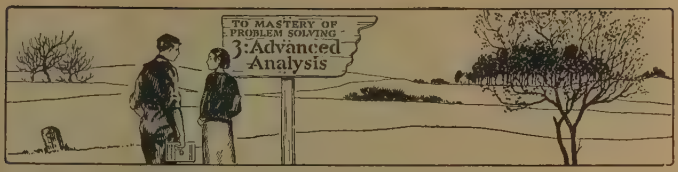
13. When it is 6:00 A.M. at 45° west, what time is it at 30° W.? 60° W.?

14. When it is 4 P.M. at Greenwich, England, what time is it at 15° E.? 15° W.?

15. Does latitude (distance north or south of the equator) have anything to do with time?

16. Complete: Places east of us have sunrise (earlier, later) than we. Places west of us have sunset (earlier, later) than we.

17. With the aid of a map or globe can you find the latitude and longitude of the place where you live?



Help No. III. Problem Analysis

1. A truck carries a load of bricks weighing $2\frac{1}{2}$ tons. The bricks are of the usual size, 8″ by 4″ by 2″. If the bricks weigh 100 lb. to the cubic foot, how many bricks are there in the load?

ANALYSIS

$$\text{No. bricks} = \frac{\text{no. cu. in. in load}}{\text{no. cu. in. in 1 brick}}$$

$$\text{No. cu. in. in load} = \text{no. cu. ft. in load} \times 1728$$

$$\text{No. cu. ft. in load} = \frac{2\frac{1}{2} \text{ tons}}{100 \text{ lb.}} = \frac{2\frac{1}{2} \times 2000}{100} = 50$$

$$\text{No. cu. in. in load} = 50 \times 1728 = 86{,}400$$

$$\text{No. cu. in. in brick} = 8 \times 4 \times 2 = 64$$

$$\text{No. bricks} = \frac{86{,}400}{64} = ?$$

2. The freight rate on wheat shipped from Minneapolis to Chicago was 10 cents a hundred pounds. What was the freight charge on 600 bu.? (A bushel of wheat weighs 60 lb.)

ANALYSIS

$$\text{Freight charge} = \text{No. of 100 lb.} \times \$.10$$

$$\text{No. of 100 lb.} = \frac{600 \times \text{weight of 1 bu.}}{100} = ?$$

$$\text{Freight charge} = ? \times \$.10 = ?$$

3. Find the number of bricks in a 3-ton load of bricks. The bricks are 8″ by 4″ by 2″ and weigh 80 lb. to the cubic foot.

4. Compute the freight charge at \$.15 per 100 pounds on 1000 bushels of wheat.

Underwood & Underwood

5. The Girls' Harmonica Band is raising money to buy uniforms. They have \$96.77 in their treasury. There are 32 girls in the band. The dresses they want cost \$2.75 each, and the hats \$.75 each. The company from which they are buying allows a 5% discount on orders for 25 or more uniforms. How much more money must the girls raise in order to pay for their uniforms?

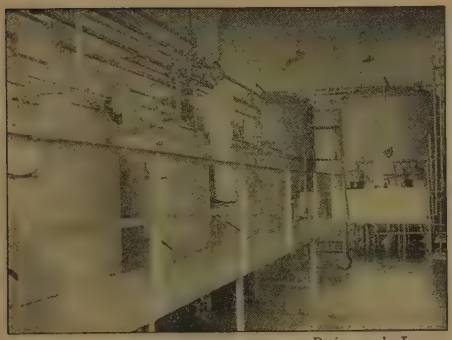
6. Find the cost of excavating a cellar 20′ x 18′ x 8′, at \$3 per cubic yard.

7. How many gallons of water will be needed to fill a swimming pool to within 1 foot of the top if the pool is 40 ft. long, 20 ft. wide, and 7 ft. deep on the average?

8. At \$12.50 per ton, how much will the fertilizer for a rectangular field 40 rods by 60 rods cost if $1\frac{1}{2}$ tons of fertilizer are needed per acre?

9. The laundry to which Mrs. Wood sends her clothes charges 4¢ a pound for washing them. The flat pieces are ironed, for which the laundry makes an additional charge of 6¢ a pound. The rest of the clothes are sent back unironed. How much will Mrs. Wood have to pay for 29 pounds of laundry, of which 11 pounds are unironed?

10. A workman and his helper worked from 8:30 until 11:45 on some house repairs. The workman received 60¢ an hour and his helper 35¢ an hour. The materials used cost \$2.19. How much did the repairs cost all together?



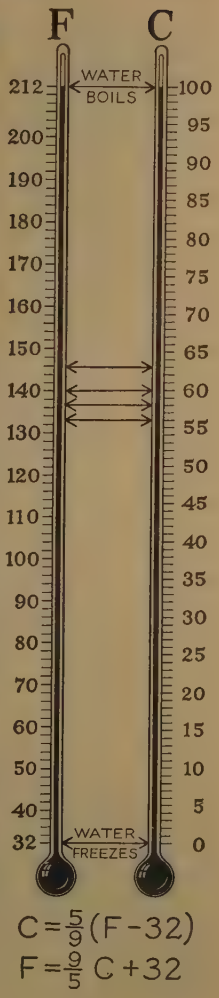
Dairymen's League

A Modern Pasteurization Plant

Mary Louise was reading a magazine article on the *pasteurization* of milk. The article stated that bacteria in milk are destroyed by pasteurization. "Pasteurization" means heating the milk to a temperature between 61° and 63° Centigrade and holding it at that temperature for not less than 30 minutes. This chart was included in the article to explain to the reader how the Centigrade scale is related to the familiar Fahrenheit scale.

The next two pages explain the relation between the two thermometers and show how the Centigrade reading which corresponds to any given Fahrenheit reading may be found by simple arithmetic. Can you find out how to do this without turning the page?

Perhaps your science teacher will lend you the two kinds of thermometer.



1. 0° C. (Centigrade) corresponds to __?__ degrees F. (Fahrenheit).

2. 100° C. corresponds to __?__ degrees Fahrenheit.

3. Water freezes at __?__ degrees C. or __?__ degrees F.

4. Water boils at __?__ degrees C. or __?__ degrees F.

5. The typhoid germ in milk is killed at 58° C. or about __?__ degrees F.

6. The tuberculosis germ in milk is killed at 140° F. or __?__ degrees C.

7. The septic sore throat germ in milk is killed at 56° C. or __?__ degrees F.

8. The diphtheria germ in milk is killed at 54 degrees Centigrade, which is the same as __?__ degrees Fahrenheit.

9. To kill germs of all the above-mentioned diseases, milk would have to be heated to at least __?__ degrees Centigrade, which is the same as __?__ degrees Fahrenheit.

10. To insure thorough pasteurization, milk is usually held at a temperature a little higher than absolutely necessary for killing the germs. That is, milk is pasteurized at 63° C. or __?__ degrees F.

11. Mary Louise thought: "Those formulas at the bottom of the chart would probably help one to change a Fahrenheit reading into a Centigrade reading and vice versa. Let's see. $C = \frac{5}{9}(F - 32)$ means that in order to find the Centigrade reading when you know the Fahrenheit reading, you subtract 32 degrees from the Fahrenheit reading and take $\frac{5}{9}$ of the result."

Do you think Mary Louise interpreted the formula correctly?

12. Mary Louise decided to make use of the chart on page 141 to see if her understanding of the formula $C = \frac{5}{9}(F-32)$ was correct. This is the way she changed from 50° Fahrenheit (the temperature below which milk and its products should be kept) to Centigrade:

$C = \frac{5}{9}(F - 32°)$ $\quad F = 50°$

$C = \frac{5}{9}(50° - 32°)$

$C = \frac{5}{9} \times 18° = 10°$ (Centigrade).

Does her result agree with the chart reading?

13. Show how she changed 95° F., 59° F., and 104° F. to Centigrade readings.

14. Give an interpretation of the formula $F = \frac{9}{5}C + 32$.

15. This is the way Mary Louise changed from 20° Centigrade to Fahrenheit:

$F = \frac{9}{5}C + 32$ $\quad C = 20°$

$F = \frac{9}{5} \times 20° + 32°$

$F = 36° + 32° = 68°$ (Fahrenheit).

Does her result agree with the chart reading?

16. Change from Fahrenheit to Centigrade the temperature at which water boils.

In most scientific work where temperatures are measured, the Centigrade scale is used instead of the Fahrenheit scale.

17. 149° F. = ___?___° C.

18. 15° C. = ___?___° F.

19. 55° C. = ___?___° F.

20. 167° F. = ___?___° C.

21. 91° C. = ___?___° F.

22. 98° F. = ___?___° C.

23. 73° F. = ___?___° C.

24. 36° C. = ___?___° F.

25. Make a graph to show the relation between Fahrenheit and Centigrade readings, from 0° to 100° C. (Measure Centigrade horizontally.)

1. How many cubic yards of earth would have to be removed in digging a cellar 9 ft. long, 6 ft. deep, and $7\frac{1}{2}$ ft. wide?

2. A coal bin is 9 ft. long, 6 ft. wide, and $5\frac{1}{2}$ ft. deep. How many tons of coal will it hold, allowing 90 lb. of coal to the cubic foot and 2000 lb. to a ton?

3. The base of a pyramid is a 12-inch square; its altitude is 12 in. What is its volume in cubic inches? in cubic feet?

4. Find the capacity in cubic feet of a cylindrical silo 24 ft. high and 10 ft. in inside diameter.

5. Find the number of square yards in the inner curved surface of the silo in Ex. 4.

6. A conical drinking cup is 4 inches deep and 3 inches in diameter. How many times could it be filled with a quart of fruit juice? (1 gal. = 231 cu. in.)

7. What is the surface of a sphere whose radius is 6 inches? What is its volume?

Compute the surfaces and volumes of the following spheres:

8. $r = 2$ (in.)

9. $d = 2$ (ft.)

10. $r = 20$ (in.)

11. $d = 20$ (in.)

12. A metal sphere 4 inches in diameter weighs how many times as much as one 2 inches in diameter, made of the same metal?

13. What is the volume of a cylindrical gasoline can 10 inches in diameter and 16 inches deep? How many gallons does it hold?

14. A cylindrical bucket 8 inches deep and 10 inches in diameter holds how many cubic feet?

15. What would it cost, at \$2.75 a cubic yard, to excavate a cellar 3 yd. long, $2\frac{2}{3}$ yd. wide, and 6 ft. deep?

16. A swimming tank 20 yd. long, 8 yd. wide, and filled to an average depth of 4 ft. contains how many gallons of water? (1 cu. ft. = 7.5 gal.)

17. What should be the depth of a conical drinking cup that is to be 3 inches in diameter in order to have it hold $\frac{1}{2}$ pint? (1 gal. = 231 cu. in.)

18. Compute the cost, at \$.90 a square yard, of painting the curved surface of a gas tank that is 46 ft. in diameter and 24 ft. high.

19. How many gallons of water will a cylindrical boiler hold if it is 10 ft. in diameter and 30 ft. long? (See Ex. 16.)

20. Compute the entire surface of the boiler in Ex. 19.

21. A spherical granite ball is 6 ft. in diameter. What is its volume? If a cubic foot of granite is about 8 times as heavy as a cubic foot of water, what will the granite ball weigh? (1 cu. ft. water weighs 62.5 lb.)

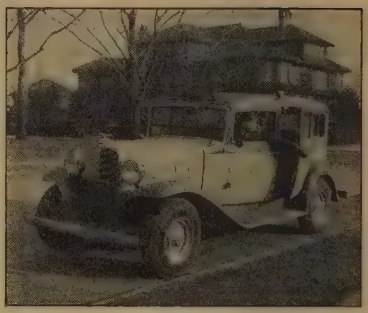
22. How many bushels will a rectangular bin hold if it is 6 ft. long, 5 ft. wide, and 4 ft. deep? (1 cu. ft. = .8 bu.)

23. Which is larger, a cylinder 4 ft. in diameter and 4 ft. long or a sphere 4 ft. in diameter? How much larger?

24. A cylindrical well 24 ft. deep and 4 ft. in diameter is $\frac{2}{3}$ full of water. How many gallons does it contain? (Ex. 16.)

25. How many gallons of water fell on a rectangular lawn that was 50 ft. wide and 66 ft. long, during a 1-inch rain?

26. To what depth would the water (Ex. 25) fill a cylindrical well 4 ft. in diameter?



1. Harry Martin, who worked as a taxicab driver, wished to go into business for himself. He resolved to save on the average 40 cents out of each dollar he earned. If he worked 9 weeks at $10.50 a week, 20 weeks at $12.25 a week, and 23 weeks at $14.00 a week, how much would he save? (He succeeded in saving that amount.)

2. Harry's employer was favorably impressed with his industry, the quality of his work, and his thrift. He lent Harry enough money so that with his savings he could buy a 750-dollar taxi and a 100-dollar garage. How much did Harry borrow? How much interest would he have to pay for the use of this money for a year, at 6%?

3. Harry found he could drive 150 miles on a 10-gallon tank of gasoline and $1\frac{1}{2}$ qt. of oil. This was __?__ miles per gallon of gasoline and __?__ miles per quart of oil.

4. During Harry's first year he drove his car 14,600 miles. At 15 cents a gallon for gasoline, and 20 cents a quart for the oil, how much did he spend for these items?

5. He spent $32.75 for insurance, $37.50 for repairs, and $16.50 for other expenses. He estimated that the car had depreciated about 60% in value, or about __?__ dollars. He estimated that the total cost of tires for the year was $57.

6. Harry's receipts for the year were $1624.80. Compute his profit for the year. (Assume no depreciation on the garage.) Did it pay Harry to go into business? What do you think his experience was worth to him?

1. *Manager:* We have investigated your references and have decided to offer you a position. We will start you in at $1000 a year and then you will be given your choice of (1) an increase of $50 per six months every six months, or (2) a raise of $200 per year every year. Which do you prefer? Salaries are paid each half year.

Applicant: Of course I choose the second plan. Who wouldn't?

Manager: Well, I think we should secure someone else for the position. Good day!

Very few pupils are able to solve this problem.

2. Can you find what is wrong in this computation? A young man applied for a position in a business house. He said his services were worth $1200 a year. The manager said this salary was too high, and argued as follows:

Q. There are 365 days in a year? *A.* Yes.

Q. You sleep 8 hours a day or 122 days a year. This leaves 243 days? *A.* That's right.

Q. You rest 8 hours a day, or 122 days a year, leaving only 121 days? *A.* Yes, I do.

Q. There are 52 Sundays a year. You don't work Sundays. 52 from 121 is 69? *A.* That seems straight.

Q. You never work Saturday afternoons. Isn't that 26 days off of 69? *A.* Yes.

Q. Well, then, do you think we can pay $1200 for 43 days' work?

Do you see the fallacy in the argument?

3. *Curious boy* (*at Zoo*): How many birds and how many beasts have you in this menagerie?

Attendant: I just noticed that there are 30 heads and 100 feet. Of course you can figure out for yourself how many beasts and how many birds there are.

1. Six times a certain number plus 4 equals 30. Find the number. (Use an equation.)

Solve and check each of these equations:

2. $m + 2\frac{1}{4} = 10$
3. $\frac{2y}{3} = 10$
4. $100 = y - 75.5$
5. $.04\ m = 20$
6. $\frac{s}{4} = \frac{2}{3}$
7. $2\frac{1}{2}\ y = 100$

8. Harry wanted to find the weight of his dog. He put the dog in a box and placed it on the scales. Dog and box together weighed 13 lb. Then he weighed the box alone. It weighed $2\frac{5}{8}$ lb. How much did his dog weigh?

9. Express 45,876,328 to the nearest million.

10. 35% of 18 = ?
11. $\frac{1}{4}$% of 90 = ?
12. 12 = ?% of 14.
13. 1.25 = ?% of 25.
14. 2% of ? = 40.
15. 200% of ? = 900.

16. A school paper stated that the contributions to the Red Cross in the school amounted to \$75 and that this was 50% above the amount of the contributions the previous year. How much were the contributions the previous year?

17. Find the area of a triangle whose base is 8 ft. and altitude 6 ft.

18. Find the volume of a cylinder, the radius of whose base is 4 ft. and whose altitude is 8 ft.

19. A rectangle $2' \times 4'$ was enlarged by doubling both its length and its width. Its old area was what per cent of its new area? Draw a diagram to illustrate.

Standards: A, 15. B, 12. C, 9.

CHAPTER V

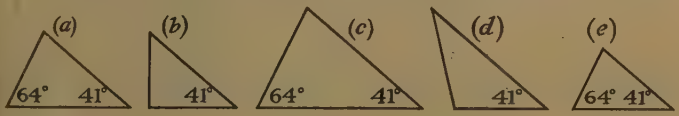
INDIRECT MEASUREMENT

Similar Triangles

By Ewing Galloway, N. Y.

Some Boy Scouts who were camping on the bank of a river wished to know how wide the river was. Can you think of a way to measure the distance across the river without crossing it? If not, the following exercises will help you learn how.

1. Which of these triangles appear *similar* (shaped alike)?

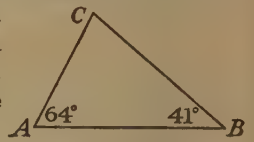


2. Would you say that any two triangles are similar if one angle of one triangle equals one angle of the other? (Draw some triangles to illustrate your idea.)

Would you say that any two triangles are similar if two angles of one triangle are equal to two angles of the other? You will now have an experiment to find out.

An experiment to find out whether two triangles are similar if two angles of one triangle are equal to two angles of the other

(1) Draw a line segment, AB, of any convenient length. At B construct, with the protractor, an angle of 41°. At A construct an angle of 64° and extend the sides of the two angles so as to form a triangle. Call the vertex angle C. Your work should look like the figure shown here.



(2) Compare the shape of the triangle ABC that you have drawn with the shape of those drawn by your classmates.

It is helpful to have the triangles cut out and displayed before the class.

(3) Do all the triangles drawn by the class seem to have the same shape? If the smaller ones were enlarged, could they be made to fit exactly upon the larger ones?

(4) What was done in Ex. (1) that caused all members of the class to get triangles of the same shape?

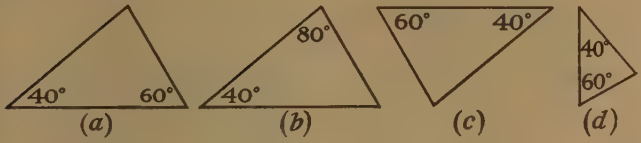
(5) Compare your angle C with angle C in the triangles drawn by your classmates. Show that in these triangles any angle C ought to be equal to any other angle C. (Remember that the sum of the angles of a triangle is 180°.)

(6) Are the triangles drawn by the class necessarily of the same size? Are any two necessarily of the same size?

Result of experiment. **If two angles of one triangle are equal to two angles of another triangle, the triangles are similar.**

1. Sketch a triangle ABC, in which $\angle A = 80°$ and $\angle B = 70°$. Now sketch a triangle XYZ, in which $\angle Z = 30°$ and $\angle Y = 70°$. Would you say that the triangles are similar? Why?

1. Which of these triangles do you consider similar? Why?



2. If two angles of one triangle are equal to two angles of another triangle, are the third angles equal? Why?

3. Max asked: "If you are told that two triangles are similar, do you know that the three angles of one triangle are equal to the three angles of the other triangle?" How would you have answered his question?

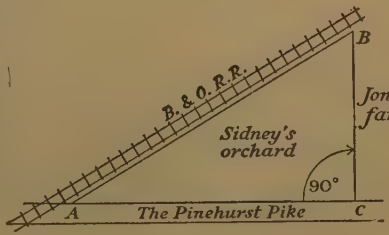
4. The mathematical symbol for "is similar to" is $\sim$. The symbol $\triangle$ means "triangle." Thus $\triangle ABC \sim \triangle XYZ$ is read: "Triangle ABC is similar to triangle XYZ."

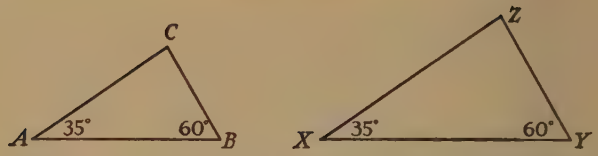
5. Complete: $\triangle XYZ \sim \triangle ABC$ if $\angle X = \angle A$ and if $\cdots$

6. The base of one triangle is 2 inches; its angles are 30°, 60°, and 90°. The base of another triangle, similar to the first, is 4 inches. How large are its angles?

7. In right triangles ABC and XYZ, acute angle A equals acute angle X. Can you tell whether the triangles are similar?

8. This diagram shows Sidney's orchard. Sidney wishes to draw a triangle in his notebook that will have the same shape as the outdoor triangle. Show how he might do it.





1. Is $\triangle ABC \sim \triangle XYZ$? Why? Does AB appear to be about $\frac{1}{2}$, or $\frac{2}{3}$, or $\frac{3}{4}$ as long as XY? Use your metric scale to check your judgment. Compare BC and YZ. Now compare AC and XZ. What do you notice concerning the corresponding sides of these similar triangles? (*Corresponding sides* are sides opposite equal angles.)

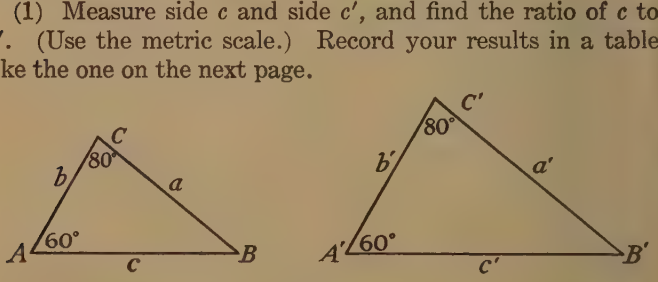
2. What is the ratio of AB to XY in Ex. 1? (Ratio of AB to XY means the length of AB divided by the length of XY.) What is the ratio of BC to YZ? the ratio of AC to XZ? What can you say concerning the ratios of corresponding sides of these similar triangles?

Do you think that the ratios of the corresponding sides of any two similar triangles are equal?

An experiment to find out whether in any two similar triangles ABC and A′B′C′ as shown here, $\frac{a}{a'} = \frac{b}{b'} = \frac{c}{c'}$

($A'B'C'$ is read "A-prime, B-prime, C-prime.")

(1) Measure side c and side c', and find the ratio of c to c'. (Use the metric scale.) Record your results in a table like the one on the next page.



(2) Measure a, a', b, and b' and record your results.

c = ? mm.	a = ? mm.	b = ? mm.
c' = ? mm.	a' = ? mm.	b' = ? mm.
$\frac{c}{c'}$ = ?	$\frac{a}{a'}$ = ?	$\frac{b}{b'}$ = ?

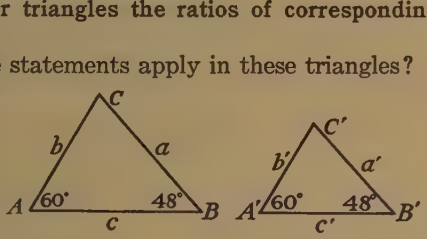
What do you think is true concerning the ratios of corresponding sides of similar triangles?

In any two similar triangles the ratios of corresponding sides are equal.

3. Which of these statements apply in these triangles?

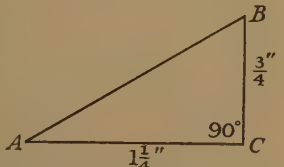
(1) $\triangle ABC \sim \triangle A'B'C'$.

(2) The ratio of a to a' equals the ratio of b to b'.



(3) $\frac{a}{a'} = \frac{b}{b'}$ (4) $\frac{a}{a'} = \frac{c}{c'}$ (5) $\frac{b}{b'} = \frac{c'}{c}$

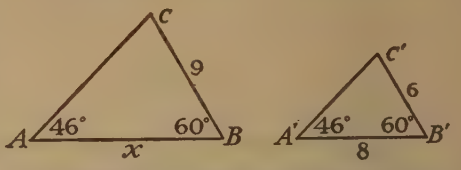
4. Here is a triangle which Sidney drew to represent his orchard (p. 151). AC is $1\frac{1}{4}$ inches long. The line outdoors that AC represents is 500 feet. How long is BC? How many feet does it represent?



5. Ben said: "I have learned that in similar triangles the angles are equal and the sides are equal." Frank said: "In similar triangles the angles are equal and the ratios of the sides are equal." Lewis said: "In similar triangles corresponding angles are equal and the ratios of corresponding sides are equal." Who gave the best answer?

6. If you are told that $\triangle ABC \sim \triangle A'B'C'$, what do you know about $\angle A$ and $\angle A'$? $\angle B$ and $\angle B'$? $\angle C$ and $\angle C'$? Does $\frac{AB}{A'B'} = \frac{BC}{B'C'}$? Does $\frac{AB}{A'B'} = \frac{AC}{A'C'}$? Does $\frac{AC}{A'C'} = \frac{BC}{B'C'}$?

This figure shows two similar triangles. The length of the line AB is unknown. Can you find it? You probably see that the ratio of AB to 8 equals the ratio of 9 to 6. Let us use x to represent the number of units in AB. Then x is easily found if these equal ratios are written in the form of the equation: $\frac{x}{8} = \frac{9}{6}$. By solving this equation, you can find the length of AB.



1. Write as an equation: The ratio of x to 4 is equal to the ratio of 2 to 3. What is the value of x?

2. One of the principles often used in solving equations is that we may *multiply both sides of the equation by the same number*. By what would you multiply both sides of the equation $\frac{x}{2} = \frac{3}{5}$ to get the equation $x = \frac{6}{5}$?

3. Solve the following equations and tell in each case by what you multiply both sides:

(*a*) $\frac{x}{6} = \frac{7}{12}$ (*f*) $\frac{11}{2} = \frac{p}{4}$ (*k*) $\frac{x}{5} = 2\frac{1}{2}$

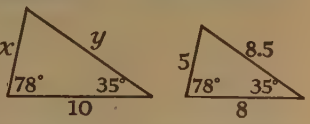
(*b*) $\frac{n}{3} = \frac{5}{4}$ (*g*) $\frac{n}{100} = \frac{3}{10}$ (*l*) $\frac{x}{4} = 2\frac{1}{2}$

(*c*) $\frac{3}{4} = \frac{x}{3}$ (*h*) $\frac{r}{5} = \frac{3}{10}$ (*m*) $\frac{p}{25} = \frac{1}{5}$

(*d*) $\frac{h}{10} = \frac{6}{15}$ (*i*) $\frac{x}{4} = \frac{7}{2}$ (*n*) $\frac{n}{8} = 4\frac{1}{2}$

(*e*) $\frac{y}{4} = \frac{5}{2}$ (*j*) $\frac{y}{3} = \frac{2}{3}$ (*o*) $\frac{x}{10} = 3.6$

4. Look at this figure and complete: The ratio of x to 5 equals the ratio of 10 to ___?___. The ratio of y to 8.5 equals the ratio of ___?___ to 8.



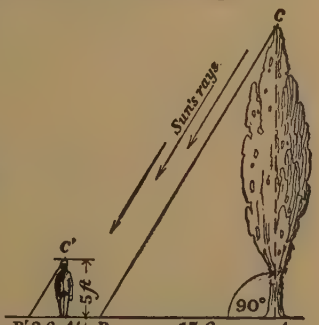
5. Write each of the statements in Ex. 4 as an equation.

6. Solve the following:

(*a*) $\frac{x}{5} = \frac{10}{8}$ (*b*) $\frac{y}{8.5} = \frac{10}{8}$

7. Tell what your results for Ex. 6 mean with reference to the figure above.

8. (*a*) The boy in the picture is 5 feet tall. His shadow is 3 feet long. The shadow of the tree is 15 feet long. Compare the triangles in the picture and see if they can be called similar triangles.



(*b*) In general science we learn that the sun's rays are parallel. In this figure, this means that angle B' is equal to angle ___?___.

(*c*) Show that $\triangle ABC$ is similar to $\triangle A'B'C'$.

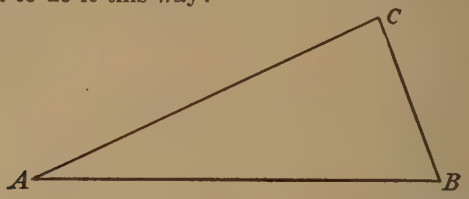
(*d*) Complete: If the shadow of the tree is 5 times as long as the shadow of the boy, then the tree must be ___?___ times as tall as the boy. How tall is the tree?

(*e*) If t represents the number of feet in the height of the tree, show that $\frac{t}{5} = \frac{15}{3}$. Find the value of t.

9. Why is measurement such as that used in Ex. 8 called "indirect measurement"?

10. History suggests that man learned very early how to use "shadow reckoning." What reason can you give for this?

Do you know how to make an exact copy of triangle ABC? Fred tried to do it this way:



(1) He measured AB and drew a line $A'B'$ equal to AB.

(2) He measured angle A and drew an angle at A′ equal to the angle A.

(3) He measured angle B and drew an angle at B' equal to the angle B. He called the point where the lines met C'.

(4) He cut out the triangle $A'B'C'$ and tried to fit it over triangle ABC. Do you think his triangle was an exact copy of triangle ABC?

An experiment

1. Use Fred's method to make a copy of triangle ABC.

2. Cut out your triangle and see if you can make it fit exactly over triangle ABC.

3. Place the triangles in such a position that the class can examine them. (For this purpose it is helpful to paste or pin a considerable number of the triangles to a card or to a large sheet of paper.)

4. Choose any two of the triangles and see whether one of them can be placed upon the other, so that every part of the one fits exactly over the corresponding part of the other.

5. Just what did you do to get two triangles that are exactly alike in size and shape?

6. Do all the triangles have the same shape? same size?

Two triangles that are exactly alike in size and shape are said to be ***congruent.***

Two triangles that have two angles and the included side of one equal respectively to two angles and the included side of the other are *congruent*.

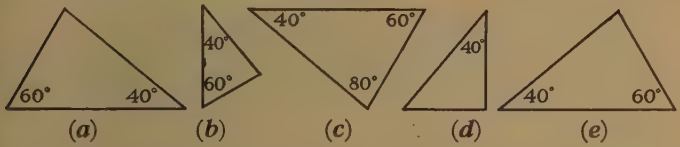
Corresponding parts of congruent triangles are equal.

1. Would you mark these statements right or wrong?

(*a*) I can draw two *similar* triangles if I make two angles of one equal to two angles of the other, even if I do not use the same base.

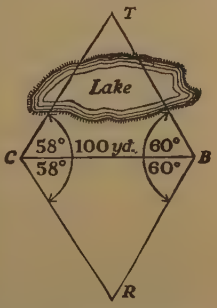
(*b*) I can draw two *congruent* triangles if I make two angles and the included side of one triangle equal to two angles and the included side of the other.

2. Which of these triangles are congruent?



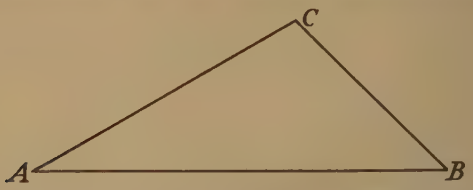
(*a*) (*b*) (*c*) (*d*) (*e*)

3. At a Boy Scout camp a target was set up beyond a small lake. Two Scouts wished to know how far the target T was from their camp C, as shown in the figure. How could they find out?



They paced off 100 yards. They measured the angle TCB with a large protractor (called a *field protractor*) and found it to be 58°. They constructed on the opposite side of CB a 58° angle. They measured angle CBT and found it to be 60°, and then constructed an angle on the opposite side of CB equal to it. They "ran these lines" until they intersected at a point R. Does that make $\triangle CBR$ congruent to $\triangle CBT$? Which line in the triangle on dry land corresponds to CT? How could the boys find the distance CT?

What method have you just learned for making an exact copy of a triangle? Alice tried the following method of making an exact copy of a triangle. Read it carefully.



(1) She measured AB and drew a line $A'B'$ equal to AB.

(2) She measured angle A and drew an angle at A' equal to angle A.

(3) She measured AC and laid off $A'C'$ equal to AC.

(4) She connected C' and B', making triangle $A'B'C'$.

(5) She cut out triangle $A'B'C'$ and tried to fit it over triangle ABC.

Do you think she made an exact copy of triangle ABC?

An experiment

1. Use Alice's method to make a copy of triangle ABC.

2. Cut out your triangle and see if you can make it fit exactly over triangle ABC.

3. Compare your triangle as to shape and size with those drawn by your classmates.[1]

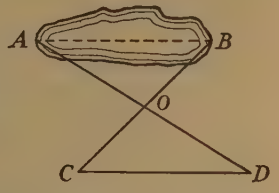
4. Would you consider this statement true?

Two triangles have the same shape and size if $AB = A'B'$, $AC = A'C'$, and $\angle A = \angle A'$.

Two triangles are congruent if two sides and the included angle of one are equal to the corresponding parts of the other.

[1] It is helpful to display about ten of these triangles before the class. They may be pinned over each other to suggest the correct conclusion.

1. Doris and Ann wished to find the length of AB, which they could not measure directly. Do you see how they could do it, using congruent triangles?



(*a*) They set a stake at O, extended AO to D so that OD equaled OA,[1] and extended BO to C so that OC equaled OB. Then they measured CD. Was CD equal to AB?

(*b*) Doris said: "Yes, of course $CD = AB$. They are corresponding parts of congruent triangles. The triangles are congruent because two sides and the included angle of one are equal to two sides and the included angle of the other." Do you agree?

(*c*) Ann asked: "What are the equal included angles?" Doris replied: "Angles AOB and COD. Don't you remember that we learned last year that when two lines intersect, the vertical angles are equal?"

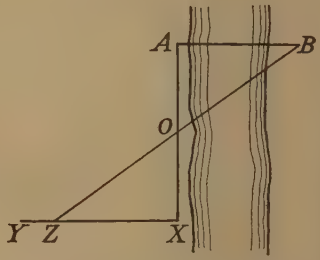
The preceding problem illustrates an important use of congruent triangles—the indirect measurement of a distance which cannot be measured directly. The girls laid out congruent triangles so that AB, which they could not measure directly, would equal CD, which they could measure directly.

2. In this figure $AO = OC$ and $BO = OD$. Are the triangles congruent? Why? Are AB and CD equal? Why?

3. Would you say that two congruent triangles are necessarily similar? that two similar triangles are necessarily congruent?

[1] One end of a tape was fastened to the stake at O. Ann held the tape at a point X such that OX was equal to OA. Doris sighted from point A through point O and had Ann move point X of the tape till it was in line with A and O. At this point the stake D was driven.

1. Some boys wished to find the distance between two trees A and B on opposite sides of a river. They thought they could do it by using congruent triangles. Do you see how it could be done?



(a) They set a stake at O and made OX equal to OA.

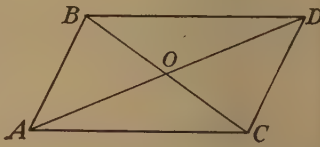
(b) They sighted from A to B and measured angle A with a field protractor. They staked out XY, making angle X equal to angle A, and placed Z in line with O and B.

(c) Then they measured XZ. Do you think the triangles are congruent? If so, then the length of XZ equals the length of AB. Why?

2. How did these boys know that triangles XOZ and AOB are congruent? (See page 157.)

3. Would you say that the boys used indirect measurement? Why?

4. In this figure $OB = OC$ and angle OAB = angle ODC. Are triangles AOB and COD congruent? Why? Does $AB = CD$? Why?

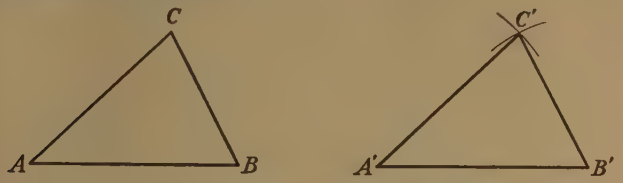


5. Construct two triangles ABC and XYZ, in which $AB = XY$, angle A = angle X, and $AC = XZ$. Does $BC = YZ$? Why?

6. State two ways of making congruent triangles.

7. Would you say that the ratios of the corresponding sides of congruent triangles are equal? What is the ratio?

George wanted to make an exact copy of triangle ABC. He used this method:



(1) He laid off $A'B'$ with his compass equal to AB.

(2) With A' as a center and with a radius equal to AC, he drew an arc above $A'B'$.

(3) With B' as a center and with a radius equal to BC, he drew an arc intersecting the first arc at C', as shown.

He believed triangle $A'B'C'$ to be an exact copy of triangle ABC because the sides of one triangle were equal respectively to the sides of the other. Do you think it was?

1. Using the above method, construct a triangle ABC, in which $AB = 2\frac{1}{2}$ in., $AC = 2$ in. and $BC = 3$ in., and a triangle $A'B'C'$, in which $A'B' = 2\frac{1}{2}$ in., $A'C' = 2$ in., and $B'C' = 3$ in.

2. Cut out triangles ABC and $A'B'C'$ (Ex. 1) and see if you can fit them together. Are they congruent?

3. Are all the triangles ABC of the class congruent?

4. What do the results of Exs. 1 to 3 suggest regarding congruent triangles?

Two triangles are congruent if the sides of one are equal respectively to the sides of the other.

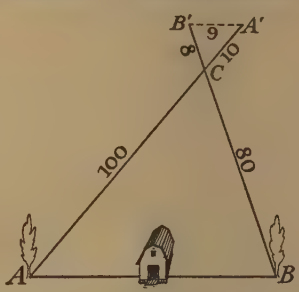
5. Would you say that the triangles which you drew in Ex. 1 are similar?

6. Construct triangles whose sides are:

(*a*) 3 in., $2\frac{7}{8}$ in., $2\frac{5}{16}$ in. (*c*) $2\frac{7}{8}$ in., $3\frac{3}{4}$ in., $2\frac{11}{16}$ in.

(*b*) 4 cm., 3.6 cm., 2.9 cm. (*d*) 6.2 cm., 5.8 cm., 4.6 cm.

Rex and Jim saw their Uncle Henry staking out a triangle like the triangle $A'B'C$ shown in this figure. They asked him what he was doing. He said: "What I really want to measure is the distance from the tree at A to the tree at B. But I can't measure that line directly, because there is a barn in the way. I can find the length of that line by studying triangle $A'B'C$."



He said that his two triangles ABC and $A'B'C$ were similar because the ratio of 8 to 80 and the ratio of 10 to 100 are equal. He explained that any two triangles are similar if the ratio of one pair of corresponding sides equals the ratio of another pair of corresponding sides, and the included angles are equal. Do you think this is true? The experiment that follows will help you decide.

An experiment to find out whether two triangles are similar

(1) Draw a line AB 2 inches long. At A draw an angle of 50°. On the line you obtained when drawing the 50° angle, lay off a segment 1 inch long. Call this AC. Connect C and B, giving you triangle ABC.

(2) Draw a line $A'B'$ 3 inches long. At A' draw an angle of 50°. On the line obtained when drawing the 50° angle, lay off a segment $1\frac{1}{2}$ inches long. Call this $A'C'$. Connect C' and B', making triangle $A'B'C'$. (See figures, page 163.)

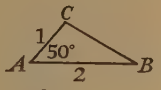
(3) Cut out the triangles ABC and $A'B'C'$.

(4) Fit triangle ABC over triangle $A'B'C'$.

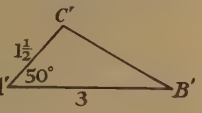
Do the triangles seem to have the same shape? AB is what part of $A'B'$? AC is what part of $A'C'$? Measure BC and $B'C'$. BC is what part of $B'C'$? What did we do that made the triangles similar?

Result of experiment. The members of a certain class were asked to write what they had learned from this experiment. Three of their answers were:

James: We made the ratio of AB to $A'B'$ equal to the ratio of AC to $A'C'$ and the included angles equal, thus making the triangles similar.



Fred: We got two similar triangles by drawing the figures so that $\frac{AB}{A'B'} = \frac{AC}{A'C'}$ and $\angle A = \angle A'$.



Louise: Two triangles are similar if the ratio of one pair of corresponding sides equals the ratio of another pair of corresponding sides, and the included angles are equal.

Who wrote the best statement?

Two triangles are similar if the ratio of one pair of corresponding sides equals the ratio of another pair of corresponding sides, and the included angles are equal.

Let us now return to the triangles that the boys' Uncle Henry drew. (See preceding page.)

1. Is the angle ACB equal to the angle $A'CB'$? Check your answer by measuring the angles with the protractor.

2. The ratio of AC to $A'C$ is ___?___ to ___?___.

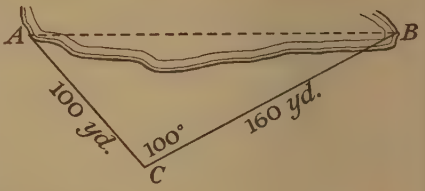
3. The ratio of BC to $B'C$ is ___?___ to ___?___.

4. Show that the small triangle $A'B'C$ is similar to triangle ABC.

5. Uncle Henry measured $A'B'$ and found it to be 9 feet. Therefore the distance from A to B is ___?___ feet.

6. Draw a triangle XYZ. Extend XZ through Z to X', making $ZX' = \frac{1}{2} XZ$. Extend YZ through Z to Y', making $ZY' = \frac{1}{2} YZ$. Are triangles XYZ and $X'Y'Z$ similar? Why?

Catherine and Grace wished to know the length AB of their swimming course. They knew how to find this distance by the use of a ***scale drawing***. Do you know how?



(1) Catherine drove a stake into the level ground at C. (See the figure.) Then AC was measured and found to be __?__ yards. BC was found to be __?__ yards. The angle between the two lines was measured with a field protractor or a compass. It was __?__ degrees.

(2) What parts of the outdoor triangle did they know?

(3) Grace then drew a small triangle to represent the outdoor triangle. Do you know how she did it?

(*a*) She let 1 in. represent 40 yd., and drew a line __?__ inches long to represent AC.

(*b*) How large was angle C in Grace's triangle? How long did she make the line which represented CB?

(4) Now draw a triangle like the one Grace drew to represent the outdoor triangle.

(*a*) Draw a line to represent AC, using 1 in. to represent 40 yd. At C draw an angle of 100°. Is the last line, as you have it now, definite or indefinite in length? On this line lay off a distance to represent 160 yd. Call this CB. Draw AB.

(*b*) If your work is accurately done, you now have a triangle which is __?__ to the outdoor triangle. Why?

(*c*) The line AB in your triangle is __?__ inches long. Since 1 inch represents 40 yards, the line AB represents __?__ yards. Hence the distance on the lake, between A and B, is __?__ yards.

(5) In the preceding exercise you used a scale in which 1 in. represents 40 yd. If you had used 1 in. to represent 25 yd.,

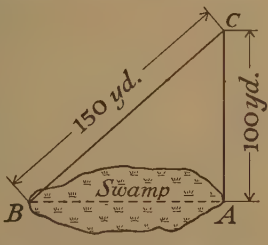
would your scale drawing have been larger or smaller? What kind of drawing would you get if you used the scale in which 1 in. on paper represents 1 in. outdoors?

(6) In making the scale drawing of the swimming-course triangle, which of the following principles was used?

(*a*) Two triangles are similar if two angles of one triangle are equal to two angles of the other.

(*b*) Two triangles are similar if the ratio of one pair of corresponding sides equals the ratio of another pair of corresponding sides, and if the angles included between these sides are equal.

(7) Joe wished to measure the distance from A to B across a swamp. Would you know how to find the distance by using a scale drawing? If not, read on.

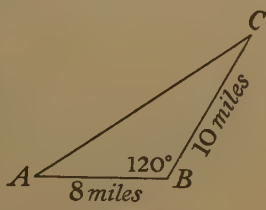


Joe drove a stake into the level ground at C. Then he measured AC and BC and found them to be 100 yd. and 150 yd. He found the angle at C to be 48 degrees.

He then drew the figure shown here to represent the outdoor triangle. Can you tell how he constructed it? How many yards did he let 1 in. represent? How large did he make angle C? How can you find out from his triangle the distance across the swamp?

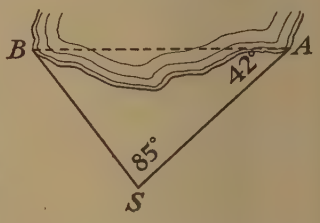
Is this scale drawing similar to the outdoor triangle? Why?

1. The lines AB and BC in this figure represent a country road from A to C. What scale was used in making this drawing? Can you find by measuring AC how much shorter it would be to walk straight across country from A to C than to follow the road?



We have learned that if we make two angles of one triangle equal to the corresponding angles of a second triangle, the two triangles are similar. In finding an unknown distance by means of a triangle drawn to scale, it is not enough to make the scale drawing *similar* to the triangle which it represents. We must also make one side of the scale drawing one tenth, or three hundredths, or five thousandths, or some known part of the corresponding side in the outdoor triangle. Hence, we must measure at least one side in the outdoor triangle.

1. (*a*) In the map of a portion of Long Lake, B is a small cottage; A is a camp; angle SAB is 42°; and angle ASB is 85°. Can you draw a triangle similar to triangle ABS? Would it be possible, with these facts, to make a scale drawing of triangle ABS, or would it be necessary to have additional information?



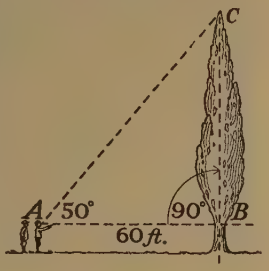
Suppose you measured AS with a tape and found it to be 300 yd. How many parts of the triangle would you then know? Would you then have enough facts to make a scale drawing of triangle ABS?

(*b*) Make a scale drawing of triangle ABS, in which AS represents 300 yd. Let 1 in. represent 40 yd. Complete: I find AB on my paper to be __?__ in. If 1 in. represents 40 yd. in the outdoor triangle, then each $\frac{1}{4}''$ represents 10 yd. and hence AB represents __?__ yd. Therefore the distance from the camp at A to the cottage at B is __?__ yd.

(*c*) Could the problem be solved if BS, $\angle S$, and AS were known?

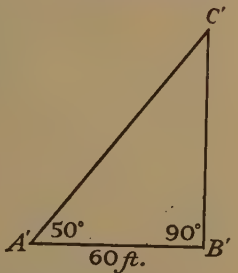
(*d*) Do you think the problem could be solved if $\angle B$, BS, and $\angle S$ were known?

This figure shows how Herbert and John measured the height of a tree by measuring two angles and a side of a triangle. One side of the triangle extends from one of John's eyes to the top of the tree. How are the other two sides formed?



The boys found with a field protractor that angle A was 50 degrees. The angle at B was ___?___ degrees. Why? The distance from John's eye to the tree is ___?___ feet.

1. The figure at the right is a copy of a triangle drawn by the boys to represent the outdoor triangle. They decided that they would let 1 inch represent 60 feet. By this scale any line $1\frac{1}{4}$ inches long represents ___?___ feet outdoors. They made this triangle similar to the outdoor triangle. How does the figure show this?



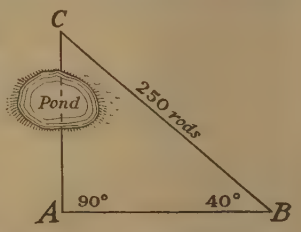
Scale: 1 in. = 60 ft.

2. Since the triangle which they drew is similar to the outdoor triangle, $B'C'$ is a scale drawing of the height of the tree. Measure accurately with a ruler the line $B'C'$. How many feet does this represent? Assume that John's eye is 5 feet above the ground. Complete: The tree is ___?___ feet plus 5 feet, or ___?___ feet high.

3. Could John have measured the slanting line AC from his eye to the top of the tree? Measure $A'C'$ on the scale drawing. Complete: The line $A'C'$ represents ___?___ feet.

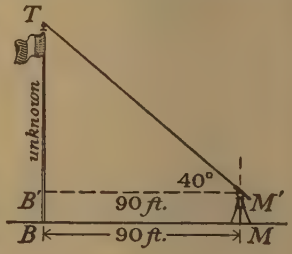
4. Did Herbert and John use congruent triangles? similar triangles?

1. Mary's father owns a triangular farm, the shape of which is shown in this figure. He wishes to know the length of AC, but cannot measure it directly, on account of the large pond. Show how to find the length of AC by means of a scale drawing, using the scale: 1 cm. = 20 rods.



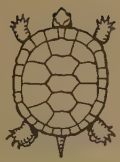
Complete: Angle C = ? degrees; AC = ? rods; AB = ? rods; the area of the farm = ? square rods, or ? acres.

2. Tom Holt wishes to know the height of a flagpole, as shown here. A scale drawing will help him. For example, he measures a straight line of any length on the ground from the base of the flagpole. He calls this line BM. Suppose he makes the line 90 feet long.



Then he sets at M a *transit*, or a homemade protractor, with which he can measure the angle between a horizontal line and the *line of sight* from M' to the top of the pole. He knows also that the angle B' is a right angle. So he knows the length of the line $B'M'$ and the size of the angle at M' and of angle B'.

He makes a scale drawing to represent the known length and the known angles. From this drawing he "scales" or "measures" the height of the flagpole from B' to T and finds it to be ? feet. He knows the portion of the flagpole BB' to be $4\frac{1}{2}$ feet; therefore the height of the pole is ? feet.



Have you ever noticed the pleasing arrangement of the parts of certain natural objects? Would you say that these figures shown above are well *balanced?* A figure that possesses balance is said to be *symmetrical* or to have ***symmetry.***

1. Do you think that this figure is symmetrical? Could the two parts divided by the dotted line be made to *coincide* (exactly fit together) by folding on the line? If so, the figure possesses *line symmetry* and the line of folding is the ***axis of symmetry.***

Does each of the figures above have an axis of symmetry? If so, tell where you think it is located in the figure.

2. Can you show that a square has four axes (pronounced *axees*, plural of "axis") of symmetry?

3. How many axes of symmetry does a rectangle have?

4. How many axes of symmetry has an equilateral triangle (a triangle with three equal sides)? Make a sketch to illustrate your belief. How many axes of symmetry has an isosceles triangle (one with two equal sides)? a scalene triangle (one having no equal sides)?

5. Would you say this figure is symmetrical? If so, how many axes of symmetry can you find? Copy the figure and see whether you can draw the axes.

1. Which of these capital letters illustrate line symmetry?

A B C D E F G

2. Copy the symmetrical letters, and sketch the axis or axes of symmetry in each.

3. Betty's mother taught her how to make these symmetrical figures by folding and cutting sheets of paper. How many axes of symmetry are there in Fig. 1? in Fig. 2? Fig. 3? Fig. 4?

FIG. 1

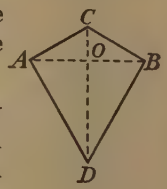
FIG. 2

FIG. 3

FIG. 4

4. How large an angle is formed by the axes of symmetry in Fig. 2? Fig. 3? Fig. 4?

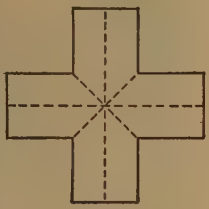
5. Do you think that a kite-shaped figure (two isosceles triangles having a common base between them) is symmetrical? Why?



6. In the kite-shaped figure, would you say that AB and CD are perpendicular? Do you think that CD bisects AB? How could you test your belief?

7. Show that by connecting points a kite-shaped figure may be formed in the construction for bisecting an angle (page 112), and also in the construction for dropping a perpendicular from a point to a line (page 112).

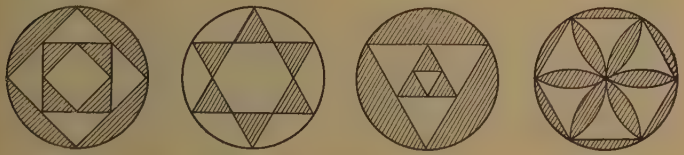
1. This geometrical figure (a cross) is used by the American Red Cross. It has four axes of symmetry. Construct an enlarged copy of the cross, making each arm 1 inch wide and 1 inch long. Use ruler and protractor.



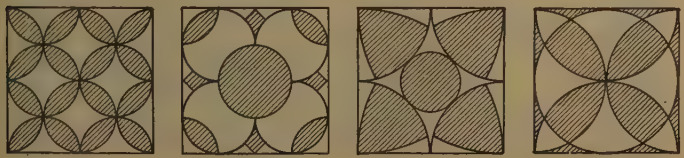
2. Construct a kite-shaped figure like the one shown on the preceding page, making *AO* equal 1 in., *OC* equal 1 in., and *OD* equal 2 in.

3. Construct an equilateral triangle and draw dotted lines to show all its axes of symmetry.

4. Can you construct enlarged copies of each of these symmetrical figures? In each construction use a circle having 1-inch radius.



5. Can you construct enlarged copies of these symmetrical figures?



6. Many pupils enjoy making original symmetrical figures. They often use color to make the designs more decorative. An exhibit of the best of the designs on the bulletin board of the classroom is interesting and worth while.

"GOD ETERNALLY GEOMETRIZES"

—PLATO

(A Pupil's Talk to a School Assembly)

A boy said he thought that a very wise man came to the early people and told them all about geometry. It didn't happen that way. The early people noticed many geometric forms in nature. As a matter of fact, these forms are all about us and we are sure to notice them if we will only open our eyes.

Some of these forms are the *cone*, the *sphere*, the *hemisphere*, the *cylinder*, and the *prism*. Here is a natural shell (see border), which you see has the form of a *cone*. Here are some berries that suggest a *sphere*. Other kinds of fruit, such as the orange, also suggest the sphere. This half of a coconut suggests a *hemisphere*, and so does this little bird's nest. If you were standing out on a prairie, the sky would look as if it came down in the form of a *hemisphere*. Then, here is a section of a birch tree. It has the form of a *cylinder*. There is a kind of stone called basalt that has the form of a *prism*.

There are three stars in the sky that form a group called "*the Triangle*." And natural crystals sometimes have triangular faces.

If I were to drop a pebble into a pool of water, many *circles* would appear, first small ones and then large ones. Or, if I should run my finger around the inside of this coconut section, I should have a circle. Of course, the sun and the moon suggested circles even to the early people.

Then there are the five-, six-, and eight-sided figures called the *pentagon*, the *hexagon*, and the *octagon*. The skeleton of this starfish suggests a pentagon. Some snow crystals have the form of a hexagon. The petals of flowers are arranged so as to outline various *polygons*.

The trees and the paths of falling objects are *vertical*. The trunks of two vertical trees are *parallel*. A tree is *perpendicular* to the ground, and this suggests to us the *right angle*. As you know, the *right angle* is very important.

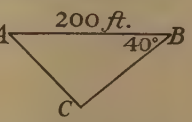
The geometric relations are *equality*, *congruence*, *similarity*, and *symmetry*.

Notice this shell. You would have to look closely to see the dots on it; there are a lot of them, and they are equally distant from each other, and that illustrates *equality*. Then observe this portion of a shell. Its other half fits right on it, and when two things fit together, just like that, it illustrates *congruence*. Here is a butterfly, just the same as the big butterfly and similar in shape and in looks, but different in size. They are said to be *similar* to each other, and that illustrates *similarity*. If you have observed a butterfly with its wings folded, you have seen how the wings on one side exactly fit the wings on the other side. That illustrates *symmetry*. Symmetry is also seen in a leaf, or a fern like this one.

These are some of the many different geometric forms that are all about us, and which led Plato to say: "God eternally geometrizes."

1. A baseball diamond is 90 ft. square. Make a scale drawing to find the distance a catcher must throw the ball from home plate to second base. Let 1 cm. represent 10 ft., or let 1 in. represent 16 ft.

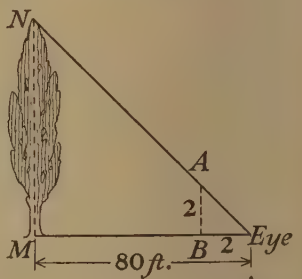
2. (*a*) John knows that AB, shown here, represents 200 ft. and that $\angle B = 40°$. With this information, can he construct a scale drawing of the triangle? Give reasons for your answer.



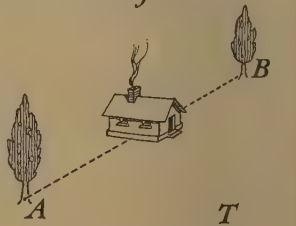
(*b*) What other fact should he get?

3. Do you know what a blueprint is? What has it to do with a scale drawing?

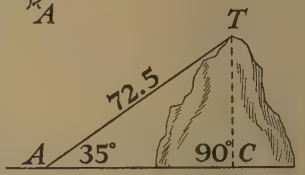
4. To measure the height of a tree, Fred held a 2-foot ruler (represented by AB in this figure) in a vertical position, with the bottom 2 feet from his eye and so as just to "cover" the tree. The tree was 80 feet from Fred's eye. What was its height?



5. A surveyor wishes to find the exact distance between the two trees shown in this figure. The line AB passes through a building. How can he measure this distance?



6. Some Girl Scouts measured the height of a mound. They stretched a string from A to T, as shown, finding AT to be 72.5 ft. Angle A was 35°. How high was the mound?



1. This problem was given in a test: When a post 10 ft. high casts a shadow 4 ft. long, the shadow of a neighboring church steeple is 48 ft. long. How high is the steeple?

Here are the equations which some pupils wrote on their test papers. Each pupil used x as the height of the steeple.

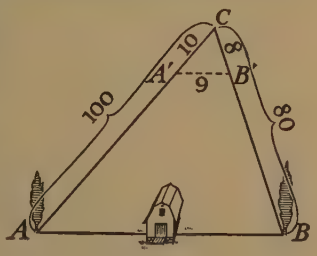
Mabel wrote: $\frac{x}{10} = \frac{48}{4}$ Sam wrote: $\frac{x}{4} = \frac{48}{10}$

Felix wrote: $\frac{x}{48} = \frac{4}{10}$ May wrote: $\frac{48}{4} = \frac{x}{10}$

Arthur wrote: $\frac{x}{48} = \frac{10}{4}$ Grace wrote: $\frac{4}{x} = \frac{10}{48}$

Which pupils were "on the right track"?

2. Eleanor said that her Uncle Henry could have set the stakes A' and B' as shown in this figure, and could then have solved the problem. Was she right? (See page 162.)



3. Study the figure and complete: The ratio of AB to 9 equals the ratio of ___?___ to ___?___.

4. Write the statement in Ex. 3 as an equation, and solve the equation.

Scale drawings are used by surveyors and engineers to (1) estimate distances and angles; (2) show clearly how the parts of an outdoor figure are related; (3) check results obtained by other methods.

5. Solve: (*a*) $\frac{x}{10} = \frac{36}{48}$ (*d*) $\frac{y}{18} = 1.4$ (*g*) $\frac{p}{8} = \frac{4}{5}$

(*b*) $\frac{a}{6} = \frac{12}{7}$ (*e*) $\frac{b}{5} = \frac{2}{3}$ (*h*) $\frac{x}{3} = \frac{7}{8}$

(*c*) $\frac{n}{4} = \frac{6}{8}$ (*f*) $\frac{c}{3} = \frac{5}{6}$ (*i*) $\frac{l}{6} = \frac{7}{12}$

Help No. IV. Telling What You Must Do

Tell how you would solve each of the following kinds of problems, assuming that you knew all the necessary facts.

1. How do you find the average mark your class makes on a mathematics test?

2. How do you find the margin and the net profit made in any business for a given period?

3. State how Mrs. Curtis can find what per cent of her monthly income she spends for rent.

4. How do you find a number when you know a certain per cent of it?

5. How do you find what per cent one number is of another?

6. How can a common fraction be changed to a decimal fraction? to per cent?

7. How do you find the net cost of an article on which a discount is allowed?

8. How do you decide whether you should purchase a large or a small quantity of some goods?

9. What would you want to know about two tires of different makes in order to decide which of them is the more economical to buy for your car?

10. How do you find the total discount on a bill of goods when two discounts are allowed?

11. How do you find a *per cent* of *increase?* a *per cent* of *decrease?* Illustrate each.

1. A farmer sold 55 bushels of Roman Beauty apples at $4.75 a barrel. Each barrel contained 2 bushels and 3 pecks. How much did he receive for the apples?

2. Sarah and Ellen bought berries at 80 cents a peck. They put them in quart boxes and sold them at 15 cents a box. What did they clear on each peck, if they paid 2 cents apiece for the boxes they used?

3. What would you have to pay for enough varnish to cover a floor 18 ft. wide and 20 ft. long if a pint covers 25 sq. ft. and the varnish costs 31 cents a pint or 54 cents a quart?

4. James and his sister Madge decided to plant popcorn in their garden. The rows were to run crosswise and be 2 ft. 6 in. apart and the hills 2 ft. apart in the rows. The outer rows were 1 ft. from the edge of the garden. Their garden was 52 ft. wide and 62 ft. long. How many hills of popcorn did they need to plant?

5. Alice made a frame for her flower bed. The frame was something like a box without a top or a bottom. She wanted the inside dimensions to be 22 in. by 30 in. The frame was made from a board 1 in. thick, 8 in. wide, and 12 ft. long. How much of the board was not used?

6. How many garden plots, each 20 ft. by 36 ft., can be made on a rectangular lot that is 220 ft. by 396 ft.?

7. How much change should I receive from a 5-dollar bill after paying for $5\frac{1}{2}$ pounds of coffee at 38 cents a pound and $2\frac{1}{4}$ pounds of honey at 36 cents a pound?

8. Mr. Donovan receives a salary of $2500 a year. His rent costs him $40 a month, and his other expenses are $1200. He saves the remainder. What per cent of his salary does he save?

9. Tim is 5 feet tall. How tall will he be two years from now if he grows 3 % of his present height the first year and 3 % of his height a year from now during the second year?

10. Mrs. A, Mrs. B, and Mrs. C sent for 2 barrels of apples (1 barrel = $2\frac{3}{4}$ bushels). Mrs. A took 2 bushels, Mrs. B took $1\frac{1}{2}$ bushels, and Mrs. C took the rest. How much should each pay if the apples cost $5.60 a barrel and 50 cents a barrel was spent on cartage?

11. A boy began work Wednesday morning, March 12, and worked every day of the week except Sunday until the evening of May 12. How much did he earn at $1.25 a day?

12. An eighth-grade class consisting of 36 pupils plans to give an ice-cream party. One quart of ice cream is allowed for 6 pupils. At 70 cents a gallon, the ice cream will cost ___?___ dollars.

13. A train travels a distance of 121 miles in 5.5 hours. At the same rate of speed how long would it take the train to make a run of 181.5 miles?

14. John rode 6 miles on his bicycle in 48 minutes. He rode at the rate of ___?___ miles an hour.

15. William's step is about 2 ft. 6 in. long. If he takes 2 steps a second, how long will it take him to walk a mile?

16. Two boys agreed to strip the sod from a tennis court for 3 cents a square foot, and to roll the ground carefully for $.005 a square yard. The tennis court was 78 feet by 36 feet. How much did they earn if paid at these rates?

17. One-half gallon of ice cream will serve 16 guests. How many quarts would be needed to serve 40 guests?

18. At a special "one-quarter-off" sale, a dress sold for $36. What was the regular price of the dress?

1. When the shadow of a boy $4\frac{1}{2}$ ft. tall is 6 ft. long, the shadow of a tree is 60 ft. long. How tall is the tree?

2. Two triangles, ABC and $A'B'C'$, are similar. $AB = 8$, $A'B' = 6$, and $A'C' = 5$. How long is AC?

3. Sketch two triangles, ABC and XYZ, in which $\angle C = \angle Z$, $AC = 40$, $BC = 44$, $XZ = 30$, and $YZ = 33$. Is XY equal to $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{2}{3}$, $\frac{3}{4}$, or $\frac{4}{11}$ of AB? Why?

4. Are two triangles congruent if two angles of one are equal to two angles of the other? Illustrate your answer with a sketch.

5. The ratio of x to 8 equals the ratio of 25 to 40. What is the value of x?

6. In $\triangle ABC$ and XYZ, $\angle A = \angle X$ and $AB = XY$. Sketch the triangles. Are they congruent? Are they similar? Why?

7. Make a sketch to show how you could use congruent triangles to find the distance from A to B, even though you could not walk from A to B.

Solve and check these equations:

8. $3x = 18$

9. $x - 5 = 9$

10. $\frac{x}{3} = \frac{4}{5}$

11. $10 = x + 2$

12. $\frac{x}{2} = \frac{3}{8}$

13. $\frac{x}{7} = \frac{1}{2}$

14. Find the area of a triangle whose base is 4 and whose altitude is 3.

15. Write the formula for finding the circumference of a circle when the diameter is known.

16. 16% of 24 is __?__.

17. 18 is __?__ % of 27.

18. 12 is 50% of __?__.

19. 12 is 200% of __?__.

20. 1.5% of 14 is __?__.

21. 8 is 75% of __?__.

1. When the shadow of a pole $6\frac{3}{4}$ ft. tall is 18 ft. long, the shadow of a tree is $58\frac{1}{2}$ ft. long. How tall is the tree?

2. In $\triangle ABC$ and XYZ, $\angle A = \angle X$, $\angle C = \angle Z$, $AB = 20$, and $XY = 15$. Compare AC and XZ; BC and YZ.

3. In $\triangle ABC$, $AB = 96$, $AC = 80$, and $BC = 72$. D is on AC and E is on BC. $CD = 12$ and $CE = 10.8$. Find DE.

4. Sketch two similar triangles, ABC and $A'B'C'$, in which $AB = 2\,A'B'$. Compare their areas. (You may need to measure the altitudes of the triangles.)

5. Make a sketch to illustrate how you would use congruent triangles to find the distance across a river. What parts of the triangles would you have to know?

6. The ratio of segment a to segment b is 3 to 4. If segment b is 10 inches long, how long is segment a?

7. A freight train leaving Albany for New York (150 miles) at 8 A.M., ran at the rate of 25 miles an hour. At 11 A.M. a fast express followed at the rate of 50 miles an hour. Did the express train overtake the freight train? If so, in how many hours?

8. 3.5% of 12 is __?__.

9. $\frac{1}{2}$% of 24 is __?__.

10. 300% of 16 is __?__.

11. 18 is 75% of __?__.

12. 14 is __?__% of 12.

13. 16 is 7.5% of __?__.

Solve and check these equations:

14. $\frac{x}{3} = \frac{2\frac{1}{2}}{4}$

15. $6y - 2 = 4\frac{1}{2}$

16. $\frac{2y}{4} = \frac{8}{1}$

17. $3p + .5p = 350$

18. Write the formula for finding the circumference of a circle when the radius is known.

1. From AC to BC of a $\triangle ABC$ draw a line DE parallel to AB. Is $\triangle DEC$ similar to $\triangle ABC$? Why?

2. If D, in Ex. 1, is midway between A and C, will E be midway between B and C? Why?

3. The area of $\triangle DEC$ (Ex. 2) is what part of the area of $\triangle ABC$?

4. A treasure is buried in a garden. It is 10 ft. from the east side of the garden. Is it definitely located? Why? If you knew that the treasure is 16 ft. from the southeast corner of the garden and 10 ft. from the east side, could you locate it precisely? How?

5. A treasure is buried 18 ft. from one tree and 20 ft. from another tree. The trees are 25 ft. apart. Make a scale drawing to show how you would locate the treasure.

6. A rectangle is 6 ft. wide and 8 ft. long. The width of a similar rectangle is 2 ft. What is its area?

Solve the following equations:

7. $\frac{2x}{5} = \frac{7}{8}$ 8. $\frac{3y}{4} = 2\frac{1}{2}$ 9. $\frac{5y}{2} = 15$

10. The base of one triangle is $\frac{3}{4}$ of the base of the other. They have equal areas. Compare their altitudes.

11. In two similar triangles, ABC and $A'B'C'$, $AB = 6\frac{1}{2}$, $A'B' = 8\frac{3}{4}$, and $AC = 7\frac{1}{4}$. How long is $A'C'$?

12. Make a sketch to show how you would use an isosceles right triangle to find the height of a flagpole.

13. 165% of 30 = ___?___.

14. $\frac{1}{4}$% of 28 = ___?___.

15. 16 is 3.5% of ___?___.

16. 16 is ___?___% of 13.

17. 18 is 150% of ___?___.

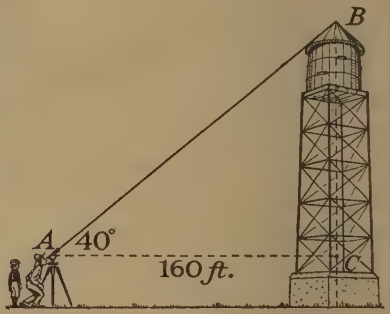
18. 3.5 is ___?___% of 6.5.

CHAPTER VI

USING THE RIGHT TRIANGLE

Finding the Height of a Tower

By Ewing Galloway, N. Y.



Tom went with his Uncle Mack to measure the height of a tower, as shown above. They measured angle A and the ground line AC. Angle A was 40° and AC 160 ft. How large was $\angle C$?

Uncle Mack then looked in his mathematics book and said: "In a right triangle the side opposite a 40-degree angle is .84 as long as the side next to it. So the height of this tower, BC, is .84 as long as AC, or .84 $\times$ 160 ft."

Tom: That is interesting. Do you mean that in *any* right triangle having a 40-degree angle, the side opposite the 40-degree angle is *always* .84 as long as the side next to the 40-degree angle?

Uncle Mack: Tom, you need to experiment a little. Draw some right triangles of various sizes, making sure that each has an angle of 40°. Measure the side opposite the

40-degree angle and the side next to it. Then divide, and see what you get. You must learn to discover facts for yourself.

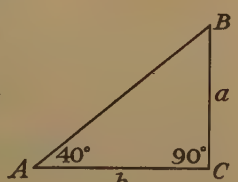
1. Measure BC and AC in the right triangle on the preceding page. Then divide the length of BC by the length of AC. What result do you get?

2. Draw a right triangle ABC, in which $\angle A = 40°$ and $\angle C = 90°$. Choose any convenient length for AC. Then find the ratio of BC to AC (divide the length of BC by the length of AC). Compare your result with the results of your classmates. Do your results agree rather closely with Uncle Mack's?

3. Were your triangles (Ex. 2) similar? Why? Should your results in Ex. 2 agree even though you used different-sized triangles?

4. Are all 40-degree right triangles similar? Do the results of Ex. 2 agree with what you have learned previously regarding the ratios of corresponding sides of similar triangles?

5. Would you say without measuring that side a in this triangle is .84 as long as side b? Why?



6. Which of these statements apply to the figure? (1) $a = .84\,b$ (2) $\frac{a}{b} = .84$ (3) $\frac{b}{a} = .84$ (4) $b = .84\,a$

7. In right triangle ABC, $\angle A = 40°$ and $\angle C = 90°$. (See the figure above.) If b is 100, how long is a?

8. If b (Ex. 7) is 8.4, what is a?

9. If a (Ex. 7) is 168, what is b?

10. If a (Ex. 7) is 4.2, what is b?

1. In a right triangle ABC, $\angle A = 40°$ and $\angle C = 90°$. If AC is 50, what is BC? If AC is 125, what is BC?

2. If BC in Ex. 1 is 200, what is AC? What is AC if BC is 60?

3. *An experiment.* Construct a right triangle ABC, in which $\angle A = 30°$ and $\angle C = 90°$. Choose any convenient length for AC. Without measuring, would you say that BC is about $\frac{1}{2}$, $\frac{1}{4}$, or $\frac{3}{5}$ of AC? Now measure AC and BC and find the ratio of BC to AC. Compare your result with the results of your classmates. (Should all of you obtain closely agreeing results? Why?)

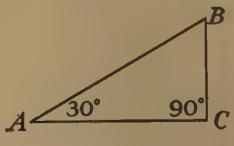
4. Find the average of the results obtained by members of the class.

If your work in Ex. 3 was carefully done, you found that the side opposite a 30-degree angle in a right triangle, regardless of the size, is .58 as long as the side next to the 30-degree angle.

5. Which of these statements apply to Ex. 3?

(1) $\frac{BC}{AC} = .58$ (2) $BC = .58\ AC$ (3) $\frac{AC}{BC} = .58$

6. If AC in this figure represents 100 yards, how many yards does BC represent?



7. If BC represented 100 yards, what length would AC represent?

8. Sketch a triangle PQR, in which $\angle R = 30°$, $\angle Q = 90°$, and $RQ = 28$. Find PQ.

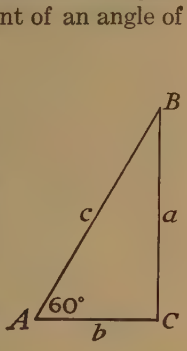
9. Sketch a triangle DEF, in which $\angle D = 30°$, $\angle E = 60°$, and $DF = 12.6$. Find EF.

10. Would you say that the side opposite a 30° angle in *any* triangle is .58 as long as the side next to it?

We have learned that in a right triangle the ratio of the side opposite a 40-degree angle to the side next to it is .84. For brevity we use the word *tangent* in place of "ratio of side opposite · · · to side next to · · · " Thus the tangent of an angle of 40° = .84. What is the tangent of an angle of 30°?

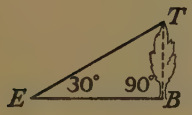
1. Would you say that the tangent of 60° is less than 1 or more than 1?

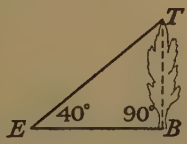
2. In order to find the tangent of $\angle A$ in this figure, you would need to measure what lines? Measure them, and find the ratio of a to b. What is the tangent of a 60-degree angle?

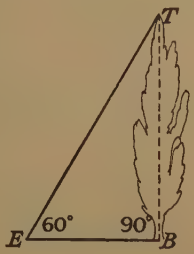


3. The tangent of 60°, to the nearest hundredth, is 1.73. How close were you?

4. In which of these cases would you say that the height of the tree BT is .84 as long as the ground line EB? .58 as long? 1.73 as long?







5. Find BT in each figure, assuming the length of EB to be 80 ft.; 60 ft.; 115 ft.

6. Complete: If the length of EB does not change, and the height of the tree increases, then the tangent of angle E __?__.

Using a Table of Tangents

The tangents of angles of various sizes have been carefully computed and recorded for reference. Here is a table of tangents from 1° to 89°, correct to hundredths. (For very accurate work, we would need the tangents correct to five or six decimal places.)

TABLE OF TANGENTS

ANGLE	TAN-GENT	ANGLE	TAN-GENT	ANGLE	TAN-GENT	ANGLE	TAN-GENT	ANGLE	TAN-GENT
1°	.02	19°	.34	37°	.75	55°	1.43	73°	3.27
2	.03	20	.36	38	.78	56	1.48	74	3.49
3	.05	21	.38	39	.81	57	1.54	75	3.73
4	.07	22	.40	40	.84	58	1.60	76	4.01
5	.09	23	.42	41	.87	59	1.66	77	4.33
6	.10	24	.44	42	.90	60	1.73	78	4.70
7	.12	25	.47	43	.93	61	1.80	79	5.14
8	.14	26	.49	44	.96	62	1.88	80	5.67
9	.16	27	.51	45	1.00	63	1.96	81	6.31
10	.18	28	.53	46	1.03	64	2.05	82	7.12
11	.19	29	.55	47	1.07	65	2.14	83	8.14
12	.21	30	.58	48	1.11	66	2.25	84	9.51
13	.23	31	.60	49	1.15	67	2.36	85	11.43
14	.25	32	.62	50	1.19	68	2.48	86	14.30
15	.27	33	.65	51	1.23	69	2.61	87	19.08
16	.29	34	.67	52	1.28	70	2.75	88	28.64
17	.31	35	.70	53	1.33	71	2.90	89	57.29
18	.32	36	.73	54	1.38	72	3.08		

1. What is the tangent of 30°? 40°? 50°? 60°? 80°?

2. What is the angle whose tangent is .27? .36? 2.75?

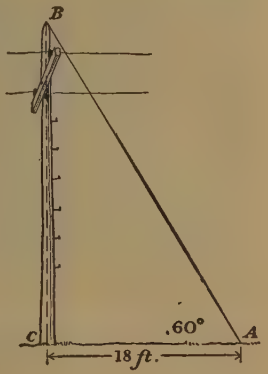
3. Estimate the size of the angle whose tangent is 1.05; 1.30; 6.

4. Why do you suppose the table does not include the tangent of 90°?

5. If an angle is doubled, would you say that the tangent of the angle is also doubled? Check for 15°; for 20°; for 40°.

In each of the following problems, be sure to sketch the right triangle that is suggested by the problem. You must know the size of one acute angle. Write an equation using the tangent. Look up the tangent in the table. Solve the equation.

1. The brace wire AB of a telephone pole BC, shown here, makes with the ground an angle of 60°. It enters the ground 18 ft. from the foot of the pole. Find the height of the pole BC.



SOLUTION

(1) $\frac{BC}{AC} = \text{tangent } 60°$

(2) $\frac{BC}{18} = 1.73$

(3) $BC = 18 \times 1.73$

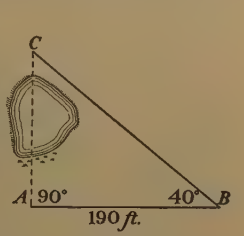
(4) $BC = 31.14$

(*a*) Where do we get the 1.73?

(*b*) How does equation (3) follow from equation (2)?

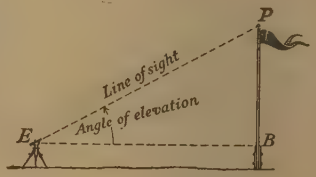
2. The brace wire of a telephone pole makes with the ground an angle of 50°. It enters the ground 20 ft. from the pole. Find the height of the pole.

3. Mary's father owns a triangular-shaped lot, as shown here. He found the length of AC by means of the tangent ratio. Can you? This is like a problem which you solved by using a scale drawing. Note that you now have a far better method.



4. If angle A in Ex. 3 were 5° larger, how much larger would AC be?

1. The figure below shows what a surveyor has in mind when he talks about an *angle of elevation*. In order to measure the angle of elevation of the tip of a flagpole, a transit is placed at any point E (see the figure). The telescope is first pointed horizontally toward the flagpole and the point B marked on the flagpole on a level with the transit. The telescope is then turned until the top of the flagpole, P, is in the line of sight. The angle PEB is the *angle of elevation* of point P from point E, the point of observation.



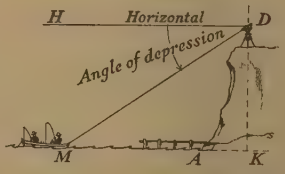
Illustrate an angle of elevation in your classroom.

What is PB if EB is 50 and angle E is 35°?

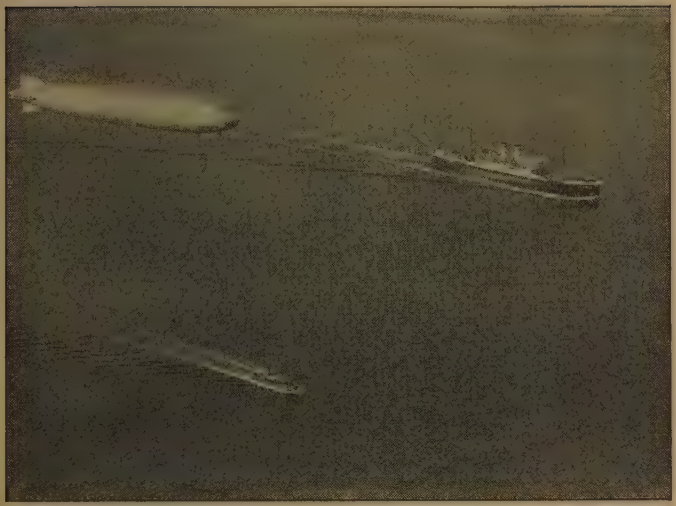
2. When the angle of elevation of the sun is 35°, a building casts a shadow 80 feet long on level ground. How high is the building?

3. What is the angle of elevation of the sun when a boy 4 ft. tall casts a shadow 6 ft. long? (What is the angle whose tangent is $\frac{4}{6}$ or .67?)

4. The angle at D in this figure shows what is meant by an *angle of depression*. You are to imagine yourself at D, an elevated point, with an instrument for measuring angles. When the telescope is turned from the horizontal position until it points toward the men in the boat, it turns through the angle of depression, HDM.



5. Mr. Guard knows that the bluff at D (Ex. 4) is 140 ft. above water. The angle of depression of the boat as seen from D is 42°. How far from A is the boat if AK is 12 ft.?



By Ewing Galloway, N. Y.

1. When the *Graf Zeppelin* in the picture was directly over the small ship, an observer on the large ship, 460 yd. away, observed the angle of elevation of the nose of the *Zeppelin* to be 35°. How high was the *Zeppelin?*

2. A boat passes a tower on the shore. The searchlight in the tower is 240 ft. above sea level. Find how far the boat is from the shore, if the angle of depression of the boat from the top of the tower is 50°.

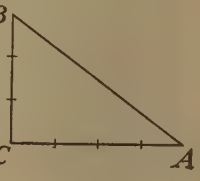
Solving problems by using *tangents* is one of the methods of *trigonometry*. (Trigon-o-metry = *triangle measurement*.) Trigonometry includes a study of other relations between the angles and the sides of a triangle. It helps us in the study of electricity, astronomy, surveying, and mechanics. It is an interesting and practical part of high school and college mathematics. Most pupils enjoy trigonometry very much.

The exercises that follow will help you to understand one of the great truths of mathematics, which enables us to find one side of a right triangle when the other two sides are known.

Pythagoras (Pythăg'oras), a Greek mathematician, who lived about 2500 years ago, was the first one to prove it. It is called the ***Rule of Pythagoras.***

1. (*a*) Draw a right triangle, as shown here, in which BC is 3 units long and AC is 4 units long. Make your measurements very carefully.

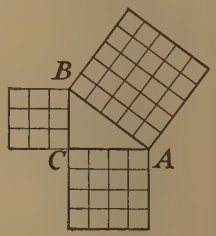
(*b*) Measure the length of the *hypotenuse* AB, using the same unit of measure that you used in laying off BC and AC. This shows that AB is ___?___ units long.



The *hypotenuse* of a right triangle is the side that is opposite the right angle.

2. (*a*) Using the triangle you obtained in Ex. 1, construct a square with BC as one of its sides. Construct another square with AC as one of its sides, as in the next figure.

(*b*) Now construct a square with AB as one of its sides.



3. Complete the following:

The square on BC contains ___?___ square units.

The square on AC contains ___?___ square units.

In these two squares together there are ___?___ square units.

The square on AB contains ___?___ square units.

4. Compare the number of square units in the square on the hypotenuse with the sum of the numbers of square units in the other two squares.

5. Do you think that in any other right triangle a square constructed on the hypotenuse would equal the sum of the squares constructed on the other two sides? How could you test your answer?

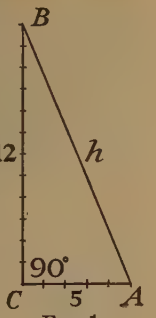


FIG. 1

6. (*a*) Construct a right triangle, as shown in Figure 1, in which AC is 5 units and BC is 12 units. Then measure the hypotenuse AB. If you are an accurate worker, you will find that AB is exactly a whole number of units long.

(*b*) In Figure 1, what is the square of AC? the square of BC? of AB? Show that $(AC)^2 + (BC)^2 = (AB)^2$.

7. See if you can find the length of the hypotenuse AB in Figure 2.

8. Which of the following equations apply to the triangle shown in Figure 2?

(1) $h^2 = 6^2 + 8^2$
(2) $h^2 = 100$
(3) $h = 10$

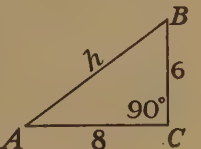


FIG. 2

9. The results obtained in your experiments will help you to decide which of the following statements applies to the triangle shown in Figure 3:

(*a*) $h^2 = a^2 + b^2$ (*b*) $h = a + b$

The preceding exercises suggest the following general truth:

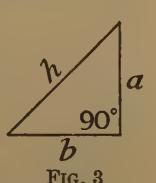


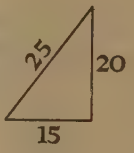
FIG. 3

In a right triangle, the square of the hypotenuse equals the sum of the squares of the other two sides.

This is called the Rule of Pythagoras. It applies only to right triangles. This rule is used to find one side of a right triangle when the other two sides are known.

You have just learned that in any right triangle $h^2 = a^2 + b^2$. It is true also that *a triangle in which the sum of the squares of two sides equals the square of the third side is a right triangle.*

This second relation may be used to test any triangle to see whether it is a *right* triangle. For example, to test the triangle shown here, we should see whether $25^2 = 15^2 + 20^2$. Does it? If so, the triangle is a __?__ triangle.



1. The sides of a triangle are 9, 12, and 15. See whether the triangle is a right triangle.

2. The sides of a triangle are 2, 3, and 4. Test it to see whether it is a right triangle.

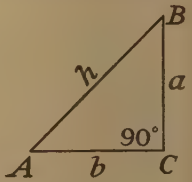
3. A triangle is formed by fastening together three sticks, 12″, 16″, and 20″ long. Is it a right triangle?

4. Test the triangle the sides of which are 8, 10, and 12 to see whether it is a right triangle.

5. Think of h, a, and b as the three sides of a triangle. If you know that $h^2 = a^2 + b^2$, then it follows that the triangle is a __?__ triangle. If you know that a triangle is a right triangle with h as the hypotenuse, then it follows that $h^2 = a^2 + b^2$.

6. Examine the following equations to see which apply to the triangle shown here.

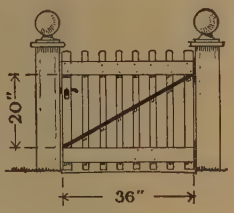
(1) $a^2 + b^2 = h^2$
(2) $a^2 = h^2 - b^2$
(3) $b^2 = h^2 - a^2$



7. What can you do to the equation $a^2 + b^2 = h^2$ to obtain the equation $a^2 = h^2 - b^2$?

8. How can you change the equation $a^2 + b^2 = h^2$ into the equation $b^2 = h^2 - a^2$?

Ralph wished to buy an iron brace for a gate that he was planning to make. He knew that the brace would form the hypotenuse of a right triangle, the base of which was to be 36 in. and the height 20 in. Do you know how to *calculate* the length of the brace? This is the way Ralph found how long the brace should be:



(1) $h^2 = 36^2 + 20^2$ (3) $h^2 = 1696$

(2) $h^2 = 1296 + 400$ (4) $h = 41.2+$

How did he know that $h^2 = 36^2 + 20^2$? After he knew that $h^2 = 1696$, how did he find that $h = 41.2$? Ralph knew a principle in mathematics which you may have learned. He could find a number which, multiplied by itself, gives 1696. Does $41.2 \times 41.2 = 1696$? The 41.2 is called the ***square root*** of 1696 to the nearest tenth.

We say that 4 is the square root of 16 because $4 \times 4 = 16$. Similarly, 8.5 is the square root of 72.25 because $8.5 \times 8.5 = 72.25$. The square root of h^2 is h, because h times $h = h^2$.

What is the square root of 25? of 49? of 81? 144? $\frac{1}{4}$? .25? .64? 900? 2500?

We have a sign, $\sqrt{\ }$, for the square root of a number. Thus, $\sqrt{25}$ is read, "the square root of 25."

The square root of any given number is that number which, when multiplied by itself, produces the given number.

1. What is the square root of 4? 169? 225? h^2?

2. Complete: Does 28×28 equal 784? If so, then 28 is the __?__ of 784.

3. Complete: Does 8.2×8.2 equal 67.24? If so, then 8.2 is the __?__ of 67.24.

4. What is the square root of 400? 1600? 3600? 4900? 6400? 8100?

1. Refer to the Table of Squares and Square Roots of Numbers on the following page to find—

(*a*) the square of 21.

(*b*) the square root of 21. (The table gives the square roots to the nearest thousandth.)

(*c*) the square of 69.

(*d*) the square root of 69.

(*e*) the square of 150.

(*f*) the square root of 150.

2. To find the square of 2.5 (not given in the table), look for the square of 25. Show that the square of 2.5 is 6.25. (Use common sense in placing the decimal point.)

3. What is the square of 3.5? 4.6? 12.4? 14.8?

4. What is the square root of 1024? (Find 1024 in the second column.) the square root of 7225? of 7921?

5. Find, to the nearest hundredth, each of the following: (*a*) $\sqrt{28}$ (*b*) $\sqrt{140}$ (*c*) $\sqrt{19}$ (*d*) $\sqrt{85}$ (*e*) $\sqrt{46}$

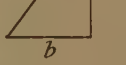
6. If $n^2 = 2209$, what does n equal?

7. What is n if $\sqrt{n} = 4.243$?

8. What is n when $n^2 = 20{,}164$? when $n^2 = 2401$?

9. What is the side of a square whose area is (*a*) 1089? (*b*) 4761? (*c*) 7744? (*d*) 11,664?

10. In a right triangle, as shown here, what is h when $a = 4$ and $b = 3$? What is h when $a = 6$ and $b = 8$? when $a = 4.5$ and $b = 6$?



11. Mary said that the length of one side of a square whose area is 10 square inches is 3.16 inches. John thought that one side was 3.162 inches. Who was right?

12. Find the perimeter of a square whose area is 50 square inches.

Table of Squares and Square Roots of Numbers

No.	Square	Square Root	No.	Square	Square Root	No.	Square	Square Root
1	1	1.000	51	2,601	7.141	101	10,201	10.050
2	4	1.414	52	2,704	7.211	102	10,404	10.100
3	9	1.732	53	2,809	7.280	103	10,609	10.149
4	16	2.000	54	2,916	7.348	104	10,816	10.198
5	25	2.236	55	3,025	7.416	105	11,025	10.247
6	36	2.449	56	3,136	7.483	106	11,236	10.296
7	49	2.646	57	3,249	7.550	107	11,449	10.344
8	64	2.828	58	3,364	7.616	108	11,664	10.392
9	81	3.000	59	3,481	7.681	109	11,881	10.430
10	100	3.162	60	3,600	7.746	110	12,100	10.488
11	121	3.317	61	3,721	7.810	111	12,321	10.536
12	144	3.464	62	3,844	7.874	112	12,544	10.583
13	169	3.606	63	3,969	7.937	113	12,769	10.630
14	196	3.742	64	4,096	8.000	114	12,996	10.677
15	225	3.873	65	4,225	8.062	115	13,225	10.724
16	256	4.000	66	4,356	8.124	116	13,456	10.770
17	289	4.123	67	4,489	8.185	117	13,689	10.817
18	324	4.243	68	4,624	8.246	118	13,924	10.863
19	361	4.359	69	4,761	8.307	119	14,161	10.909
20	400	4.472	70	4,900	8.367	120	14,400	10.954
21	441	4.583	71	5,041	8.426	121	14,641	11.000
22	484	4.690	72	5,184	8.485	122	14,884	11.045
23	529	4.796	73	5,329	8.544	123	15,129	11.091
24	576	4.899	74	5,476	8.602	124	15,376	11.136
25	625	5.000	75	5,625	8.660	125	15,625	11.180
26	676	5.099	76	5,776	8.718	126	15,876	11.225
27	729	5.196	77	5,929	8.775	127	16,129	11.269
28	784	5.292	78	6,084	8.832	128	16,384	11.314
29	841	5.385	79	6,241	8.888	129	16,641	11.358
30	900	5.477	80	6,400	8.944	130	16,900	11.402
31	961	5.568	81	6,561	9.000	131	17,161	11.446
32	1,024	5.657	82	6,724	9.055	132	17,424	11.489
33	1,089	5.745	83	6,889	9.110	133	17,689	11.533
34	1,156	5.831	84	7,056	9.165	134	17,956	11.576
35	1,225	5.916	85	7,225	9.220	135	18,225	11.619
36	1,296	6.000	86	7,396	9.274	136	18,496	11.662
37	1,369	6.083	87	7,569	9.327	137	18,769	11.705
38	1,444	6.164	88	7,744	9.381	138	19,044	11.747
39	1,521	6.245	89	7,921	9.434	139	19,321	11.790
40	1,600	6.325	90	8,100	9.487	140	19,600	11.832
41	1,681	6.403	91	8,281	9.539	141	19,881	11.874
42	1,764	6.481	92	8,464	9.592	142	20,164	11.916
43	1,849	6.557	93	8,649	9.644	143	20,449	11.958
44	1,936	6.633	94	8,836	9.695	144	20,736	12.000
45	2,025	6.708	95	9,025	9.747	145	21,025	12.042
46	2,116	6.782	96	9,216	9.798	146	21,316	12.083
47	2,209	6.856	97	9,409	9.849	147	21,609	12.124
48	2,304	6.928	98	9,604	9.899	148	21,904	12.166
49	2,401	7.000	99	9,801	9.950	149	22,201	12.207
50	2,500	7.071	100	10,000	10.000	150	22,500	12.247

(1) George wished to find the square root of 200. Could he get it from the Table of Squares and Square Roots?

(2) He noticed in the table that $\sqrt{196}$ is 14 and $\sqrt{225}$ is 15. So he decided that $\sqrt{200}$ must be nearer 14 than 15. Do you agree?

(3) He decided that $\sqrt{200}$ must be about 14.1 or 14.2 or 14.3. So he decided to square each and write the results.

$14.1^2 = 198.8$
$14.2^2 = 201.6$

(4) After he squared 14.2, he noticed that 14.1^2 is nearer 200 than 14.2^2. So he said that $\sqrt{200} = 14.1$ to the nearest tenth.

(5) Grace wished to find $\sqrt{77.6}$. She knew that $\sqrt{77.6}$ is nearer $\sqrt{81}$ than $\sqrt{64}$. How did she know that?

(6) Grace thought: "$\sqrt{77.6}$ must be about 8.7 or 8.8 or 8.9." She squared each and recorded her results. Then she decided that $\sqrt{77.6} = 8.8$ to the nearest tenth. Do you agree?

$8.7^2 = 75.69$
$8.8^2 = 77.44$
$8.9^2 = 79.21$

(7) Albert wished to find $\sqrt{1000}$. From the table he saw that $\sqrt{1000}$ is nearer __?__ than __?__. So he decided that $\sqrt{1000}$ is about 31.6 or 31.7. Compare 31.6^2 and 31.7^2 with 1000. What, then, is $\sqrt{1000}$, to the nearest tenth?

Find to the nearest tenth each of the following:

1. $\sqrt{204}$
2. $\sqrt{330}$
3. $\sqrt{621}$
4. $\sqrt{1008}$
5. $\sqrt{1610}$
6. $\sqrt{7400}$
7. $\sqrt{38.25}$
8. $\sqrt{50.48}$
9. $\sqrt{65.02}$
10. $\sqrt{6000}$
11. $\sqrt{20.38}$
12. $\sqrt{9800}$

13. How large should each side of a square be made if its area is to be 300 square feet?

14. Refer to Ralph's problem, page 193. Can you now find the length of the brace? ($\sqrt{1696} =$ __?__.)

Since you have learned the important truth known as the Rule of Pythagoras and how to find square roots of numbers, you can now solve many practical problems.

1. The bottom of a 20-foot ladder is 8 feet from the wall of a building against which the ladder is leaning. How high up the wall does the ladder reach?

2. A rope stretched from the top of a pole 27 feet high reaches the ground 10 feet from the foot of the pole. How long is the rope?

3. Find the hypotenuse of the triangle shown in Figure 1.

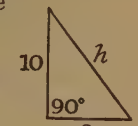


FIG. 1

4. What is the diagonal of a square whose sides are 20 ft.? (Figure 2.)

5. How much shorter is AC than $AB + BC$? (Figure 2.)

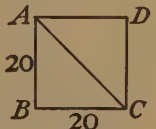


FIG. 2

6. Find the combined area of the triangle and the square shown in Figure 3.

7. The sides of a triangle are represented by $3x$, $4x$, and $5x$. Is it a right triangle? Will it be a right triangle if $x = 2$? if $x = 3$? if $x = \frac{1}{2}$?

8. What is the diagonal of a rectangular plot of ground (a) whose sides are 20 ft. and 30 ft.? (b) whose sides are 10 ft. and 30 ft.?

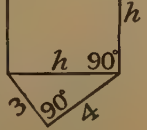


FIG. 3

9. In this chart a, b, and h refer to altitude, base, and hypotenuse, respectively, of a right triangle. What is the missing number in each column?

a =	3	6	?	?	1	30	10	5
b =	4	8	3	7	2	40	?	?
h =	?	?	5	13	?	?	20	10

10. (*a*) An extension ladder 60 feet long is placed so that the foot of the ladder is 15 feet from the base of the building. How high up does the ladder reach?

(*b*) If I move the foot of the ladder out so that it is 20 feet from the building, what happens to the top of the ladder? How high up does the ladder then reach?

(*c*) If I then move the foot of the ladder in toward the wall so that it is only 12 feet from the wall, do I raise or lower the top of the ladder? How much?

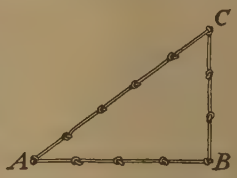
(*d*) If the street is only 25 feet wide, can I place the ladder so that it will just reach a window which is 40 feet above the ground?

11. How long is the diagonal of a square whose side is 8 feet? the side of a square whose diagonal is 8 feet?

12. The brace wire of a pole is to be attached 32 feet above the ground and to enter the ground 20 feet from the pole. How much wire will be needed for the brace wire, if 8 feet is used for fastening the wire to the pole and to the anchor?

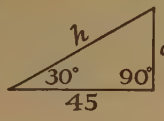
13. Could a 30-inch cane be placed in the bottom of a trunk 18 inches wide and 24 inches long, inside dimensions?

14. More than 3000 years ago the Egyptians used an interesting method of making square (right-angled) corners. They stretched a rope tightly to form a triangle. The rope was marked off into 12 equal parts, as shown here.



One side of the triangle contained 3 of the equal parts of the rope, another side 4 of the parts, and the third side 5 of them. Do you see that the method gave them a right angle? You might try this method of making a square corner.

1. (*a*) Look at this figure and write an equation to show that you know the meaning of the tangent of an angle of 30°.



(*b*) Write an equation to show that you can apply the Rule of Pythagoras to this figure.

(*c*) What will represent the area of this figure?

(*d*) What will represent the perimeter of this triangle?

(*e*) What test can be used to determine whether a triangle is a right triangle?

(*f*) If you can find the hypotenuse of the right triangle shown here, you have learned the important facts of this chapter.

2. Which of the following equations are true with reference to this figure?

(1) $\tan 35° = \frac{a}{b}$

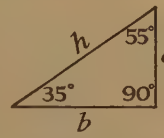
(2) $\tan 55° = \frac{b}{a}$

(3) $b^2 = h^2 - a^2$

(4) $h = a + b$

(5) $h = \sqrt{a^2 + b^2}$

(6) Area $= \frac{1}{2}\,ab$



3. A baseball diamond is a square 90 ft. on a side. Use the Rule of Pythagoras to find the distance from home plate to second base. What is the distance from first base to third base? Is your answer the same as the answer you get by reworking Ex. 1 on page 174?

4. Mr. Perry fenced a square lot that contained 10 acres. How many rods of fencing did he need? (An acre is 160 square rods.) What was the cost of the fencing at \$.36 a rod?

5. A tower 120 ft. high is built on the bank of a river. The angle of depression of a point on the opposite bank, measured from the top of the tower, is 32°. How wide is the river?

6. Measure carefully the length and the width of a sheet of paper. Imagine that a diagonal of the sheet is drawn. Use the tangent ratio to find the angle that the diagonal makes with each edge of the paper. Then draw the diagonal, and measure the angles with a protractor. Compare the results.

A freehand sketch of each figure will help you to solve the following problems:

7. At a time when the angle of elevation of the sun was 41°, the shadow of a certain building was found to be 120 ft. long. How high was the building?

8. When the angle of elevation of the sun is 45°, what relation exists between the height of an object and the length of the shadow it casts?

NOTE. In Exercises 9, 10, and 11 assume that side a is opposite angle A, etc.; also that angle C is a right angle.

9. Find a in a right triangle if $A = 48°$ and $b = 42$ ft.

10. In a right triangle, $A = 64°$ and $b = 82$ yd. Find a.

11. What is a in a right triangle if $B = 35°$ and $b = 56$ in.?

12. Sketch a square, calling a side s and the diagonal d. Which of these statements apply to the figure you drew?

(a) The area, A, $= s^2$. (b) $A = \frac{d^2}{2}$. (c) $s = \frac{1}{2}d$.

(d) The tangent of the angle formed by d and s is 1.

(e) $d = s\sqrt{2}$. (f) $d^2 = 2s^2$.

13. If d in Ex. 12 is 20, what is s? What is A?

14. Find the number of rods of fence required to enclose a square field containing 40 acres. (1 acre = 160 sq. rd.)

15. An acre which is in the form of a square contains __?__ square rods, or __?__ square feet. Each side of such an acre is __?__ rods long, or __?__ feet long.

Estimate each of the following amounts. Then *compute* the exact result for each example. If your estimate in each exercise does not differ from the exact result by more than the number that appears in parentheses after the exercise, you get one "point" credit for it. Good eighth-grade pupils have made 10 points. Can you?

1. In two similar triangles ABC and XYZ, $AB = 144$, $BC = 116$, and $XY = 71$. Find the length of YZ. (Limit of error: 1)

2. The cost of $12\frac{1}{2}$ yards of goods at 48 cents a yard. (Zero cents)

3. The area of a triangle whose base is 12.4 cm. and whose altitude is 5.3 cm. (3 sq. cm.)

4. The cost of Ellen's new dress, if the materials cost \$8.12 and the dressmaker charges \$2.50 per day for $2\frac{1}{4}$ days' work on the dress. (25 cents)

5. The cost of $19\frac{3}{4}$ dozen eggs, at 39 cents a dozen. (\$0.30)

6. The cost of a piano, list price \$495 with a 15% discount. (75 cents)

7. The cost of 74 gallons of gasoline at 18 cents a gallon. (18 cents)

8. The area of a circle whose radius is 2 ft. 5 in. (2 sq. ft.)

9. The per cent of increase on an article bought at \$26 and sold for \$35. (5%)

10. The change you should get from a 10-dollar bill offered in payment for 19 yards of cloth at 45 cents a yard. (45 cents)

11. The value of 1095 pounds of sweet potatoes at \$1.75 a bushel. Allow 55 pounds to the bushel. (20 cents)

12. The cost of a railroad ticket from New York to Chicago (880 miles) at 3.6 cents per mile. (\$2)

13. Find the value of x in the equation $\frac{2x}{3} = 13.5$. (.75)

1. In $\triangle ABC$, $\angle C = 90°$, $\angle A = 35°$, and $AC = 85$ ft. What is the length of BC? the area of the triangle?

2. The string of a kite makes an angle of 49° with the ground at A. Point B, directly underneath the kite, is 120 ft. from A. How high is the kite above B?

3. What is the angle of elevation of the sun when a boy 5 ft. tall casts a shadow $7\frac{1}{2}$ ft. long? (Do not use your protractor.)

4. The foot of a ladder 20 ft. long is 6 ft. from a wall. How high on the wall does the ladder reach?

5. Find the area of the right triangle in which $h = 15$ and $b = 9$.

6. The area of a rectangle is 72 sq. in. The base is double the height. How long are its sides?

7. Two boys started on bicycles from a certain place, one going 12 miles northwest and the other 8 miles northeast. How far apart were the boys then?

8. If $3.14\,r^2 = 100$, what does r^2 equal? r?

9. Find the radius of a circle whose area is 157 square units; 200 square units.

10. Find the circumference of a circle whose area is 50.

11. How long an umbrella can be packed in the bottom of a trunk that is 28 in. wide and 30 in. long?

12. A merchant paid \$60 for an article which he marked to sell at a margin (increase) of 40%. Later he was obliged to sell it at a reduction of 25% of the marked price. His margin was what per cent of the cost price?

13. A merchant sold a suit of clothes for \$28, at a special "$\frac{1}{3}$ off" sale. He had marked the suit to sell at an increase of 40% above what he paid for it. What did he pay for it?

1. *Dorothy:* Have you a *lucky* number?

John: Yes, I've always considered 7 my lucky number. Why do you ask?

Dorothy: Well, John, you will have plenty of them if you do this little trick:

(*a*) Write these numbers: 1, 2, 3, 4, 5, 6, 7, 9.
(*b*) Multiply your lucky number by 9. You get 63.
(*c*) Multiply 12,345,679 by 63. You get 777,777,777. Isn't that enough sevens for you?

John: That's very interesting. How would you have played the trick if 6 had been my lucky number?

Dorothy: Oh, you can work that out, John.

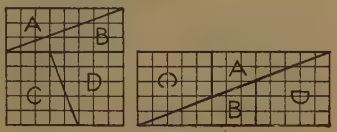
2. Can you show how sixteen trees may be set in twelve straight rows with four trees in every row? If you can, perhaps you will be able to show how sixteen trees may be set in fifteen straight rows with four trees in each row.

3. Suppose that twelve coins are arranged as shown here. I can rearrange the coins so that I have a square with five coins on each side. Can you?

4. Fred and John have a can containing 8 gallons of milk and wish to divide it equally. Their only measures are a 3-gallon can and a 5-gallon can. How can they make the division?

5. A philosopher had a window which let in too much light; the window was a yard square. He blocked up one half of it and still had a square window that was a yard high and a yard wide. Can you illustrate it?

6. Albert placed blocks *A*, *B*, *C*, and *D* in these two positions to prove that $8 \times 8 = 5 \times 13$; hence $64 = 65$!



Solve and check each of these equations:

1. $4y + 6 = 18$
2. $16 = \frac{2m}{3} + 4$
3. $.05z = 60$
4. $\frac{2y}{5} = 2.4$

5. The sale price of an umbrella is \$2. All merchandise is being sold at this sale for $\frac{4}{5}$ the regular price. What was the regular price of the umbrella? (Use an equation to solve this problem.)

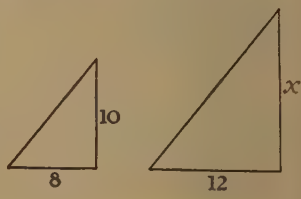
6. A rug-cleaning establishment charges 7¢ a square foot for cleaning rugs. Find the total charge for cleaning three rugs whose dimensions are $9' \times 12'$, $8\frac{1}{2}' \times 11'$, and $6' \times 9\frac{1}{4}'$.

7. From a field containing 9.8 acres, a farmer got 372.5 bu. of corn. What was the average yield per acre?

8. 16% of 16 = ?
9. 16 is ? % of 16.
10. 16% of ? = 16.
11. 150% of 24 = ?
12. 2.5 is ?% of 10.
13. 6% of ? = \$24.

14. Harry bought a bicycle for \$14. He cleaned it up, and then sold it for \$16. His margin was what per cent of the cost?

15. These two triangles are similar. Compute the length of side x.



16. Find the hypotenuse AB of right triangle ABC, in which AC is 6 in. and BC is 8 in.

17. What is the tangent of angle A in triangle ABC of Ex. 16?

Standards: *A, 14.* *B, 12.* *C, 9.*

CHAPTER VII

INTEREST AND BANKING

Finding What You Know about Interest

Persons in all walks of life need to understand banking practices and the meaning of interest and methods of computing it. You have been taught something about banking practices in previous grades. Here is a test that will enable you to determine how much you know about interest and banking. If you can solve correctly eleven of the twelve problems in the test, your teacher may let you skip from here to page 222.

Inventory Test. *Time, 30 minutes*

1. What is the interest on \$100 for 1 year at 6%?
2. Find the interest on \$200 for 3 months at 5%.
3. Compute the interest on \$1000 for 45 days at 4%.
4. Write a formula for finding interest. (Use i, p, r, and t.)
5. How much will \$400 amount to in 2 years at 6% simple interest?
6. The Interest Graph on page 208 shows that the interest on \$250 for 1 year at 4% is __?__.
7. Use the Interest Table on page 213 to find the interest on \$500 for 10 days at 6%.
8. The promissory note on page 214 is due on what date?
9. A note dated June 15, due in 90 days, matures on what date?
10. Compute the interest at 6% on a 300-dollar note made on July 1 and paid on October 1.
11. Find the bank discount on a note for \$500 at 6% for 90 days.
12. Compute the interest on \$360 for 72 days at $4\frac{1}{2}$%.

(1) Harry borrowed \$300 from Mr. Rex for 2 years at 6%. How much interest will he have to pay for the use of the money? (When the rate of interest is stated merely as a per cent, *per year* is understood.)

SOLUTION. The interest on \$300 for 1 year at 6% is .06 times \$300, or ___?___. The interest for 2 years is 2 × \$18, or ___?___.

(2) In Ex. (1), what is the *principal?* the *rate of interest?* the *time?* the *interest?* the *amount* (principal + interest)?

Find the interest in these examples, correct to the nearest cent:

	PRINCIPAL	TIME	RATE		PRINCIPAL	TIME	RATE
1.	\$400	1 yr.	6%	6.	\$1000	$\frac{1}{3}$ yr.	3.5%
2.	\$500	2 yr.	5%	7.	\$1260	4 mo.	$5\frac{1}{2}$%
3.	\$750	3 yr.	4%	8.	\$1400	3 mo.	$4\frac{1}{2}$%
4.	\$825	$\frac{1}{2}$ yr.	3%	9.	\$1875	$\frac{1}{4}$ yr.	$3\frac{1}{2}$%
5.	\$975	6 mo.	5%	10.	\$2000	8 mo.	$4\frac{1}{4}$%

11. A college boy borrowed \$500, agreeing to pay interest at 6% per year. How much would he owe at the end of 1 year?

Mr. Jackson asked his class to write the rule for finding interest. Four of the answers were:

A. Interest equals principal times rate times time (in years).
B. Interest = Principal × rate × time.
C. $i = p \times r \times t$.
D. $i = prt$.

(*a*) Are all the above statements of the rule correct?
(*b*) Which statement is the briefest?
(*c*) In statement *C*, what does i stand for? p? r? t?
(*d*) In statement *D*, what does prt mean?
(*e*) A statement in the form of an equation, in which letters are used to represent numbers, is called a ___?___.
(*f*) State the meaning of the formula $i = prt$.

1. Use the formula $i = prt$ to find the interest on \$500 for 4 months at $4\frac{1}{2}\%$.

SOLUTION. $p = 500$, $r = \frac{45}{1000}$, $t = \frac{1}{3}$ yr.

$i = prt$

$$i = \cancel{500}^{1} \times \frac{\cancel{45}^{15}}{\cancel{1000}_{2}} \times \frac{1}{\cancel{3}_{1}} = \frac{15}{2} = 7.50 \quad \textit{Ans.}\ \$7.50$$

Use the interest formula to find the interest when —

2. $p = \$1$, $r = .06$, and $t = 1$ year.

3. $p = \$600$, $r = 3\frac{1}{2}\%$, and $t = 3$ months.

4. $p = \$250$, $r = 5\frac{1}{2}\%$, and $t = 8$ months.

5. $p = \$800$, $r = 4\frac{1}{2}\%$, and $t = 4$ months.

6. What is the interest on \$75 for 45 days at 6% per annum? (Think of 45 days as $\frac{45}{360}$ of a year.)

7. What is i, if $p = \$50$, $t = 75$ days, and $r = .045$?

8. Copy and complete the following table. Alert pupils often discover how to do this with a small amount of computing.

If	p is	\$100	\$100	\$100	\$100	\$200	\$50
	r is	.06	.06	.06	.06	.06	.06
	and t is	1 yr.	3 mo.	4 mo.	6 mo.	6 mo.	6 mo.
then i is							

9. A man borrowed p dollars at 6%. What does $.06\,p$ represent? $.03\,p$? $.12\,p$?

10. Compute the amount (principal + interest) when —

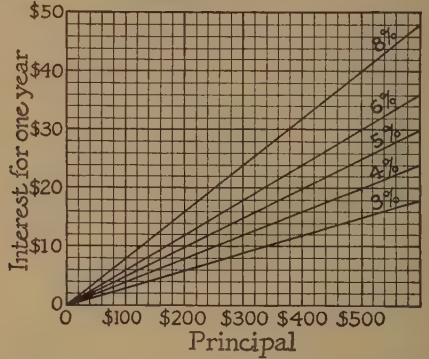
(*a*) $p = \$600$, $r = .06$, and $t = 1$ yr.

(*b*) $p = \$1500$, $r = .04$, and $t = 8$ mo.

(*c*) $p = \$360$, $r = .02$, and $t = 9$ mo.

This graph shows the interest on various sums of money at various rates.

INTEREST GRAPH



1. From the graph, read the interest on \$400 for 1 year at 3%; 4%; 5%; 6%; 8%.

2. What line on the graph pictures the equation $i = .03\,p$? $i = .04\,p$? $i = .05\,p$? $i = .06\,p$?

3. Consider the equation $i = .06\,p$. It represents the interest on \$ ___?___ for ___?___ year at ___?___ %.

4. In the equation $i = .06\,p$, what is i if p is \$200? What is i if p is \$300? if p is \$400? if p is \$600? if p is \$750?

Complete this table for the equation $i = .06\,p$:

If p is.....	\$100	200	300	400	650	750	1000
then i is....	\$ 6						

5. Consider the relationship between the principal and the interest in the completed table of Ex. 4. Is it represented by any one of the lines in the graph? By which one?

6. Make a copy of the graph. Extend each line on the graph so that you can read the interest on principals up to \$1200.

From the graph, find the interest for 1 year—

7. on \$250 at 6%.
8. on \$750 at 4%.
9. on \$150 at 5%.
10. on \$500 at 3%.
11. on \$800 at 8%.
12. on \$1200 at 6%.

13. On the graph for Ex. 6 draw a line from which you can read the yearly interest on various principals at 7%.

14. Estimate what principals will yield \$20 interest in 1 year at the different rates of interest shown in the graph on the preceding page.

15. The graph shows that the interest on \$600 at 6% is \$ ___?___. If we change the \$600 to \$60, then the interest becomes \$ ___?___. If we change the \$600 to \$6, the interest becomes \$ ___?___. If we change the \$600 to \$6000, the interest becomes \$ ___?___.

16. Make an interest graph to show the yearly interest on various principals up to \$1000 at $3\frac{1}{2}$%; at $4\frac{1}{2}$%; at $5\frac{1}{2}$%; and at $6\frac{1}{2}$%.

17. I should like to receive \$50 interest each year from my investments. Can you find out from your graph for Ex. 16 how much I should have to invest at $4\frac{1}{2}$%? at $5\frac{1}{2}$%? at $6\frac{1}{2}$%?

Use the interest formula to solve each of the following:

18. At 6%, the interest on \$1 for $16\frac{2}{3}$ years will be \$ ___?___.

19. At 6%, the interest on \$1 for ___?___ years will be \$1.

20. How long will it take a dollar to earn a dollar at 4% interest? at 6% interest? 5% interest? 2% interest? 3% interest?

21. Would you say that the interest formula, generally speaking, is more useful than an interest graph? Why so?

Can you explain this method of finding the interest on \$300 for 2 months and 20 days at 6%?

$$\frac{\overset{1}{\cancel{300}}}{1} \times \frac{2}{\cancel{6}} \times \frac{2}{\cancel{80}} = 4 \qquad \text{Interest} = \$4.00.$$

Where does the $\frac{80}{360}$ come from?

Find the interest to the nearest cent on each of the following:

	Principal	Time	Rate		Principal	Time	Rate
1.	\$200	30 da.	6%	9.	\$300	2 mo. 4 da.	4%
2.	\$450	20 da.	5%	10.	\$450	3 mo. 6 da.	3%
3.	\$600	75 da.	4%	11.	\$600	4 mo. 8 da.	2%
4.	\$250	96 da.	3%	12.	\$560	3 mo.10 da.	6%
5.	\$900	8 da.	2%	13.	\$800	5 mo. 6 da.	4%
6.	\$1075	10 da.	4%	14.	\$400	3 mo. 2 da.	$5\frac{1}{2}\%$ ($\frac{11}{200}$)
7.	\$1200	12 da.	5%	15.	\$600	5 mo. 8 da.	$3\frac{1}{4}\%$ ($\frac{13}{400}$)
8.	\$1250	24 da.	6%	16.	\$200	1 mo. 6 da.	$4\frac{1}{2}\%$

17. How much will \$500 amount to in 2 years with interest at 6% per annum?

18. How much will \$500 amount to in 8 months with interest at 4% per annum?

19. How much will \$600 amount to in 90 days with interest at 5% per annum?

20. How much will \$250 amount to in 2 mo., 10 da., with interest at 6% per annum?

21. How much will \$1000 amount to in 1 year at 4%?

1. Find the interest on $400 for 60 days at 6%.

2. Find the interest on $500 for 60 days at 6%.

3. Find the interest on $645 for 60 days at 6%.

4. Find the interest on $750 for 60 days at 6%.

5. Write the principal and beside it the interest as found in Exs. 1 to 4. What relation do you notice between the principal and the interest in these cases?

In finding the interest on any principal for 60 days ($\frac{1}{6}$ year) at 6%, you have done this multiplication:→ You could find the interest in each case by multiplying the principal by $\frac{1}{100}$, or more conveniently by .01. Multiplying a number by .01 simply moves the decimal point two places to the left.

$$\text{Principal} \times \frac{1}{\cancel{6}_{1}} \times \frac{\overset{1}{\cancel{6}}}{100} =$$

6. Find the interest in Exs. 1 to 4 for 6 days instead of 60 days. Compare the interest with the principal in each case. Can the interest on any principal for 6 days at 6% be found by multiplying by .001; that is, by moving the decimal point three places to the left?

7. The interest on $500 for 60 days at 6% is ___?___. If you know the interest for 60 days, you can find the interest for 30 days by dividing the interest for 60 days by ___?___. If you know the interest for 60 days, how can you find the interest for 90 days? 15 days? 10 days?

8. The interest on $500 for 6 days at 6% is ___?___. If you know the interest for 6 days, how can you find the interest for 12 days? 18 days? 24 days? 1 day? 2 days? 3 days? 4 days?

To find the interest on any principal at 6% for 60 days, move the decimal point in the principal two places to the left; to find the interest for 6 days, move the decimal point three places to the left. This is called the 6% method.

1. By the 6% method find the interest at 6% on —

(*a*) \$700 for 60 days; 30 days; 15 days; 20 days.
(*b*) \$450 for 60 days; 90 days; 75 days; 120 days.
(*c*) \$215 for 6 days; 1 day; 2 days; 3 days.
(*d*) \$320 for 6 days; 12 days; 9 days; 18 days.
(*e*) \$600 for 82 days. (Interest for 60 days = ?
Interest for 20 days = ?
Interest for 2 days = ?
Interest for 82 days = ?)
(*f*) \$400 for 72 days; 66 days; 96 days; 124 days.
(*g*) \$1000 for 68 days; 78 days; 88 days; 98 days.
(*h*) \$240 for 14 days; 18 days; 21 days; 22 days.
(*i*) \$115 for 10 days; 11 days; 12 days; 13 days.

2. Find the interest on \$700 for 60 days at 4%.

(HINT. Interest on \$700 for 60 days at 6% = \$7.00.
Interest on \$700 for 60 days at 4% = $\frac{2}{3}$ of \$7.00 = ?)

3. Find the interest on \$1200 for 60 days at 6%; 5%; 4%; 3%; 2%; 1%.

4. Find the interest on \$600 for 6 days at 6%; 5%; 4%; 3%; 2%; 1%.

5. Find the interest on \$300 for 90 days at 6%; 5%; 4%; 3%; 2%; 1%.

6. Find the interest on \$750 for 75 days at 6%; 5%; 4%; 3%; 2%; 1%.

7. If you wish to find the interest on any principal for 60 days at 3%, first find the interest for 60 days at 6%, and then • • •

8. Mr. Hall invested \$90 in wreaths, trees, and tree ornaments to sell at Christmas time. He borrowed the money at 6%. How much interest did he have to pay for the 9 days he kept the money?

When a banker wishes to compute interest quickly, he uses an interest table similar to the one shown below. This table reads as follows:

The interest on \$100 at 6% for 90 days is \$1.50; the interest on \$200 at 6% for 40 days is \$1.333; etc.

SIMPLE INTEREST TABLE — RATE 6%

DAYS	$100	$200	$300	$400	$500	$600	$700	$800	$900
90	$1.500	$3.000	$4.50	$6.000	$7.500	$9.00	$10.500	$12.000	$13.50
60	1.000	2.000	3.00	4.000	5.000	6.00	7.000	8.000	9.00
50	.833	1.667	2.50	3.333	4.167	5.00	5.833	6.667	7.50
40	.667	1.333	2.00	2.667	3.333	4.00	4.667	5.333	6.00
30	.500	1.000	1.50	2.000	2.500	3.00	3.500	4.000	4.50
20	.333	.667	1.00	1.333	1.667	2.00	2.333	2.667	3.00
10	.167	.333	.50	.667	.833	1.00	1.167	1.333	1.50
9	.150	.300	.45	.600	.750	.90	1.050	1.200	1.35
8	.133	.267	.40	.533	.667	.80	.933	1.067	1.20
7	.117	.233	.35	.467	.583	.70	.817	.933	1.05
6	.100	.200	.30	.400	.500	.60	.700	.800	.90
5	.083	.167	.25	.333	.417	.50	.583	.667	.75
4	.067	.133	.20	.267	.333	.40	.467	.533	.60
3	.050	.100	.15	.200	.250	.30	.350	.400	.45
2	.033	.067	.10	.133	.167	.20	.233	.267	.30
1	.017	.033	.05	.067	.083	.10	.117	.133	.15

1. Use the table to find the interest at 6% on \$300 for 20 days; on \$600 for 50 days; on \$200 for 10 days; on \$900 for 90 days.

2. Find the interest on \$500 for 15 days. (Find the interest for 10 days and for 5 days; that is, \$0.833 plus \$0.417, which equals \$1.25.) Find the interest on \$500 for 25 days.

3. Find the interest on \$300 for 63 days; on \$700 for 100 days; on \$1500 for 40 days; on \$600 for 79 days; on \$100 for 38 days.

4. Find the interest on \$250 for 55 days.

5. What is the interest on \$640 for 60 days at 3%?

6. What is the interest at 6% on \$1000 for 10 days? for 1 day? for 11 days?

Mr. Adams needed $300 to help pay for a tractor and asked his neighbor, Mr. Johnson, to lend him the money. He was willing to pay 6% interest and was confident that he could pay back the $300 in 6 months.

Mr. Johnson: I don't lend money unless the borrower gives good *security*.

Mr. Adams: I understand that, Mr. Johnson, and I have asked two of my neighbors, Mr. Smith and Mr. Fern, to sign the note. They have agreed to "go on the note" with me.

Mr. Johnson: Very well. Those men are safe. If they will sign the note, I will let you have the money.

Here is the note which Mr. Adams gave to Mr. Johnson:

$300.00 Freeport, Ill., March 15, 1936

Six months after date I promise to pay to the order of John Johnson

Three hundred and 00/100 Dollars

at Freeport, Ill., with interest at 6% per annum

Value received

Samuel Adams

Henry Smith

James Fern

A note of this kind is called a ***promissory note.***

If Mr. Adams does not pay the note at ***maturity*** (the date payment is due), Mr. Johnson has the legal right to collect the money from either of the other persons who *signed* the note. They have obligated themselves to pay the note if Mr. Adams does not.

The person who signs the note is called the ***maker.*** Who was the maker of this note?

The person to whom the written promise is made is called the ***payee.*** Who is the payee in the above note?

The amount for which the note is written is the ***face*** of the note. What is the face of the above note?

1. A note written "I promise to pay to William Owen" is payable only to Mr. Owen. A note written "I promise to pay to the order of William Owen" is payable to Mr. Owen or to anyone to whom Mr. Owen orders it to be paid. Would you wish to have a note drawn (written) payable to you only, or to your order? Why?

2. A note payable "to the order of William Owen" can be sold by Mr. Owen if he *indorses* it (writes his name on the back of the note). What would Mr. Johnson have to do to the note shown on the preceding page in case he wished to sell it?

3. A note payable to *bearer* can be collected by anyone who holds it when due. Notes are seldom drawn payable to *bearer*. Can you give a reason for this?

4. Promissory notes that can be bought and sold are said to be ***negotiable***. (What is the meaning of the word *negotiate?*) Promissory notes are negotiable when they are drawn "to bearer" or "to the order of." Would you prefer to have notes that belong to you *negotiable?* Why? Is the note shown on the preceding page negotiable?

5. Notes without the words "bearer" or "to the order of" are not negotiable. Write a non-negotiable note payable to one of your classmates, and sign it John Doe.

6. A note which reads, "*On demand* I promise to pay . . . " is called a ***demand note***. It is due any time the owner *demands* payment. What are the advantages and disadvantages of a demand note (*a*) for the holder? (*b*) for the maker?

7. Notes are sometimes made to read, "One day after date I promise to pay . . . " When would such a note be due? Such a note is practically the same as a demand note.

8. Write a demand note for $50 with interest at 6%, such as John Doe might give to Richard Roe.

Mr. R. D. Jones sold to Richard Kelley this note:

$500.00 Chicago, Ill., March 15, 1936

Six months after date I promise to pay to the order of R. D. Jones Five hundred and 00/100 Dollars at Chicago, Ill. with interest at 6% per annum. Value received

J. S. Brown

It was necessary for Mr. Jones to *indorse* the note. Three methods of indorsing notes are shown here.

1. One method is called ***indorsement in blank.*** By this method Mr. Jones, the payee and seller of the note, signs his name across the back of the note. Mr. Jones becomes liable for payment of the note if the maker fails to pay it at maturity. Blank indorsement makes the note payable to *bearer*.

R. D. Jones

2. Another method is called ***indorsement in full.*** By this method the note is payable only to Mr. Kelley or to his order. Mr. Jones is still liable for payment of the note.

Pay to the order of Richard Kelley
R. D. Jones

3. A third method is called ***indorsement without recourse.*** By this method Mr. Jones assumes no responsibility for the payment of the note.

Pay to the order of Richard Kelley without recourse on me
R. D. Jones

4. Which method of indorsing notes would you prefer (*a*) if you were buying the note? (*b*) if you were selling it?

If you hold a note dated July 15, on what day would you expect to collect it if it were due 3 months after date? 90 days after date?

To find the maturity dates of notes, the business world uses these rules:

(1) When the time to run is stated in days, find the date of maturity by counting by days from the date of issue.

EXAMPLE. A note dated July 15, due in 90 days, matures October 13. (16 more days in July, plus 31 days in August, plus 30 days in September, plus 13 days in October.)

(2) When the time to run is stated in years, or months, find the date of maturity by counting by years, or by months, from the date of issue.

EXAMPLE. A note dated July 15, due in 3 months, matures October 15.

(3) When the time to run is between two specific dates not more than a year apart, count the exact number of days between dates.

EXAMPLE. The time on a note dated March 16 and due May 4 is 49 days. (15 more days in March, plus 30 in April, plus 4 in May.) (If the number of days computed in this way should be 361, 362, 363, 364, or 365, it would be counted as 360.)

(4) To find the time between two dates more than a year apart, count the whole number of years and the remaining days.

1. Find the time between March 17 and July 4.

2. Find the time between April 16 and November 18.

3. A note for $1000, dated March 21 and due in 4 months, is due on what date? Compute the interest at 4%.

4. A note for $265, dated July 15, drawing interest at 6% per annum, was due in 90 days, or on Oct. ___?___. How much interest was due when the note matured?

Here is a device which enables us to tell quickly how many days there are from a *certain* day of a *given* month to the *corresponding* day of any other month.

From	To Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
Jan.	365	31	59	90	120	151	181	212	243	273	304	334
Feb.	334	365	28	59	89	120	150	181	212	242	273	303
March	306	337	365	31	61	92	122	153	184	214	245	275
April	275	306	334	365	30	61	91	122	153	183	214	244
May	245	276	304	335	365	31	61	92	123	153	184	214
June	214	245	273	304	334	365	30	61	92	122	153	183
July	184	215	243	274	304	335	365	31	62	92	123	153
Aug.	153	184	212	243	273	304	334	365	31	61	92	122
Sept.	122	153	181	212	242	273	303	334	365	30	61	91
Oct.	92	123	151	182	212	243	273	304	335	365	31	61
Nov.	61	92	120	151	181	212	242	273	304	334	365	30
Dec.	31	62	90	121	151	182	212	243	274	304	335	365

If the period contains Feb. 29, add one day.

1. How do you think the top line of the table is made?

2. Can you explain how the second line was obtained? the third line?

To find the number of days from September 10 to November 10 (same year), look for September on the left side and November at the top. The table shows that there are 61 days from September 10 to November 10.

3. By means of the table, find the number of days between the following dates:

(*a*) From January 17 to July 17; (*b*) from February 12 to September 12; (*c*) from June 1 to May 1; (*d*) from August 2 to November 2; (*e*) from August 2 to November 12; (*f*) from July 30 to October 4.

4. From August 5 to November 20 is __?__ days; from November 8 to July 29 is __?__ days; from July 4 to November 25 is __?__ days; from December 25 to July 4 is __?__ days; from March 1 to December 1 is __?__ days.

5. A note for $275, dated July 17 and bearing interest at the rate of 6% per annum, was paid January 9 of the following year. Compute the interest.

6. Mary went to boarding school October 7, 1935, and returned home June 2, 1936. How many days was she away from home?

7. A note made on June 18 for $600, bearing interest at the rate of 6% per annum from date until paid, was paid on August 17 of the same year. Compute the interest on the note.

8. Clarence Dunn owned a note for $250, dated October 21, 1936, bearing interest at the rate of 4% per annum from date until paid. On April 19, 1937, he offered to sell it for $256.25. Assuming that the note would be paid on demand, do you think Mr. Dunn's offer was a reasonable one?

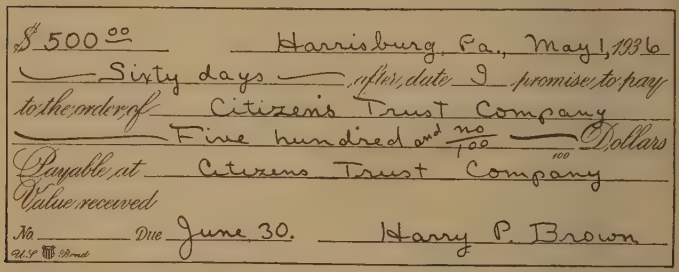
9. Use the interest table on page 213 to find the interest at 6% per annum for each of the following:

	Principal	Time		Principal	Time
(*a*)	$160.00	27 days	(*f*)	$865.00	76 days
(*b*)	$ 54.00	63 days	(*g*)	$ 10.00	35 days
(*c*)	$275.00	82 days	(*h*)	$ 75.00	42 days
(*d*)	$ 20.00	95 days	(*i*)	$135.00	106 days
(*e*)	$ 37.00	61 days	(*j*)	$590.00	180 days

10. Find the interest at 6% on a 460-dollar note dated Feb. 10 and paid the following Feb. 8. (In using the day table, any number of days between 360 and 365 should be considered as 1 year.)

11. A high school boy agreed to work for a farmer during the summer season for $.80 a day (Sundays included). If he began on the morning of June 17 and worked until the evening of September 28, how much did he earn?

Mr. Brown, a merchant of good financial standing, needed $500 with which to buy some goods. He arranged to borrow the money from a local bank for 60 days. The promissory note which he gave the bank is shown below:



$500.00 Harrisburg, Pa., May 1, 1936

Sixty days after date I promise to pay to the order of Citizens Trust Company

Five hundred and no/100 Dollars

Payable at Citizens Trust Company

Value received

No. Due June 30. Harry P. Brown

U.S. Stamp

Notice that no mention of interest appears in the note. The bank collected the interest *in advance*. What is the interest on $500 for 60 days at 6%? Mr. Brown received only $495 for the note. Sixty days later, on June 30, he paid the bank $500.

Interest collected in advance by a bank is called ***bank discount***. When the bank deducted $5 interest on Mr. Brown's note, it *discounted* the note; Mr. Brown received the ***proceeds***.

1. What is the bank discount and what are the proceeds on a note for (*a*) $3000 for 60 days at 6%? (*b*) for $500 for 30 days at 6%? (*c*) for $2500 for 90 days at 6%? (*d*) for $1500 for 60 days at 4%?

2. When banks make loans, they usually collect interest at the time the note is *made;* when individuals make loans, they usually collect interest at the time the loan is *paid*.

TO THE TEACHER. You may wish your pupils to repeat at this time the Inventory Test on page 205.

1. On July 1, 1935, John Ross bought a gasoline engine from the Harding Hardware Company, giving his note for $300 at 6% due January 1, 1936. What amount would be due when the note matured?

2. The Harding Hardware Company needed the money on November 1, 1935. Could Mr. Ross be forced to pay the note then?

3. The company sold the note to the Merchants Bank and Trust Company, November 1, 1935. Should the bank have paid as much as $300 for the note? as much as $309? Why?

4. The company received $306.06 for the note. Can you explain why?

5. Here is the way the banker computed the *proceeds:*

(1) Face of note, $300
(2) Interest for 184 days (July 1 to Jan. 1) at 6%, $9.20
(3) Amount due at maturity, $309.20
(4) Number of days before maturity (from November 1 to January 1), 61
(5) Discount (interest) on $309.20 at 6% for 61 days, $3.14
(6) Proceeds, $309.20 − $3.14, or $306.06

How did the banker find the "number of days before maturity" (step 4)? How did he find the discount (step 5)? How did he find the proceeds (step 6)?

Find the discounts and the proceeds on the following notes:

	Made	Face	Due	Interest	Discounted	Discount
6.	Aug. 1	$300	Dec. 1	6%	Nov. 1	6%
7.	Jan. 1	$500	July 1	6%	Mar. 1	6%
8.	Jan. 1	$750	Aug. 1	4%	June 10	6%
9.	Mar. 1	$1000	Jan. 1	5%	Sept. 15	6%
10.	July 15	$450	Nov. 15	5½%	Nov. 1	5%

(1) The owner of a promissory note may *sue* (bring legal proceedings against) the maker of a note, if it is not paid at maturity.

(2) A promissory note does not bear interest unless the fact is so stated. If interest is specified but no rate is given, then the legal rate of the state in which the note is made is used. What is this rate in your state?

(3) An individual may lawfully receive from a corporation a higher rate of interest than the "legal rate" in a given state; but an individual may not lawfully receive interest from another individual at a rate higher than the legal rate.

(4) Promissory notes written with a lead pencil are legal.

(5) A promissory note made on Sunday, or on a legal holiday, is legal in most states. Is this true in your state?

(6) All non-interest-bearing notes bear interest at the legal rate after maturity, if they are not paid when due.

(7) If a promissory note is not paid at maturity, the holder of it has the right to sue any and all persons whose names were on the note when he received it, except those who indorsed it "without recourse."

(8) If a note is lost, stolen, or destroyed, this does not excuse the maker from paying it, provided the owner can: (*a*) prove what the face value of the note was, and (*b*) prove that the maker received the full value of the face of the note.

(9) An innocent purchaser of a note which was obtained by fraud can collect from the maker.

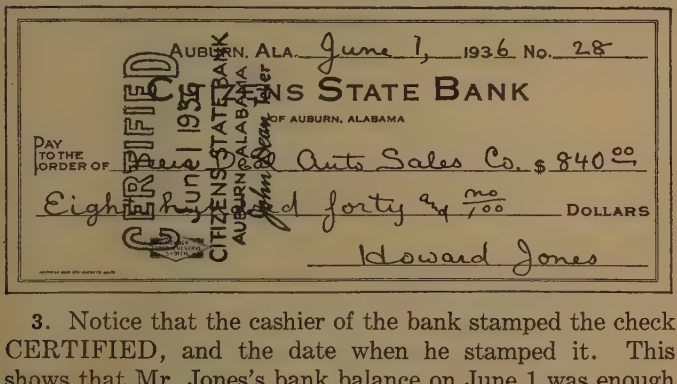
(10) A note that is signed by a minor cannot be collected from the parents of the minor.

(11) The owner of a note should pay interest regularly (semiannually or annually). Why?

(12) Everyone should know how to draw up a promissory note. Why?

1. Mr. Jones was buying an automobile for $840 from the New Deal Auto Sales Co. He had the money on deposit in a checking account in a bank; so he proposed to give the dealer his *personal check* for $840. The salesman declined to accept the check. Why do you suppose he did so?

2. The salesman said: "I am sorry, but we don't accept personal checks from persons we don't know. We ask our customers to pay in actual money, or by a *certified check*. You may obtain a certified check by going to your bank." Mr. Jones then went to his bank and obtained a certified check for the $840, as shown below.

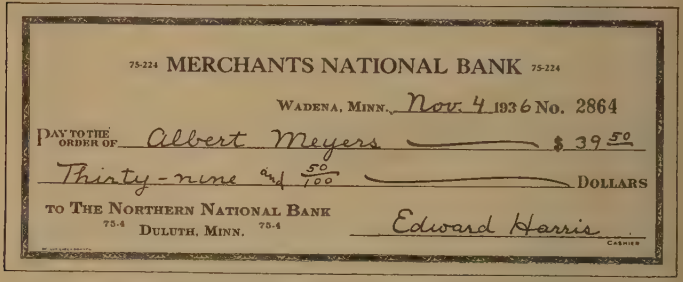

AUBURN, ALA. June 1, 1936 No. 28
CITIZENS STATE BANK
OF AUBURN, ALABAMA
PAY TO THE ORDER OF New Deal Auto Sales Co. $840 00
Eight hundred forty and no/100 DOLLARS
Howard Jones
CERTIFIED
JUN 1 1936
CITIZENS STATE BANK
AUBURN ALABAMA

3. Notice that the cashier of the bank stamped the check CERTIFIED, and the date when he stamped it. This shows that Mr. Jones's bank balance on June 1 was enough to cover this check.

4. The cashier then deducted $840 from Mr. Jones's account, and held it until the check was presented. Mr. Jones then delivered the check to the salesman in payment for the car. Do you see why the salesman had confidence in the certified check?

5. Would you say that payment by a certified check is just as convenient as payment by actual cash (*a*) for the buyer? (*b*) for the seller?

Albert Meyers of Wadena, Minn., ordered a $39.50 stove from the Household Products Stores, Inc., in Duluth, Minn. Instead of paying the bill by sending his personal check, a certified check, a money order, or currency, he sent a *bank draft*, as shown below.


75-224 MERCHANTS NATIONAL BANK 75-224

WADENA, MINN. Nov. 4 1936 No. 2864

PAY TO THE ORDER OF Albert Meyers $ 39 50

Thirty-nine and 50/100 DOLLARS

TO THE NORTHERN NATIONAL BANK
75-4 DULUTH, MINN. 75-4

Edward Harris
CASHIER

(1) Mr. Meyers purchased the draft from the Merchants National Bank in Wadena.

(2) The Merchants National Bank of Wadena had money on deposit in the Northern National Bank of Duluth, Minn.

A ***bank draft*** is a check written by one bank ordering another bank to pay a specified sum of money to the order of the purchaser of the draft.

(3) This draft was a check written by ___?___, the cashier of the Merchants National Bank, ordering the ___?___ of Duluth to pay $39.50 to the order of Albert Meyers.

(4) Mr. Meyers indorsed the check by writing on the back of it: *Pay to the order of Household Products Stores, Inc.—Albert Meyers*, and mailed it to the Household Products Stores, Inc.

(5) Since a bank draft is signed by the cashier of the bank, it carries assurance that it is genuine and that it will be paid when presented.

(6) What is the difference between a certified check and a bank draft?

1. Sometimes interest is computed on the basis of *365 days to the year*. How much interest on that basis would be due on \$5000 for 30 days at 3%? Can you solve this problem without the help of the work below?

$$\frac{\overset{10}{\cancel{\$5000}}}{1} \times \frac{3}{\underset{1}{\cancel{100}}} \times \frac{30}{\underset{73}{\cancel{365}}} = \frac{900}{73} = \$12.33$$

2. Why do you think the interest found in Ex. 1 is called *exact interest?*

3. What would be the *exact* interest on \$4000 for 15 days at 3%?

Find to the nearest cent the exact interest on:

4. \$260 for 60 days at 5%
5. \$350 for 45 days at 3%
6. \$280 for 50 days at 4%
7. \$120 for 30 days at 3%
8. \$1200 for 90 days at 4%
9. \$1500 for 100 days at 5%
10. \$2500 for 120 days at $2\frac{1}{2}$%
11. \$1800 for 140 days at 4 %
12. \$1600 for 130 days at 2 %
13. \$1250 for 110 days at 3 %
14. \$600 for 90 days at $2\frac{1}{2}$%
15. \$500 for 112 days at 4 %

16. Counting 365 days to the year, find the interest on \$400 from April 4 to June 22 at 6%.

17. Find the exact interest on \$2,000,000 from March 3 to April 24 at 4%.

18. How much will \$800 amount to in 60 days, bearing exact interest at 4%?

19. How much will \$600 amount to in 30 days, bearing exact interest at 4%?

20. How much will \$10,000 amount to in 120 days with exact interest at 3%?

By Ewing Galloway, N. Y.

1. Mr. Jones saves some money each month, but he has little time in which to study how to invest it. He decided to open an account in a savings bank. By making inquiries he learned that banks have different rules regarding deposits, interest, etc., but that the following apply at his bank:

(*a*) Money deposited in the savings bank is carefully invested by the bank. The depositor need not worry about the safety of his money.

(*b*) Anyone may open an account by depositing a very small sum — usually as low as $1.

(*c*) Deposits may be made at any time, even by mail.

(*d*) The bank pays interest on deposits.

(*e*) Interest is figured semiannually (Jan. 1 and July 1).

(*f*) On deposits made during the first three days of any month, interest is computed from the first of the month. On deposits made after the third day, interest is computed from the first of the following month.

(*g*) Interest is not paid on parts of a dollar, on parts of a month, except as in (*f*), or on sums withdrawn between the dates on which interest is computed.

1. On January 2, 1935, Mr. Jones deposited $200 in the Home and Community Savings Bank, which paid interest at 3% per annum, payable semiannually. On March 7 he deposited $100. On September 6 he deposited $50. On November 23 he withdrew $75. Could you find what his balance was on January 1, 1936?

MR. JONES'S SAVINGS ACCOUNT

DATE	DEPOSITS	INTEREST	WITHDRAWALS	BALANCE
1935				
Jan. 2	$200	—	—	$200.00
Mar. 7	100	—	—	300.00
July 1	—	3.75	—	303.75
Sept. 6	50	—	—	353.75
Nov. 23	—	—	75	278.75
1936				
Jan. 1	—	4.17	—	282.92

2. On July 1, 1935, Mr. Jones was entitled to interest on the $200 for 6 months. Why for 6 months? How much interest is that? He was also entitled to interest on $100 for 3 months. Why 3 months? How much interest was due him on July 1? This interest helps to make up the new balance for July 1. What was the balance on July 1, 1935?

NOTE. Even if the bank is closed on January 1 or July 1, the interest is entered as though the computation were made then.

If Mr. Jones had not withdrawn the $75 during the next half year, he would have received interest on $303 for 6 months and the interest on $50 for 3 months. But because he withdrew the $75 during the interest period, he lost the interest on the 50-dollar deposit and on $25 of the previous balance.

3. Make a copy of Mr. Jones's savings account. Suppose he deposited $40 on March 3, 1936. What is his balance July 1, 1936?

4. On July 3, 1936 Mr. Jones withdrew \$50, his balance on July 1 being \$327.55. (Why would it be wise not to withdraw money from the account except at the beginning of an interest period if this can be avoided?) On October 7 he deposited \$75. What is his bank balance January 1, 1937?

5. Harold Hall opened a savings account on March 15, 1936. During that year his deposits were as follows:

March 15.....	\$100	September 30....	\$100
May 15.....	300	November 15....	200

Complete: (1) On July 1 he was entitled to interest on \$100 for 3 months; this interest at 3% was _?_ dollars.

(2) On July 1 he was entitled to interest also on \$300 for _?_ month, that interest being _?_ dollars.

(3) His balance July 1 was _?_ dollars.

(4) The interest on \$401.50 from July 1 to January 1 was _?_ dollars.

(5) The interest on \$100 from September 30 to January 1 (here counted as 3 months — why?) was _?_ dollars.

(6) The interest on \$200 from November 15 to January 1 (here counted as 1 month) was _?_ dollars.

(7) Harold's balance on January 1 was _?_ dollars.

(8) Since Harold deposited only \$700, he received _?_ dollars as interest.

You see that the interest on the savings earned interest, or, we may say, Harold received "interest on interest."

6. Ella Eastman had a balance of \$60 on January 1. Her deposits were as follows: January 11, \$20; February 2, \$10; May 3, \$12. What was her balance on July 1, if she received interest at 3% per annum, payable semiannually?

7. On January 1 Arthur Hardy deposited \$100 in a savings bank that paid interest at the rate of 3% per annum, computed semiannually. What would his balance be (*a*) on July 1? (*b*) on the following January 1?

Ralph Taylor deposited \$800 in the County Savings Bank on January 2, 1935. The bank pays 3% interest January 1 and July 1.

(1) During the first six months Ralph's principal would earn \$__?__. On July 1, 1935, his balance was \$800 + 1½% of \$800, or \$812. (Why 1½% of \$800?)

(2) How much interest would his \$812 earn during the second six-month period? (1½% of \$812 = ?)

(3) Show that his balance on January 1, 1936, was \$812 + \$12.18.

(4) How much interest would his \$824.18 earn during the next six-month period? (Remember that in computing interest, savings banks disregard cents in the principal.)

(5) Show that Ralph's balance on July 1, 1936, was \$836.54.

(6) Show that Ralph's \$800 would grow to \$849.08 by January 1, 1937; to \$861.81 by July 1, 1937; and to \$874.72 on January 1, 1938.

When the interest on a principal is left to draw interest on itself, the principal is said to be drawing ***compound interest.***

(7) In one year Ralph's \$800 *amounts to* \$824.18. Therefore the *compound* interest on \$800 at 3% *compounded semiannually* is \$824.18 − \$800, or \$__?__. How does that compare with the simple interest on \$800 at 3% for one year?

(8) Find to what amount a savings deposit of \$1000 would grow in 1 year at 3% interest compounded semiannually.

(9) Mr. Green has \$4000 to invest. He can lend it for a year to his friend, Mr. Black, who will pay him 2% simple interest, or he can deposit it for a year in a savings bank which pays 2% interest compounded semiannually. Which would be the better investment? How much better?

Mr. Mercer deposited $1000 in a savings bank at 3% interest compounded semiannually. How much did this amount to in 2 years?

SOLUTION. Each yearly increase in the principal has to be worked out separately, as shown in the sample solution. You will see that the amount is $1061.36. The interest is $61.36. Study the solution. Cover it and see whether you can solve the problem without referring to it.

Remember that savings banks do not pay interest on part of a dollar.

Compute the interest, compounded semiannually, on the following. (In computing interest, disregard a fraction less than one-half cent; call a fraction one-half cent, or greater, a full cent.)

1. $3000, 2 yr., 4%
2. $2000, $1\frac{1}{2}$ yr., 4%
3. $2150, 2 yr., 2%
4. $1850, 2 yr., 6%
5. $10,000, 3 yr., 4%
6. $420.50, 2 yr., 4%
7. $100, 2 yr., 3%
8. $550, 2 yr., 3%
9. $600, 3 yr., 3%
10. $400, 2 yr., 4%
11. $2000, 2 yr., $2\frac{1}{2}$%
12. $1000, 2 yr., $3\frac{1}{2}$%

SAMPLE SOLUTION

$1000.00 = first principal
$.01\frac{1}{2}$

$15.00 = int. first 6 mo.
1000.00

$1015.00 = amt. after 6 mo.

$1015.00 = prin. for next 6 mo.
$.01\frac{1}{2}$

$15.23 = int. second 6 mo.
1015.00

$1030.23 = amt. after 1 yr.

$1030.00 = prin. for next 6 mo.
$.01\frac{1}{2}$

$15.45 = int. third 6 mo.
1030.23

$1045.68 = amt. after $1\frac{1}{2}$ yr.

$1045.00 = prin. for next 6 mo.
$.01\frac{1}{2}$

$15.68 = int. fourth 6 mo.
1045.68

$1061.36 = amt. after 2 yr.
1000.00 = first principal

$61.36 = comp'd int. for 2 yr.

1. The table below shows that \$1 in 1 yr. compounded annually at 4% will amount to \$1.04000; in 2 yr. it will amount to \$1.08160; in 10 yr. it will amount to \$1.48024; in 35 yr. it will amount to \$3.94609. (Check each of these by referring to the table.)

NOTE. The results given in the table do not correspond exactly with compound interest as paid by banks, because the table counts interest on parts of a dollar, while savings banks do not.

YEARS OR PERIODS	RATE OF INTEREST					
	1%	2%	3%	4%	5%	6%
1	1.01000	1.02000	1.03000	1.04000	1.05000	1.06000
2	1.02010	1.04040	1.06090	1.08160	1.10250	1.12360
3	1.03030	1.06121	1.09273	1.12486	1.15763	1.19102
4	1.04060	1.08243	1.12551	1.16986	1.21551	1.26248
5	1.05101	1.10408	1.15927	1.21665	1.27628	1.33823
6	1.06152	1.12616	1.19405	1.26532	1.34010	1.41852
7	1.07214	1.14869	1.22987	1.31593	1.40710	1.50363
8	1.08286	1.17166	1.26677	1.36857	1.47746	1.59385
9	1.09369	1.19509	1.30477	1.42331	1.55133	1.68948
10	1.10462	1.21899	1.34392	1.48024	1.62889	1.79085
11	1.11567	1.24337	1.38423	1.53485	1.71034	1.89830
12	1.12683	1.26824	1.42576	1.60103	1.79586	2.01220
13	1.13809	1.29361	1.46853	1.66507	1.88565	2.13293
14	1.14947	1.31948	1.51259	1.73168	1.97993	2.26090
15	1.16097	1.34587	1.55797	1.80094	2.07893	2.39656
16	1.17258	1.37279	1.60471	1.87298	2.18287	2.54035
17	1.18430	1.40024	1.65285	1.94790	2.29202	2.69277
18	1.19615	1.42825	1.70243	2.02582	2.40662	2.85434
19	1.20811	1.45681	1.75350	2.10685	2.52695	3.02560
20	1.22019	1.48595	1.80611	2.19112	2.65330	3.20714
35	1.41660	1.99989	2.81386	3.94609	5.51602	7.68609

2. With interest compounded annually, to what amount will \$1 grow in (*a*) 5 yr. at 4%? (*b*) 5 yr. at 3%? (*c*) 10 yr. at 5%? (*d*) 20 yr. at 4%? (*e*) 15 yr. at 3%? (*f*) 20 yr. at 5%? (*g*) 35 yr. at 6%?

Do you see how the Compound-Interest Table on the preceding page can be used to find the amount to which any number of dollars, as \$1000, will grow in 10 years, at 4% compounded *semiannually?*

SOLUTION. 4% a year is 2% a half year; in 10 years there are 20 half-year periods. From the table, \$1 in 20 periods at 2% will grow to \$1.48595; hence \$1 in 10 years at 4% compounded semiannually will grow to \$1.48595. Hence, \$1000 will grow to $1000 \times \$1.48595$, or \$1485.95.

1. Show how to solve Exs. 1 to 6 on page 230 by means of the table.

2. Find by the table what you would have possessed on your 5th birthday if on the day of your birth your father had deposited \$200 for you in a savings bank paying 4% compounded semiannually. (How many interest periods are there in 5 years?)

3. How much would you have had on deposit on your 6th birthday? on your 7th birthday?

4. How much would you have had on deposit on your 8th birthday? on your 10th birthday?

5. To what amount will \$500 grow in 4 years, at 6% interest compounded annually?

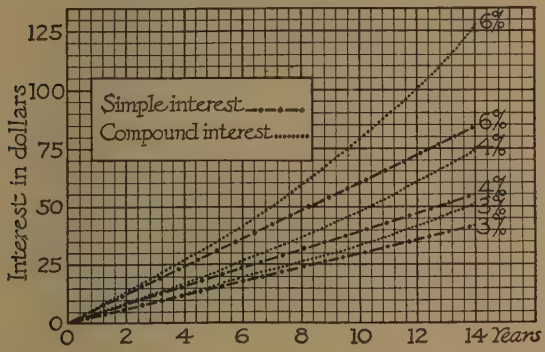
6. To find the compound interest on \$500 at 6% per annum for 4 years, you must, of course, subtract \$500 from the amount found in Ex. 5. Find the compound interest on \$500 at 6% for 4 years, compounded annually.

7. What is the compound interest on \$1000 at 4% per annum for 3 years, compounded semiannually?

8. To what amount will \$750 grow in 10 years at 4% interest compounded semiannually? What is the compound interest?

Many persons neglect to invest small sums collected as interest; in short, they do not *reinvest* their money. They get simple interest instead of compound interest. In time, the difference is very great. This fact is easily seen by this graph. The straight lines show the growth in simple interest on $100 invested at different rates of interest during 14 years. The curved lines show the growth of compound interest on $100 compounded annually over a 14-year period.

GRAPH SHOWING THE CORRESPONDENCE BETWEEN SIMPLE AND COMPOUND INTEREST ON $100



1. Which kind of interest grows more rapidly?

2. What is the difference in the two kinds of interest at 6% per annum on $100 for 10 years? for 12 years? 14 years?

3. You can draw a compound-interest curve, using the table on page 231. On a sheet of graph paper mark the horizontal scale in year units. Use the vertical scale as the interest scale (in cents). Then take the decimal fractions from the table for 6% and mark points that show how much interest is due at the end of 1 year, 2 years, 3 years, etc., up to 20 years. Draw a smooth curve through these points.

4. Make a graph that could be used to compute interest at 5% compounded annually.

5. On the same sheet of graph paper that you used for Ex. 4, draw a straight line showing simple interest at 5%.

6. By the aid of your graph answer the following questions: What is the difference between simple interest and compound interest at 5% on $100 at the end of 5 years? 10 years? 15 years? 20 years?

7. See if you can draw a curve showing the growth of compound interest on $1000 at 6% per annum compounded semiannually for 15 years.

8. If the number of periods is greater than 20 (the largest shown in the table on page 231), you find the amount for 20 periods and use this as the new principal for finding the amount for the next 10 periods.

EXAMPLE. Suppose you wish to find the compound interest on $200 for 16 years at 4% interest compounded semiannually. The amount due on $200 for 10 years (20 periods) is $200 \times \$1.48595$, or $\$\_\_?\_\_$. The amount due on $297.19 for 6 years (12 periods) is $\$297.19 \times \$1.26824$, or $\$\_\_?\_\_$.

9. To what sum would $100 at 4% interest compounded semiannually amount in 20 years?

10. Mr. Crane deposited $100 in a savings bank for his daughter Mary on her 10th birthday. When Mary was 20, the $100 had grown to about $134. Can you tell what rate of interest the savings bank paid? (Use the table.)

11. According to a newspaper story, a certain man forgot that he had $200 in a savings bank that paid 4% interest compounded semiannually. Thirty years later the bank located the man. To about what sum had his principal grown? (Use the table.)

On the day that Susan was born, her parents began to invest $50 a year for her at 6% interest compounded yearly. Would you know how to compute the amount there would be in Susan's fund on her 17th birthday (no deposit being made on that day)? Guess how large it would be!

TABLE SHOWING WHAT ONE DOLLAR A YEAR WILL AMOUNT TO AT INTEREST COMPOUNDED ANNUALLY

PERIODS	2%	3%	4%	5%	6%
1	1.020	1.030	1.040	1.050	1.060
2	2.060	2.091	2.122	2.153	2.184
3	3.122	3.184	3.246	3.310	3.375
4	4.204	4.309	4.416	4.526	4.637
5	5.308	5.468	5.633	5.802	5.975
6	6.434	6.662	6.898	7.142	7.394
7	7.583	7.892	8.214	8.549	8.897
8	8.755	9.159	9.583	10.027	10.491
9	9.950	10.464	11.006	11.578	12.181
10	11.169	11.808	12.486	13.207	13.972
15	17.639	19.157	20.825	22.657	24.673
17	20.412	22.414	24.645	27.132	29.906
20	24.783	27.677	30.969	34.719	38.993
30	41.379	49.003	58.328	69.761	83.802
40	61.610	77.663	98.827	126.840	164.048
50	86.271	116.181	158.774	219.815	307.756

The table above will help you. It should be read:

(1) One dollar deposited at the beginning of each year and allowed to grow at 6% compounded annually will amount at the end of the 5th year to $5.975. (Where shown in table?)

(2) One dollar deposited at the beginning of each year, for 40 yr. at 5% compounded annually, will amount to $126.840.

(3) One dollar deposited at the beginning of each half year for 20 yr., at 4% compounded semiannually, will amount to $61.600 (2% for 40 periods).

1. Show that Susan's fund on her 17th birthday would amount to 50 × $29.906, or ___?___.

1. When Arthur learned how rapidly regular yearly deposits grow at compound interest, he resolved to deposit \$3.65 (1¢ for each day) at the beginning of each year for many, many years. Show by the table that if he did so, and if he got 6% interest compounded annually, at the end of 10 years he would have about $3.65 \times \$13.972$. How could you tell the amount which he would have at the end of 20 years? About how much would it be?

2. Arthur's friend, John, said that any ambitious person should save at least \$52 a year (\$1 a week). Show that if he invested that sum regularly at the beginning of each year at 4% compounded annually, at the end of 20 years he would have $52 \times \$30.969$, or \$__?__. Estimate what he would have at the end of 40 years.

3. Caroline is investing \$10 a year at 4% interest, compounded annually. Show that at the end of 10 years her investments will amount to $10 \times \$12.486$, or \$__?__.

4. Show that if Caroline continues her investment, at the end of 20 years she will have $10 \times \$30.969$, or \$__?__. How much more would she have at the end of the 20th year than at the end of the 10th year?

5. Maurice had been spending 10 cents a day for between-meal sweets. One day his father said: "Maurice, if you keep up this sweet-eating habit of yours for the next 20 years, it will cost you over \$1400. Do you think you can afford that?" How do you suppose Maurice's father arrived at his result?

6. How does the table on page 235 differ from the table on page 231?

Many persons make regular weekly or monthly bank deposits. You might wish to look for a table showing how weekly or monthly deposits accumulate. The savings bank in your community may have such tables.

(1) Have you ever wondered how a bank can afford to equip and maintain its building, pay the salaries of its officers and employees, render free service to its depositors by keeping records of their deposits and withdrawals, provide depositors with free bank books, deposit slips, check books, etc., and pay interest at 2% or 3% on all deposits in savings accounts? To do all this, it is necessary for the bank to be earning money, isn't it? Let us see how a typical small bank, the Oakville Peoples Bank, supports itself.

(2) A group of business men in Oakville realized the need for a bank in the community, and also saw in the establishing of the bank an opportunity to make a good investment. They banded together under the banking laws of the state and invested $50,000 in establishing the bank.

(3) Since these men invested the $50,000 with the hope of making money, they expect the bank to earn for them a fair rate of income on their investment. If the bank could earn $5000 a year for them, would they be earning 10% on their money? Would you consider that a fair rate of income? a high rate of income?

(4) The Oakville Peoples Bank opened for business and soon had deposits totaling $1,000,000. The bank kept about 10% of this amount on hand, or $100,000. The remaining $900,000 it invested and loaned on good security at an average rate of 5%. Hence the bank's annual income from its investments is $45,000.

(5) The bank's profit for one year was its income of $45,000 less its expenses of $41,000 (salaries, rent, interest on deposits, etc.), or $4000.

(6) This profit of $4000 belongs to the owners. Since they had invested $50,000, their rate of income on their investment for the year was $\frac{4000}{50000}$, or $\frac{?}{}$%.

(1) Each year banks issue bank statements showing in detail their *assets* and *liabilities*. A bank's assets are (*a*) the property which it owns, (*b*) the funds which it has on hand, and (*c*) the money which other people owe to it. In general, a bank's liabilities are the money it owes other people.

(2) Tell which of the following amounts would be listed as assets and which as liabilities:

(*a*) The amount of cash in its vaults.
(*b*) The amount of money loaned by the bank to individuals and corporations.
(*c*) The value of the bank building and equipment.
(*d*) The amount owed to depositors.
(*e*) The amount of money invested in securities (stocks, bonds, mortgages, notes, etc.).

(3) Banks usually pay about 2% or 3% interest on money deposited with them and charge 6% or 7% on money that they loan. Can you explain why they do not pay the same rate of interest as they charge?

(4) Sometimes banks use 365 days to the year in computing interest they pay on deposits and 360 days to the year in computing interest they charge on loans. Can you explain why?

(5) The amount of money that banks make as profits fluctuates or varies from year to year in much the same way as profits in other businesses.

(6) Does your state have officials who supervise the banks in the state? Why should banks be under official supervision?

(7) What does it mean to say that a bank has *failed?*

(8) The Federal government now guarantees the safety of bank deposits of $2500 or less.

(9) In times of serious business depression, banks will not, or are unable to, make loans. Then they make no profit. In such times they may charge their depositors for their services.

By Ewing Galloway, N. Y.

(1) Mr. Brown wished to buy his daughter a piano. He had noticed that she and her friends enjoyed playing and singing together at the Clarks' home.

(2) He had only $100 in cash. The dealer was willing to sell a piano for $100 "down" and installments of $20 a month for 7 months, with interest at 6%. Mr. Brown bought the piano on these terms on December 20, paying $100 down and giving promissory notes of $20 each for the remainder. (How many notes did he give?)

(3) The first note was due January 20. When was the second one due? the third one? the last one?

(4) Each note drew interest at 6% per annum. On January 20 Mr. Brown paid the first note, with interest on it for 1 month. How much did he pay?

(5) On February 20 he paid the second note, with interest on it for 2 months. How much was that?

(6) Make a table showing how much Mr. Brown had to pay each month until the debt was paid.

(7) What was the total amount of money Mr. Brown paid out?

(8) Mrs. Brown thought that they shouldn't have bought the piano until they were able to pay cash for it. What do you think?

(9) Mr. Brown said that he was really borrowing the $140 ($240 − $100) from the piano dealer at 6%. Do you agree?

1. Mr. and Mrs. Nelson paid \$2000 cash on a farm costing \$8000. They agreed to pay \$500 every 6 months, plus interest at 6% per annum on the unpaid balance.

(*a*) How long would it take them to pay for the farm?

(*b*) At the end of 6 months they paid \$500 plus \$180 interest on \$6000. (Why \$180?)

(*c*) At the end of 12 months they paid \$500 plus interest on \$5500. How much interest?

(*d*) What should they pay at the end of 18 mo.? 24 mo.?

(*e*) How does this problem differ from the piano problem on the preceding page?

2. Mr. Work bought a house and lot for \$7500. He paid \$500 cash and agreed to pay \$500 every 6 months, with interest at 6% per annum on the unpaid balance.

(*a*) At the end of the first 6 months he paid \$500 plus 6 months' interest on \$7000 at 6% per annum, making a total payment of \$ ___?___.

(*b*) At the end of the second 6 months he paid \$500 plus 6 months' interest on \$6500, or a total of \$ ___?___.

(*c*) At the end of the third 6 months he paid \$500 plus 6 months' interest on \$ ___?___, or a total of \$ ___?___.

(*d*) How much will he still owe at the end of 5 years?

(*e*) How long will it take him to pay for the house if he keeps up his payments regularly?

3. George has \$100 in cash and can save about \$30 a month. He has found a car which he likes and may pay either of the following:

(*a*) \$100 cash and \$20 a month for 12 months, no interest.

(*b*) \$90 cash and \$20 a month for 12 months, with interest on each payment at 6% per annum.

If George chooses (*b*), his payment at the end of the first month will be \$20 plus interest at 6% per annum on \$20 for 1 month, or a total of \$ ___?___. Which is the better offer for George? How much better is it?

1. An eighth-grade class wanted to buy a radio. One could be bought for $150 cash or by paying $25 down and $25 a month for 6 months. Is the installment price more or less than the cash price? How much?

2. The class would have liked to borrow the money from a bank to pay cash for the radio, but it had nothing to offer as security for the loan; so on October 1 it bought the radio on the installment plan.

(1) Is it true that the extra cost of the installment price above the cash price ($25) may be considered as the interest on the monthly payments? What rate of interest do you think the class was paying?

$25 for 1 mo. (Oct.)
$25 for 2 mo. (Nov.)
$25 for 3 mo. (Dec.)
$25 for 4 mo. (Jan.)
$25 for 5 mo. (Feb.)
$25 for 6 mo. (Mar.)
$25 for 21 mo.

(2) Let us find the interest on the monthly payments at 6% per annum, the customary rate charged by banks.

(3) The interest on the payment made at the end of the first month (Oct.), plus the interest on the second payment made at the end of two months (Nov.), plus the interest on each of the other payments (as shown in the table above), is equivalent to the interest on $25 for 21 months. At 6%, the interest on $25 for 21 months is ___?___.

(4) But we have seen in (1) that the interest that is being paid on the 6 monthly installments is not $2.625 but $25. The rate of interest being paid on the radio is therefore $\frac{\$25}{\$2.625} \times 6\%$, or 57.1%. Does that seem to be a high rate?

3. A seventh-grade class also wanted a radio. The cash price of the set they wanted was $130. The installment price was $30 down and $10 a month for a year. Show that if the class chose the installment plan, the yearly rate of interest would be $30\frac{1}{3}\%$. (Use the plan shown above.)

On the installment plan, the buyer is really borrowing money from the seller.

A committee was appointed in an eighth-grade class to prepare a report on the subject, "Putting Your Dollar to Work." Some of the high spots in the committee's report are suggested here.

1. Compound interest has a magic-like power to make money grow. For example, one dollar, at 4% interest compounded semiannually, will grow to two dollars in $17\frac{1}{2}$ years; in 35 years it will grow to four dollars; in $52\frac{1}{2}$ years it will grow to eight dollars; in 70 years it will grow to sixteen dollars. In other words, at 4% money doubles itself every $17\frac{1}{2}$ years. (To what sum do you think \$100 deposited in a savings account at 4% interest compounded semiannually would amount in $17\frac{1}{2}$ years? in 35 years? in $52\frac{1}{2}$ years? in 70 years?)

2. At 6% interest compounded semiannually a dollar will grow to two dollars in approximately 12 years; in 24 years it will grow to four dollars; in 36 years it will grow to eight dollars; in 48 years it will grow to __?__ dollars. That is, at 6% it doubles itself every __?__ years. (To what sum do you think \$100, invested so as to make 6% interest compounded semiannually, will grow in 12 years? in 24 years? in 48 years? in 96 years?)

3. A good way to make your money earn money is to deposit it in a savings bank and leave it there. Most people do not appreciate the importance of *leaving it there*. (Explain what is meant.)

> *"If you want to know whether you are destined to be a success or not, you can easily find out. The test is simple and it is infallible. Are you able to save money? If not, drop out. You will lose. You may think not, but you will lose, sure as fate, for the* seed of success *is not in you."*
>
> JAMES J. HILL

1. The Newton School has a school bank. The school bank deposits the savings of its pupils as one account in a regular savings bank. If the school deposits \$146.50 Mar. 1, and the bank pays 3%, how much interest is due on the following July 1?

2. Mrs. Potter deposits \$30 on the first business day of each month in a savings bank that pays 3% per annum compounded on January 1 and July 1, subject to the usual rules. If she opened her account on the second of January, what will her balance be on the following January?

3. If simple interest is collected when due and at once invested, the investor practically gets compound interest even though we never speak of such an investment as yielding compound interest. Many persons fail to collect and reinvest their interest promptly when due. How much interest will be lost if Mr. Nelson, who has loaned \$5000 at 6% with interest payable semiannually, neglects to reinvest any of his interest during the first two years?

1. A note for \$1780, dated August 1, drew interest at 6%. How much interest was due on the following December 19?

2. Mr. Brown wished to buy a new wagon. He could choose either of these two plans for paying for it:

(*a*) \$120 cash, or (*b*) \$130 at the end of 6 months, without interest.

Should he borrow the money at 6% and pay cash, or should he pay \$130 at the end of 6 months?

3. Bernice bought 12 dozen oranges at 50 cents a dozen, less 10% for cash. She sold 80 of them at 2 for 15 cents, and the rest at 4 for 25 cents. What was her margin?

4. Earning simple interest at 6% per annum, how long would it take \$1000 to earn \$60 interest? to earn \$30? \$15? \$120? \$8? \$1?

5. A merchant bought a bill of goods for \$1000. The terms were "2% off for cash" or "net in 30 days." Should he borrow enough money, at 6% per year, in order to pay cash, or should he pay the full amount in 30 days?

6. How much money should be invested at 6% so that the simple interest would be \$1200 a year?

7. Marie went to a summer camp on the morning of July 2 and returned home late in the evening of September 1. Her total expenses were \$265, or an average of how many dollars a day?

8. For manufacturing a small advertising folder, a printing house charges \$5.50 for the first hundred and \$1.50 for each additional hundred. Make up a formula by which you could find the price of any number of folders of that size.

9. A merchant paid \$3 for a pair of shoes, which he wished to sell at a profit of 10% of the selling price. His overhead expenses averaged 15% of the amount of his sales. At what price should he mark the shoes?

1. A merchant paid \$2.50 for a hat and sold it at an increase of 40% of the cost price. What was his margin? His margin was ___?___ per cent of the selling price.

2. A farmer paid \$50 for a plow. Two years later he sold it for \$20. This was a decrease of ___?___ per cent of the cost price.

3. Find the interest on each of these principals for 60 days at 6%:

(*a*) \$150	(*c*) \$676	(*e*) \$90	(*g*) \$2.00
(*b*) \$265	(*d*) \$25	(*f*) \$1.00	(*h*) \$5.00

4. With simple interest at 6%, how long will it take a dollar to earn a dollar?

5. What is i if $p = \$645$, $r = 4\frac{1}{2}\%$, and $t = 90$ days?

6. Miss Evans had a balance of \$80 in her savings account on January 1. During the next 6 months her deposits were: January 15, \$20; March 1, \$20; June 1, \$40. The bank paid 4% per annum compounded semiannually. What was her balance on July 1?

7. Find the hypotenuse of a right triangle when its two sides are 6″ and 8″ respectively.

8. Find the area of a circle whose radius is 5′.

9. Solve and check these equations:

(*a*) $4y + 5 = 15$ (*b*) $.06p = 100$ (*c*) $\frac{2y}{3} = 18$

(*d*) $4y - 5 = 15$ (*e*) $.6p = 100$ (*f*) $\frac{3y}{2} = 18$

10. A farmer wishes to double the area of his rectangular chicken yard. If he doubles its present length and width, will it be the desired size?

11. What number increased 50% of itself equals 30?

12. What number decreased 50% of itself equals 30?

1. Many pupils think that $\frac{1}{4}\%$ of 1000 is 250. How do you account for this?

2. At 4% a year, how long would it take \$1000 to earn \$60 interest? to earn \$20? \$5? \$1?

3. Earning simple interest, what principal at 6% will earn (*a*) \$9 in a year? (*b*) \$15 in a year? (*c*) \$1.50 in 60 days? (*d*) \$1 in a day?

4. If Mr. Norton deposited \$10,000 in a savings bank that pays 4% interest compounded semiannually, what would his balance be at the end of 5 years? (Use a table.)

5. Solve and check the following equations:

(*a*) $\frac{3}{4}c = 21.60$	(*d*) $m - \frac{1}{10}m = 45$
(*b*) $c + \frac{1}{4}c = 105$	(*e*) $c + \frac{1}{5}c = 48$
(*c*) $c - .20c = 48$	(*f*) $c + \frac{1}{10}c = 44$

6. A clothing dealer sold a suit for \$21.60, at a loss of 25% of the cost price. What was the cost price? (See Ex. 5(*a*).)

7. A merchant marked a radio to sell at \$105, which was an increase of 25% of the cost price. What did it cost? (See Ex. 5(*b*).)

8. Which contains more ice cream, a Mellow Block, which is a prism $3'' \times 2'' \times \frac{3}{4}''$, or a Cream Roll, which is a cylinder with a length of $1\frac{1}{4}''$ and a radius of $1''$?

9. In triangle ABC, angle B equals 90 degrees. The ratio between which two sides of the triangle expresses the tangent of angle A?

10. How many feet of fence would be needed to enclose a square lot whose area is 120 square feet?

11. If a man saved \$365 a year for 10 years, and received 4% interest compounded annually, what amount would he have at the end of the 10-year period? (Use the table.)

1. Make a formula which represents the annual earnings (E) of a man who receives a salary of \$100 a month and a commission of 3% of his sales (S). Then make a graph showing the relation between E and S.

2. A farmer wished to buy a truck, the price of which was \$600. He could pay for it in 3 months without interest, or he could get 2% off for cash. Which plan should he take, if money is worth 6%?

3. Mr. Hassler gave each of his two sons, Joe and Henry, \$1000 when they were 21 years of age. Joe deposited his \$1000 in a savings bank that paid 4% interest compounded semiannually. Henry invested his in a 40-acre farm, which he rented to a neighbor for \$2.50 an acre per year. His expenses (taxes and upkeep) averaged \$60 a year. At the end of 4 years he sold the farm for \$30 an acre. Which son profited more from the use of the \$1000? How much more?

4. Mr. Crawford pays \$55 rent each month. He thought of buying a house. A real-estate dealer told him that his yearly cost of living in his own house would be about 8% of the purchase price. This included interest on the investment, upkeep, and depreciation. What could he afford to pay for a house in order to keep his rent the same?

5. In right triangles ABC and XYZ, angle A and angle X equal 45°. Can you prove the triangles similar? If you know that AB equals XY, can you prove them congruent?

6. Find the total area of a cylindrical column with a diameter of 18 inches and a height of 10 feet. (Include the area of the two ends of the column.)

7. Construct an isosceles triangle ABC, in which $AC = BC$. Would you say that the bisector of $\angle A$ would also bisect side BC? that the bisector of $\angle C$ would bisect side AB?

CHAPTER VIII

INSURANCE AND TAXES

Life Insurance

Mr. Kline: Good morning, Mr. Walters. My name is Kline. I represent the Reliable Life Insurance Company. I have come to see if you are interested in taking out some life insurance.

Mr. Walters: I have been thinking that I should make some financial provision for my wife and baby in case of my death; so I shall be glad to talk the matter over with you.

Mr. Kline (handing Mr. Walters a folder containing the table shown on the next page): This little folder shows you four kinds of insurance which my company sells, and the yearly rates per $1000 of insurance at various ages.

YEARLY INSURANCE RATES PER $1000 OF INSURANCE

AGE	ORDINARY LIFE	20-PAYMENT LIFE	20-YEAR ENDOWMENT	TERM
20	$18.69	$27.66	$47.80	$10.80
25	20.14	30.12	48.15	11.40
30	22.85	32.87	48.83	12.10
35	26.35	36.22	49.85	13.19
40	30.94	40.38	51.48	15.02
45	37.08	45.73	54.22	18.40
50	45.45	52.87	58.81	24.50

Mr. Walters (examining the folder): Let's see. I am 25 years old. If I take out an *ordinary life policy* for $1000, the insurance company promises to pay Mrs. Walters $1000 if I should die. I'd have to pay the company $20.14 a year.

Mr. Kline: Yes. As the *insured*, you make Mrs. Walters the *beneficiary*, and $1000, the *face* of the policy, would be paid to her at your death. The *premium* on that policy is $20.14.

The folder, which Mr. Walters read, contained the following explanation of the four different kinds of policies.

In most forms of insurance part of the premium goes to pay for the insurance and part is paid in to the company in the nature of an investment. This second part accumulates as a fund called the *cash surrender value* of the insurance. Whenever insurance is discontinued, this fund is paid back to the one who was insured.

(1) *Term* insurance has no investment feature. All of the premium goes to pay for the insurance from year to year, and if the insurance is dropped there is no refund.

(2) *Ordinary life* insurance is insurance with a small investment feature. For example (see the table above), at age 20 the term insurance premium is $10.80 a thousand. The ordinary life premium is $18.69 a thousand. Of this, $10.80 goes to pay for the cost of insurance and the difference ($18.69 − $10.80, or $7.80) is invested with the company. The cash

surrender value of an ordinary life policy any number of years after the policy was taken out is the sum of these investments plus compound interest on them at about 3%.

(3) A *20-payment life* policy is the same as an ordinary life policy except that larger premiums are paid so that the insurance may be fully paid for in 20 years. The premium is greater and the cash surrender value is built up faster.

(4) A *20-year endowment* policy is the same as a 20-payment life policy except that the premium is large enough to make the cash surrender value equal to the face of the policy at the end of 20 years.

Mr. Walters: Do you think it would be better for me to take out a 20-year endowment policy?

Mr. Kline: Not now. Since you are interested primarily in protecting your wife and baby if you die, the ordinary life policy is excellent. If you should wish to use insurance as a method of saving or investment, I would recommend the 20-year endowment. At your age the yearly premium on insurance for $1000 is __?__ more on an endowment policy than on an ordinary life policy. [See the table, page 249.] The endowment policy forces you to save that $28 each year, just as though you were depositing it in a bank, and the insurance company is paying you interest on it.

Mr. Walters: Well, I believe the ordinary life policy is the one I should take. That will protect my wife in the event of my death. I want to put all my other savings for the next few years into a house I am paying for, anyway.

Mr. Kline: I think that is a wise decision. For how many thousand dollars do you wish to insure yourself, Mr. Walters?

Mr. Walters: I think I'll take out a policy for $5000.

Mr. Kline: Good! I'll make an appointment for you with the company's medical examiner.

Mr. Walters: Thank you, Mr. Kline.

Here are the important terms used in life insurance:

(1) The person whose life is insured is called the ***policyholder*** or the ***insured***.

(2) The person to whom the insurance is to be paid is called the ***beneficiary***.

(3) The contract between the company and the insured is called the life insurance ***policy***.

(4) The regular payments which the insured makes to the company are called ***premiums***. (The amount of the premium varies with the age of the person and the kind of insurance.)

That evening the discussion of life insurance continued at the Walters home.

Mr. Walters: I filled out an application for a 5000-dollar life insurance policy today, Anna. I have been wondering a great deal lately about what would become of you and the baby if something should happen to me.

Mrs. Walters: Well, let's hope you will live for many years. I'm glad you took out the insurance, though. It will give me a feeling of security. What does the policy cost?

Mr. Walters: My premium will be about $100 a year. I figure we will have to set aside $8.50 each month to meet that expense. [Is Mr. Walters' estimate correct?]

Tom Walters (13-year-old nephew): I never could understand this life insurance business. You pay the company $100 a year, and if you die the day after you take out your policy, the company hands over $5000. I don't see how they can afford to do that.

Mr. Walters: True, the company can't predict when any individual whose application it accepts is going to die, but it can predict the *average* length of life of the great number of people it insures. It determines this by the *law of averages*.

Tom: I don't understand.

Mr. Walters: Well, perhaps this *mortality table* which Mr. Kline gave me will help you to understand what I mean. [See the table on the next page.] After studying the length of life of a great many people over a long period of years, the insurance companies have computed that under ordinary circumstances, out of every 100,000 children 10 years of age, 749 may be expected to die within the year. Of the 99,251 children in the group who reach 11 years of age, 746 may be expected to die during the next year, etc.

Tom: I see. Of the 98,505 who reach 12 years of age, ___?___ may be expected to die before reaching 13 years.

Mr. Walters: How many of the 100,000 persons may be expected to reach the age of 50? How many may be expected to die that year? How many may be expected to reach the age of 95? How many may be expected to die that year?

[Answer Mr. Walters' questions and discuss further the meaning of the table with the class.]

Tom: I understand the table, but how do the insurance companies use it?

Mr. Walters: I am coming to that. Suppose that each person in this group of 100,000 10-year-olds who lives to be 40 years of age wishes to take out a 1000-dollar insurance policy. According to the table there would be 78,106 such persons, and 765 of them would be expected to die during the year. Therefore, to meet the insurance for those 765 persons, a fund of $765,000 must be accumulated by the insurance company ($765 \times \$1000$). Hence, if the 78,106 persons share the cost equally, each will have to pay \$9.80 ($\$765{,}000 \div 78{,}106 = \$9.794$, or \$9.80).

If the 77,341 surviving members wish to be insured for \$1000 each for another year, they will have to contribute \$774,000 ($774 \times \$1000$). If they share that amount equally, each will have to contribute \$10.01 ($\$774{,}000 \div 77{,}341 = \$10.01$).

American Experience Table of Mortality

Illustrating the rate at which 100,000 persons, normally situated, will die. Beginning at age 10.

Completed Age	Number Surviving at Each Age	Deaths in Each Year	Completed Age	Number Surviving at Each Age	Deaths in Each Year
10	100,000	749	53	66,797	1,091
11	99,251	746	54	65,706	1,143
12	98,505	743	55	64,563	1,199
13	97,762	740	56	63,364	1,260
14	97,022	737	57	62,104	1,325
15	96,285	735	58	60,779	1,394
16	95,550	732	59	59,385	1,468
17	94,818	729	60	57,917	1,546
18	94,089	727	61	56,371	1,628
19	93,362	725	62	54,743	1,713
20	92,637	723	63	53,030	1,800
21	91,914	722	64	51,230	1,889
22	91,192	721	65	49,341	1,980
23	90,471	720	66	47,361	2,070
24	89,751	719	67	45,291	2,158
25	89,032	718	68	43,133	2,243
26	88,314	718	69	40,890	2,321
27	87,596	718	70	38,569	2,391
28	86,878	718	71	36,178	2,448
29	86,160	719	72	33,730	2,487
30	85,441	720	73	31,243	2,505
31	84,721	721	74	28,738	2,501
32	84,000	723	75	26,237	2,476
33	83,277	726	76	23,761	2,431
34	82,551	729	77	21,330	2,369
35	81,822	732	78	18,961	2,291
36	81,090	737	79	16,670	2,196
37	80,353	742	80	14,474	2,091
38	79,611	749	81	12,383	1,964
39	78,862	756	82	10,419	1,816
40	78,106	765	83	8,603	1,648
41	77,341	774	84	6,955	1,470
42	76,567	785	85	5,485	1,292
43	75,782	797	86	4,193	1,114
44	74,985	812	87	3,079	933
45	74,173	828	88	2,146	744
46	73,345	848	89	1,402	555
47	72,497	870	90	847	385
48	71,627	896	91	462	246
49	70,731	927	92	216	137
50	69,804	962	93	79	58
51	68,842	1,001	94	21	18
52	67,841	1,044	95	3	3

Notice that the charge is a little higher than it was the previous year. This is true because (1) there are fewer persons to share the cost (765 persons out of the group die at the age of 40) and (2) the number of death claims that must be paid is slightly higher the second year.

The 76,567 persons of the group who reach 42 years of age would have to contribute $785,000 to meet the death claims that year. [Show that the contribution of each for the year would have to be $10.26.]

[Tom figured that the rate for the 43-year-old group was $10.52 a year; for the 44-year-old group, $10.83; and for the 45-year-old group, $11.17. How did he get those rates?]

Mr. Walters: That shows how the mortality table helps an insurance company to calculate the cost of insurance. Of course, the company does not change the rate each year. It figures the cost for the "expected lifetime" of the insured and averages the premiums so that the premium of a policy is the same from year to year. The expense of running the company raises the rates slightly, and the interest earned by the money contributed lowers the rates; but we need not consider that just now.

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1. Why does an insurance company require a person to pass a medical examination before issuing him a policy?

2. Why does a person 50 years old have to pay a much higher premium when he takes out a life insurance policy than a man 25 years old?

3. What are the four main kinds of insurance policies mentioned in this discussion?

4. What is the chief difference between an ordinary life policy and a 20-year endowment policy?

5. Explain each of these terms as used in life insurance: premium, beneficiary, face of policy, policyholder.

6. What does life insurance insure against?

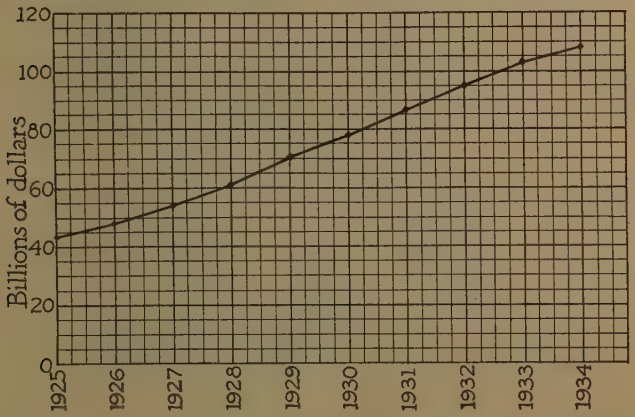
7. Using the ordinary life rates given on page 249, find the yearly premiums for these policies taken out at the given ages:

(*a*) \$1000, age 20 (*d*) \$4000, age 35 (*g*) \$7000, age 40
(*b*) \$2000, age 25 (*e*) \$5000, age 40 (*h*) \$6000, age 25
(*c*) \$3000, age 30 (*f*) \$6000, age 45 (*i*) \$7500, age 35

8. Using the table on page 249, find the yearly premiums for these policies:

	Face	Age	Kind		Face	Age	Kind
(*a*)	\$3000	20	Term	(*e*)	\$10,000	20	20-yr. end't
(*b*)	\$5000	25	20-pay. life	(*f*)	\$25,000	30	Term
(*c*)	\$2500	30	20-yr. end't	(*g*)	\$50,000	40	Ordinary life
(*d*)	\$5000	35	Ordinary life	(*h*)	\$1500	50	20-pay. life

Life Insurance in Force in U. S. during a Recent Period



9. This graph shows that during a recent 9-year period the amount of life insurance carried in the United States increased from about __?__ billions of dollars to about __?__ billions. Can you write these two amounts of money in figures?

During the recent depression, when people were losing their savings in stocks, bonds, building and loan associations, banks, mortgages, etc., they, almost without exception, lost nothing of what they had saved in life insurance. As the worst of the depression passed, the insurance companies found that they had approximately as much insurance in force as there had been at the end of 1929, when the depression began.

Not only did the insurance companies hold their customers during the hard times, but they paid out in claims eleven billion dollars during the years 1929 to 1933. Moreover, their resources increased during those years from seventeen billions of dollars to twenty-one billions. Do you ask: "How could they possibly do this when business all around them was losing money?"

Consider and discuss the following reasons why the insurance companies stood so firmly and, as you have seen, even grew during the depression:

(1) They were assured of a large income both from past customers and from a steady stream of new customers.

(2) They always underestimate their income and overestimate their expenses in order to "play safe."

(3) They have the most skillful advisers in the country to direct their investments, and they spread these investments out among many good types of enterprises. In other words, they do not "carry all their eggs in one basket."

(4) Finally, they can foretell their obligations accurately, with the help of the American Experience Table of Mortality, and these obligations are spread over such a long period of time that they seldom find it necessary to sell any security or property when it will bring a low price. They can just hold on to such property until the price goes up again. When securities must be sold to meet their obligations, they can sell those upon which they need lose no money at the time of sale.

Insurance often causes large sums of money to be paid to persons who have had but little experience in using money wisely. Temptations to invest so as to "get rich quick" are very great. A banker with a fine public spirit has urged that the President of the United States issue the following warning and that it be fastened to each life insurance policy:

United States Government statistics, as well as figures and facts obtained from insurance companies, banks, trust companies, and other reliable sources throughout the United States, show that millions of dollars are lost annually by those who make unwise and unsafe investments. The sad part is that most of the money is lost by those who cannot afford to lose it. This is especially true of widows and others who receive insurance money.

In many cases friends of the family, whose intentions are the very best, advise widows to invest their insurance money in securities that they think are safe, but often as a result every dollar of the money is lost. In other instances unscrupulous salesmen induce the beneficiaries to invest in worthless securities, and all the money is lost. As a result the widow is compelled to suffer hardship in the effort to support her family.

This warning is issued in the hope that all insurance companies will attach a copy to every policy issued hereafter and enclose a copy with every check sent out in settlement of a death claim.

(Signed) .....
President of the United States

To .....
(Fill in name of beneficiary)

I have read the above warning and it is my wish that you be extremely careful with the insurance money. I have taken this insurance to protect you — not to help someone else to get rich.

.....
(To be signed by policyholder)

Mention some ways in which persons inexperienced in handling funds might lose money. What effect do you think the warning would have?

Underwood & Underwood

Mrs. Martin: Have you heard that Meyer's Grocery Store in Linton was completely burned out last night?

Mrs. Finch: Oh! That is too bad. Was it insured?

Mrs. Martin: Unfortunately not!

Mrs. Finch: What a mistake!

Have you ever noticed that when property is damaged, whether it is by fire, lightning, theft, flood, tornado, or collision, the first question usually asked is, "Was it insured?"

Such insurance is an agreement between a company and the owner of property. The insured party (property owner) agrees to pay to the company a specified amount called the *premium*. On the other hand, the company agrees to pay to the insured party an amount sufficient to cover or nearly cover the loss or damage to his property from specified causes. The amount of money the company agrees to pay in such case is called the *face of the policy*. Insurance against fire is used most generally and should be understood by everyone.

On January 15 Mr. Cashman insured his store against loss by fire within one year of date to the amount of $3000. For his insurance policy he paid $.20 per $100 (that is, $.20 for each $100 of the $3000).

(1) The face of his policy was ___?___.

(2) His premium (amount he had to pay to the company) was $30 \times \$.20$, or ___?___. Why $30 \times \$.20$? (There are ___?___ hundreds in 3000.)

(3) The term (length of time during which the company assumed the risk) was ___?___ year.

(4) The insurance company insured Mr. Cashman's store for only $\frac{3}{4}$ of its value. Do you know what its value was? When Mr. Cashman took out his insurance, he could not get a policy for $4000, even though his store was valued at that. Companies are always careful to make the face of the policy less than the worth of property that is insured.

In completing Exercises 1 to 4, give reasons for your answers.

1. The cost of insuring a brick or steel building is ___?___ than the cost of insuring a frame house of the same value.

2. The rate for insuring property in a town with a poor fire department is ___?___ than the cost of insuring property in a city with a good fire department.

3. The rate for insuring property in a sparsely settled residential section of a city is ___?___ than the cost of insuring in a crowded downtown business section.

4. The rate of insurance on a candy store would be ___?___ than the rate on a garage.

5. The rate of insurance which Sam Clark pays on his house is $8 per $1000. He has his house insured for $4000. His annual premium is ___?___.

6. Mr. Miller's house was insured for $2400 at 60¢ per $100. His premium was ___?___.

7. The cost of insuring a building valued at \$30,000 for 80% of its value at the rate of 15¢ per \$100 is ___?___.

8. The cost of insuring furniture worth \$3600 for $\frac{5}{6}$ of its value at the rate of 40¢ per \$100 is ___?___.

9. How can an insurance company afford to pay a man \$2400 when his barn burns if he has paid the company only a small premium each year — let us say, \$18?

10. Mr. Green insured his \$8000 house under a 3-year policy for $\frac{4}{5}$ of its value at \$1.25 per \$100. What was his premium for the term of 3 years?

11. Mr. Green (Ex. 10) could have insured his property at the yearly rate of 50¢ per \$100. How much did he save in three years by choosing the 3-year rate?

12. Why do you think a company can give a cheaper rate for a long-term insurance policy?

13. In case of fire, when one's property is insured by an ordinary fire insurance policy, the company pays the full amount of loss up to the face of the policy. Thus, suppose an office building worth \$160,000 is insured for $\frac{3}{4}$ of its value.

(*a*) If a fire should damage that building to the extent of \$80,000, the owner would receive \$80,000 from the company.

(*b*) If the property should be damaged by fire to the extent of \$120,000, the owner would receive \$120,000.

(*c*) If the property should be damaged by fire to the extent of \$140,000, the owner would receive only \$120,000.

(*d*) How much would the owner receive if the property were totally destroyed by fire?

14. A grain elevator worth \$15,000 was insured for $\frac{2}{3}$ of its value at the rate of $\frac{5}{8}$%. What was the annual premium? A fire damaged the elevator to the extent of \$12,000. What was the owner able to collect?

By Ewing Galloway, N. Y.

A Hazardous Street Crossing

In a recent year there were in the United States 745,300 automobile accidents, in which 904,800 persons were injured and 29,000 were killed. An automobile insurance company that year made the following statements. Did they agree on the average with the facts stated above?

A death by automobile every 18 minutes.
A person injured non-fatally every 35 seconds.

The owner of a car is liable for damage to property and injury to persons caused by his car. To avoid the possibility of having to pay a large sum in such a case, it is customary for automobile owners to take out insurance against ***property damage*** and ***personal liability***. Some states have passed Motorists' Financial Responsibility Laws requiring operators of automobiles to show proof that they can meet the expense involved if they cause injury to persons or property while driving their cars. Property damage and personal liability insurance are considered to be sufficient proof in such cases.

There are four important forms of automobile insurance.

Personal liability insurance protects the owner against financial loss for personal injury to other persons by his car. The insurance company assumes the cost of immediate surgical relief and subsequent medical care for the injured, will pay for investigation, will furnish legal defense in case of lawsuit, and will pay any damages awarded against the owner to the limit of his policy. The usual minimum coverage (face of policy) is $5000 where one person is injured and $10,000 where more than one person is involved. Higher limits cost only slightly more and are advisable.

The owner of a car insured for personal liability is not protected when his car is driven by an unlicensed driver. (Why does this seem just?)

Property damage insurance protects the owner against loss on account of damage done by his automobile to the car or other property of another person to the limit of the policy. Cost of investigation and legal defense are assumed by the insurance company whether or not the driver is finally held liable. The usual coverage is from $1000 to $5000. Higher limits are obtainable.

Fire insurance protects the owner against loss on account of damage to his car by fire, or damage caused by lightning, explosion, or self-ignition on the road, in a garage or paint shop, or on train, steamboat, or ferry.

Theft insurance protects the owner anywhere against loss of the car or any part of its equipment by theft.

Collision insurance protects the owner of a car against loss from damage to his own car resulting from collisions.

Automobile insurance rates differ greatly, according to where the automobile is owned and operated and the size and age of the car. Why do you think insurance rates are higher in a city than in a small town?

On personal liability and property damage insurance some companies give lower rates to persons whose records show them to be careful drivers. Is this advisable? Why?

Below is a list of yearly automobile insurance rates on three makes of cars.

Use these rates in solving the problems below.

By Ewing Galloway, N. Y

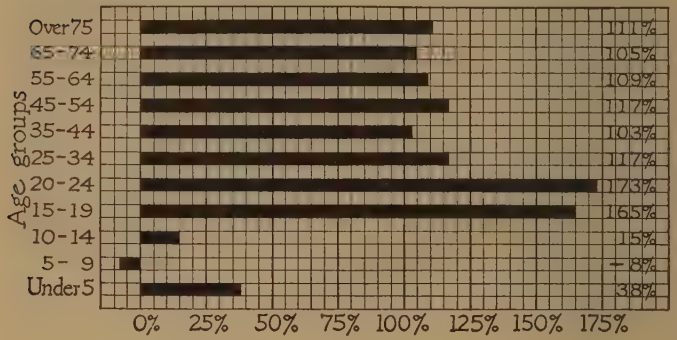
Make of Car	Fire	Theft	Personal Liability $5000-$10,000	Property Damage Liability $5000
A	$.25 per $100	$.45 per $100	$32	$11
B	$.20 per $100	$.35 per $100	$34	$11
C	$.15 per $100	$.25 per $100	$38	$12

1. Find the total annual premium for fire, theft, personal liability, and property damage insurance on a car *A*, worth $600; on a *B* car, worth $775; on a *C* car, worth $1815.

2. Mr. Green carries only personal liability insurance on his *A*, which is worth $695. What is his annual premium? Mr. Green thinks that personal liability insurance is the most necessary type of insurance. Do you agree?

You have studied about life insurance, fire insurance, and automobile insurance. These are only a few of the common kinds of insurance. What other kinds of insurance can you name? Perhaps some of the members of the class would like to make reports on the following kinds of insurance: (1) accident insurance; (2) health insurance; (3) insurance on letters and parcels sent through the mails; (4) travelers' insurance; (5) employers' liability; (6) old age; (7) rain.

PER CENT OF INCREASE IN AUTOMOBILE-ACCIDENT DEATHS BY AGE GROUPS OVER A RECENT EIGHT-YEAR PERIOD



The graph above shows how the death rate in automobile accidents has increased alarmingly over a recent eight-year period.

1. Among children under 5 years of age the death rate increased ___?___ per cent.

2. Among children between the ages of 5 and 9 the death rate decreased ___?___ per cent. Did the death rate decrease in any other age group?

3. What was the per cent of increase of death rate among persons in each of the other age groups?

4. Did the number of deaths more than double for each of the groups from 15 years up?

5. In which age group did the greatest increase in death rate occur? the next greatest?

6. Does this graph suggest that parents should restrict the use of the car by young drivers until such drivers have proved themselves extremely trustworthy and careful?

Underwood & Underwood

Do you have a Safety First Campaign in your school each year? If so, perhaps you may find some information on this page which will be helpful to you in designing posters to make you and your schoolmates more careful in automobile traffic.

In a recent year 47 per cent of all the pedestrians killed by automobiles were violating traffic rules in these ways:

(1) Crossing streets against traffic signals.

(2) Crossing streets diagonally.

(3) Crossing streets in the middle of the block.

(4) Stepping in front of cars from behind parked cars.

Consider the danger which children encounter at play when they run into the street to chase a ball or a playmate. Intent upon the game, they may forget the automobiles.

It is well to remember that snowy and rainy days are accident days. For one reason, the driver cannot see clearly in bad weather. Skidding, too, threatens the pedestrian. Likewise persons carrying umbrellas are in danger of shutting from view an approaching car, in their desire to keep dry.

The pedestrian walking along the highway is also in great danger. He should walk on the left side of the road. Then he can look ahead to cars approaching on his side of the road, and be better prepared to step from danger than he would be if the cars approached him from the rear.

1. This table shows that a car traveling 20 miles an hour is covering 29 feet a second. Check this by computation.

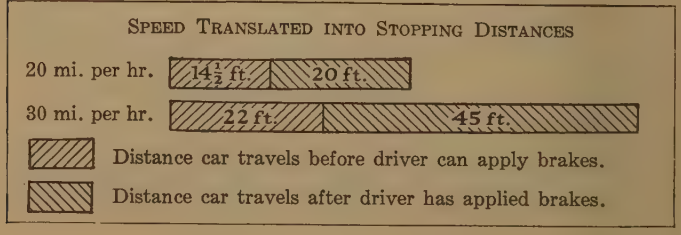
At this many miles per hour	You are traveling so many feet per second	With a destructive force equivalent to a fall of this many feet	Feet needed to stop after brakes are applied
20	29	13	20
25	37	20	31
30	44	30	45
35	51	40	61
40	59	54	80
45	66	68	101
50	73	83	125
55	81	101	151
60	88	120	180

The table also shows that if a car moving 20 miles an hour hits an object, it strikes with a force able to do as much damage as a car dropped from a height of 13 feet.

2. What does the table tell you about a car traveling at a speed of 25 miles an hour? of 30 mi.? 35 mi.? 60 mi.?

3. How does the destructive force of a car going 60 miles an hour compare with that of a car going 30 miles an hour?

4. Accurate tests have shown that the average driver needs a half second to set his brakes after he recognizes danger. During a half second a car moving at 20 miles an hour travels how many feet? (Can you tell from the table above?)


Speed Translated into Stopping Distances
20 mi. per hr.
14½ ft.
20 ft.
30 mi. per hr.
22 ft.
45 ft.
Distance car travels before driver can apply brakes.
Distance car travels after driver has applied brakes.

5. After brakes are applied, a car traveling 20 miles an hour moves 20 feet before coming to a stop. The graph on the preceding page shows that a car traveling 20 miles an hour will move __?__ feet between the time that the driver recognizes danger and the time the car comes to a stop.

6. During the half second between the recognition of danger and the applying of the brakes, a car traveling 30 miles an hour would go __?__ feet. (See table on preceding page.) After the driver's foot is applied to the brake, the car would travel 45 feet before stopping. Hence, a car traveling 30 miles an hour will go __?__ feet between the time that the driver recognizes danger and the time the car stops.

7. If a driver sees a child 40 feet ahead and applies his brakes as quickly as possible, can he stop before hitting the child, if he is driving 20 miles an hour? 30 miles an hour?

8. Mr. A figures that when he is driving 40 miles an hour, his car will go $109\frac{1}{2}$ feet from the time he recognizes danger until his car stops. Do you agree with his estimate? (Use the table on preceding page.)

9. Choose the correct word in completing each of the following sentences.

(*a*) The faster a car is traveling, the (greater, less) the damage it will do when it strikes an object.

(*b*) The faster a car is traveling, the (longer, shorter) the time required to bring the car to a stop.

10. Make a table showing the total distance needed in which to stop a car after the danger is recognized, for the various speeds from 30 miles an hour to 60 miles an hour, as shown in the table on the preceding page.

11. Make a line graph to show the relationship which is expressed in the table you made for Ex. 10. Use the graph to find the total distance a car will travel before it can be stopped, when it is going 47 miles an hour.

Some states will not grant a driver's license to anyone who cannot pass a series of eye tests. One of the tests is to determine the applicant's range or span of vision. A normal person looking straight ahead can see any moving object that comes within a span of 185° to 195° without moving his eyes. A person who has a span of less than 140° is considered a dangerous driver and should not be granted a license.

Can you think of reasons why a large span of vision is necessary for good driving? Does it aid a driver in seeing cars at intersections?

Would you like to find out your range of vision? With the aid of another pupil and your protractor, you can easily do so.

(1) Fasten a sheet of paper along one edge of a large table.

(2) Fasten, with a pin or a thumbtack, one end of a string about a yard long to the middle of the piece of paper on the table, and tie a pencil to the other end of the string.

(3) Kneel down and place your chin on the paper, with the pin directly under your nose.

(4) Keep your eyes steadily fixed on a spot on the wall straight ahead of you. Let your helper, holding the string taut, begin with the pencil out of range of your vision on the right side and move it slowly toward your range of vision, at the same time moving it up and down. Be sure that you do not move your eyes or head in order to see the pencil.

(5) As soon as the pencil comes within your range of vision, —that is, as soon as you can begin to see it move up and down, —have your helper stop moving the pencil and draw a line on the paper to indicate the direction of the string from the pin when you first saw the pencil.

(6) Do the same on the left side.

(7) The angle on your paper in front of the two lines drawn from the pin is the angle of your range of vision.

THE LONGFELLOW SCHOOL, YONKERS-ON-HUDSON, NEW YORK

Can you imagine living in a community where there were no public schools, no health department to protect the people's health, no fire department to help reduce the loss of property by fire, no police department to preserve order, no public provisions for lighting and cleaning streets, clearing and improving roads, or building bridges? We are so accustomed to having these conveniences provided by the governments of our villages, towns, cities, counties, and states, or by the Federal government, that some of us just accept them without asking how these governments can afford to provide such services.

A town must spend money every year for schools, parks, libraries, maintenance of streets, public buildings, public health, salaries, interest on borrowed money, and the like.

It is customary for a town to make a *budget* of proposed expenses for a year and to raise the money needed by *taxes*, principally on real estate (land and buildings). Persons who own buildings and rent them to tenants pay the taxes out of the rents they get from the tenants; so of course everyone has a share in paying taxes, whether or not he owns property.

1. The proposed budget for the town of Lyons estimates that about \$50,000 will have to be collected to meet the town's expenses for the coming year. The ***assessed value*** (estimated value for taxing) of all the property in Lyons is approximately \$2,000,000. Hence the \$50,000 in taxes is to be ***levied*** on the \$2,000,000 worth of property.

\$50,000 is what per cent of \$2,000,000? (\$50,000 ÷ \$2,000,000 = .025 = 2.5%.) This means that each property owner in Lyons must pay a tax of 2.5% of the value of his property. The 2.5% is called the ***tax rate***.

2. Mr. Spencer, who owns \$1000 worth of taxable property in Lyons, will have a tax bill of 2.5% of \$1000, or ___?___.

The *tax rate* in Lyons might have been expressed in any of the following ways:

(1) 2.5% of the assessed value of property.

(2) 2.5¢ on each dollar's worth of assessed value of property.

(3) 25 mills on each dollar's worth of assessed value of property. (A mill is a tenth of a cent. Does 25 mills = 2.5¢ = \$.025?)

(4) \$2.50 per \$100 worth of assessed value of property.

3. Calculate the tax on \$1500 worth of property in Lyons with the tax expressed in each of the above ways. If you need help, you may look at the following aids.

(1) Tax rate 2.5% = .025; tax = .025 × \$1500 = ___?___.

(2) Tax rate 2.5¢ per dollar; tax = 1500 × 2.5¢ = ___?___.

(3) Tax rate = 25 mills per dollar (25 mills = \$.025); tax = 1500 × \$.025 = ___?___.

(4) Tax rate = \$2.50 per \$100; tax = 15 × \$2.50 = ___?___.

4. Compute the taxes on property assessed at \$6500 when the tax rate is (*a*) \$3.50 per \$100; (*b*) 2.8%; (*c*) 1.25¢ on a dollar; (*d*) 24 mills on a dollar.

5. Mr. Hall owned a house assessed at $6500 and had taxable personal property worth $1500. What was his total tax if the tax rate was $1.80 per $100?

Find the tax on each of the properties listed below:

	Assessed Value	Rate		Assessed Value	Rate
6.	$2500	3.4%	14.	$900	3.5¢ per $1
7.	$6500	2.3%	15.	$7550	$2 per $100
8.	$2950	$2 per $100	16.	$6425	16 mills per $1
9.	$5000	$1.75 per $100	17.	$10,000	$3 per $100
10.	$2300	3.2¢ per $1	18.	$12,500	2.5%
11.	$1675	1.7¢ per $1	19.	$16,000	2.8¢ per $1
12.	$8500	22 mills per $1	20.	$18,500	25 mills per $1
13.	$3400	18 mills per $1	21.	$20,000	1.6%

22. Consult your parents to find what the local property tax rate is in your community; the county tax rate; the state tax rate. How many property tax notices are received each year by property owners in your community? What are the names of the taxes? If possible, the class should see and discuss copies of the tax notices issued in the community. Is a discount allowed for early payment of tax bills?

23. In the town of Worthington, each property owner has to pay a tax of $3.27 on each $100 of his property. How much is Mr. Bingham's tax bill if the assessed value of his property is $6500?

24. In Worthington a 1% discount is allowed if the tax is paid within 60 days. Mr. Bingham (Ex. 23) paid his bill as soon as he received it. How much did he have to pay?

25. Mr. Wilder also lives in Worthington (see Ex. 23). His property is assessed at $3500. How much is his tax bill?

26. A penalty of 2% is imposed in Worthington if a tax is not paid within 90 days. Mr. Wilder (Ex. 25) did not pay his bill within the 90 days. How much did it amount to then?

27. What is meant by the statement, "That property is going to be sold for taxes"?

28. Who is the assessor in your community? Who is the tax collector? Are you acquainted with them? Where are their offices? Perhaps they would be willing to talk to the class about their public duties.

29. One eighth-grade class gave a school-assembly program in which the members enacted the making of their town budget, the assessing of property, the working out of the tax rate, the issuing of tax bills, and the payment of bills. Would you be interested in giving such a program?

30. In addition to the property tax, there are many other local and state licenses and taxes. Among these are the gasoline tax, automobile registration fees and drivers' licenses, hunting and fishing licenses, the dog tax, etc. Name as many others as you can. The estimated income from such sources is deducted from the amount needed by the community or state in order to find out how much money must be raised by property tax.

31. Have the citizens in your community voted for any public improvements lately, such as new school buildings, roads, parks, etc., the construction of which will raise the tax rate? If so, what are they?

32. Mr. Green is planning to move his family into a new community and buy a house. Town A or Town B will be equally convenient for his work. In either town he can get for $4000 a house that suits very well the needs of his family. The tax rate in Town A is 15 mills; in Town B it is 35 mills. For what reasons might he choose the town with the higher tax rate?

1. On page 270 the tax rate in Lyons was found to be $2\frac{1}{2}\%$, or \$2.50 per \$100. How was it calculated? (The money to be raised by property tax was \$50,000; the assessed value of property was \$2,000,000.)

2. In the town of Allison \$25,600 has to be raised in property tax on property assessed at \$800,000. Compute the tax rate. Express the rate as a per cent; in dollars per \$100 worth of property; in mills per dollar.

Express the tax rates in the following examples as per cents:

	Assessed Value	Amount Needed		Assessed Value	Amount Needed
3.	\$9,627,000	\$198,000	7.	\$5,691,000	\$146,700
4.	\$6,581,000	\$186,600	8.	\$8,432,000	\$208,000
5.	\$19,620,000	\$334,300	9.	\$12,176,000	\$249,200
6.	\$1,845,000	\$ 45,000	10.	\$4,258,000	\$144,000

11. In Exs. 3 to 10 express the tax rate in mills per dollar; in dollars per \$100 worth of property.

12. The budget for Manchester calls for \$124,300 for the coming year. It is estimated that \$22,000 will be raised in special taxes. The assessed value of property in Manchester is \$4,187,000. Find the rate for the property tax.

13. Oberlin must raise \$324,000 in taxes on property assessed at \$14,952,000. What is the tax rate expressed in mills? In addition to this local tax, property owners in Oberlin have to pay a county tax of 3 mills and a state tax of 2 mills. What is their total tax rate?

14. The assessed value of property in Piedmont is \$28,390,000. The amount to be raised by property tax is \$840,000. What will be the property tax of a citizen of Piedmont who owns property assessed at \$9500?

Greater National Capital Committee—Underwood

THE UNITED STATES TREASURY

Have you ever stopped to think about the services which the United States government provides for its citizens? the cost of these services? how the government raises the money to pay these costs? Among the services rendered by the Federal government, which of the following, if any, do you consider unimportant?

(*a*) Maintaining a legislative body (Congress)
(*b*) Providing courts and law enforcement
(*c*) Standardizing weights and measures
(*d*) Aiding agriculture (Department of Agriculture)
(*e*) Operating a postal service
(*f*) Borrowing money to meet the expenses of an emergency or crisis (wars or serious economic depressions)
(*g*) Payment of interest on the public debt
(*h*) Maintaining the national executive offices
(*i*) Protection by the War and Navy departments against invasion by another country
(*j*) Providing pensions for soldiers and sailors

By Ewing Galloway, N. Y.

CUSTOMS INSPECTION

1. The two chief sources of government receipts (income) are called *internal revenue* and *customs*.

By ***internal revenue*** is meant (*a*) income taxes and (*b*) excise taxes — that is, taxes on products manufactured and sold in this country, on liquors, on the sale of securities (stocks, bonds, mortgages), amusements, luxuries, etc.

By ***customs*** is meant the tax or *duty* on articles imported from other countries. Which of the two sources would you expect to produce the most income?

2. During a recent year the government's receipts from internal revenue were \$1,604,052,691.80, and the receipts from customs were \$250,747,991.57. What were the receipts from the two sources?

3. During the same year (Ex. 2) the total receipts of the government were \$2,079,696,741.76. How much, then, was secured from sources other than those called internal revenue and customs? About what part of the total receipts came from these two sources? (Round off before dividing.)

1. Since 1913 the Federal government has taxed the incomes of its citizens. The tax is on *net incomes;* that is, the money that remains after all expenses of producing the income have been deducted. Living expenses cannot be deducted. For example, Mr. Neal, the druggist, who sells \$75,000 worth of goods which cost him \$48,000, with an expense of \$17,500 in conducting his business, has a taxable income of \$75,000 − (\$48,000 + \$17,500), or ___?___.

2. A single person with no dependents has an *exemption* of a certain amount, say \$1500. In that case Mr. Black, who has an annual income of \$2300 with no one dependent on him, pays an income tax on \$2300 − \$1500, or ___?___.

3. The head of a family has a larger exemption, say \$2500, with an additional exemption of \$400 for each child or dependent person, except his wife, supported by him. In that case Mr. Cole, with an annual income of \$4500, who has a wife and two dependent children, pays an income tax on \$4500 − (\$2500 + \$800), or ___?___.

4. Using the information in Ex. 2 and Ex. 3, compute the amount on which the tax will be paid by —

(*a*) Mr. Smith, who is married, has two children, and has an annual income of \$3500;

(*b*) Mr. Brown, who is single, has no dependents, and has an annual income of \$3100;

(*c*) Mr. Jones, who is married, has four children and an invalid father, and has an annual income of \$5200.

5. The tax rate is different for different incomes. Persons with larger incomes pay a higher rate. Why is that just?

In a recent "prosperity" year the receipts from Federal income taxes (on individuals and corporations) amounted to approximately 4 billion dollars. Four years later the receipts from such income taxes amounted to approximately 750 million dollars, a decrease of about ___?___ per cent.

There are two kinds of customs duties: *specific* and *ad valorem*. A ***specific duty*** is a tax of a certain amount per unit of the product imported, as, for example, a duty of 10¢ a bushel on apples or $2\frac{1}{2}$¢ a pound on butter. An ***ad valorem duty*** is a tax of a certain per cent of the purchase price of the article in the country from which it is imported. Thus the tariff on books from England might be 25% of the purchase price of those books in England. On certain articles both kinds of duty must be paid.

1. Mr. King imported $800 worth of tennis rackets, duty 30% *ad valorem*. The duty was ___?___, which brought the cost of the rackets up to ___?___.

2. Mrs. Smith ordered from a store in Paris an embroidered shawl worth $110. The duty was 90% *ad valorem*. What did the shawl cost her?

3. The duty on pineapples is 50¢ a crate. Find the duty on 75 crates of pineapples.

4. The duty on cigars and cigarettes is $4.50 a pound and 25%. On a purchase of 225 pounds for $650, the specific duty is ___?___; the *ad valorem* duty is ___?___; the total duty is ___?___.

5. Find the entire duty on 2536 pounds of goods costing $1565, when the duty is $5\frac{3}{4}$¢ a pound and 30% *ad valorem*.

6. Tourists returning to the United States from abroad are allowed to bring in $100 worth of goods free of duty. Miss Turner brought in some linens valued at $30 in excess of the $100 worth of goods allowed duty free. She had to pay 80% duty on the linens. What was the duty?

7. Tom's aunt brought him a penknife from England. She paid 90¢ for it there, but she had to pay 15¢ specific duty on it and 45% *ad valorem* duty. What was the total cost of the knife to Tom's aunt?

The customs which the Federal government levies on articles imported from other countries is called the *tariff*.

Whether or not tariff should exist and whether tariff rates should be high or low are points of constant disagreement among citizens and among government officials.

Those who are in favor of *high tariff* argue that it protects American workers from competing with very cheap foreign labor. For example, toys can be manufactured in Europe much more cheaply than in this country. This is largely due to the fact that wages there are so much lower than in the United States. If these toys were imported into the United States free of tariff, they could be sold for much less than those produced in this country. Hence it is claimed that our manufacturers and, in turn, their employees, would suffer. Those in favor of tariff argue that for these reasons a very high tax must be placed on all toys imported into this country, to make their cost to the storekeepers here about equal to that of domestic toys (those made in the United States).

Those opposed to tariff argue that if an article can be produced elsewhere more cheaply than it can be in the United States, then the American public should have the benefit of the resulting low prices. They say that agriculture and industry should produce chiefly those things which they can put on the market at a price equal to or lower than that at which other countries can produce them. They say this is an "age of coöperation and specialization," in which each individual community, state, or nation should specialize in producing what it can produce most economically. There should be an opportunity for *free trade* between individuals, communities, states, and nations alike.

Your class could have a most interesting debate on the subject: "High tariff rates should be abolished in the United States."

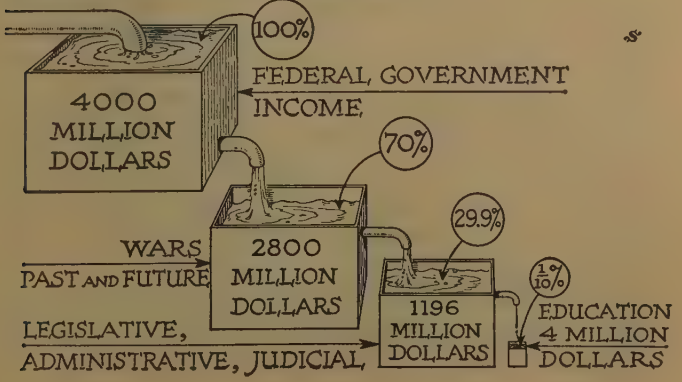
1. The principal items of cost in an annual budget of the United States government are those dealing with (1) interest on the public debt, (2) the executive and legislative departments, (3) the War and Navy departments, and (4) the Veterans' Bureau and pensions. Recently the total of these expenditures has averaged about 4 billion dollars a year.

On the average, about 30 cents of each budget dollar is spent on the public debt. How many millions of dollars is that? (The public debt has been primarily a war debt.)

2. About 20 cents of each budget dollar, on the average, is spent on the War and Navy departments. (How many dollars is that?)

3. About 20 cents of each budget dollar is spent on pensions and the Veterans' Bureau. How many cents of each budget dollar goes toward carrying the cost of the public debt, the War and Navy departments, and pensions and the Veterans' Bureau (cost of wars, past and future)? How many millions of dollars is that?

4. A diagram like this recently appeared in a magazine. What do you think its purpose was?



1. What is the cost of insuring a \$12,000 house for 80% of its value at the rate of \$.20 per \$100?

2. The assessed value of the property in a certain town was \$750,000. The estimated expenses of the town were \$25,000. How much tax would the owner of \$100 worth of property have to pay?

3. How much tax should you have to pay if you had property assessed at \$1250 and the tax rate were \$3.40 per \$100?

4. The annual tax on a 60-acre farm, assessed at \$75 an acre, is \$90. What is the tax rate per dollar? per \$100?

5. Mr. Johnson sent to London for a suit, the price of which was \$23.50. The duty was 46%. Transportation cost \$1.75. What was the total cost of the suit?

6. A merchant imported silk which cost him \$1.50 a yard plus a duty of $66\frac{2}{3}$%. At what price would he need to retail the silk in order to make a margin of 40% on the cost price?

7. A life insurance company sold a man a 2000-dollar policy for \$75 a year. After making the sixth payment, the man died. How much more was received by the beneficiary than would have been received if the premiums had been deposited in a savings bank at 4% interest compounded semiannually?

8. Late in the season skates were marked \$7.50 less 20%. George got an additional discount of 20% of the sale price on a slightly damaged pair. What did he pay for them?

9. Compute the interest at 6% on a \$625 note, dated June 7, due on the following December 24.

10. A house worth \$12,000 was insured for $\frac{4}{5}$ of its value at the rate of 40¢ per \$100 per year. What was the annual premium? A fire damaged the property to the extent of \$9000. How much was the owner able to collect?

Solve and check each of these equations:

1. $32 = 2\frac{1}{2}x + 2$ 2. $\frac{2y}{3} = 6.4$

3. How much money must be invested at 4% per annum to yield an annual income of \$500? (Use an equation.)

4. The school board at Mayville has been paying \$240 a year for pencils. Each pupil has received a pencil once every four weeks. To save money next year, each pupil will receive a pencil once every six weeks. In the 36-week school term, how much will the school board save under the new plan?

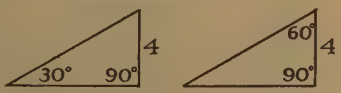
5. With water running in at the rate of 2.4 gal. per minute, how long will it take to fill a tank 40′ by 24′ by 7.5′? (1 cu. ft. = 75 gal.)

6. 25% of 25 = ? 7. 25% of ? = 25. 8. 25 is ?% of 25.

9. Find the circumference of a circle having a diameter of 5 ft. (Use $\pi = 3.14$.)

10. Of the 34 pupils in the seventh grade of the Emmett School, 27 have not been absent or tardy this month. Of the 31 pupils in the eighth grade, 25 have not been absent or tardy. Which class has the better attendance record?

11. Are these two triangles congruent? Give the reason for your answer.



12. How much will \$500 amount to in 2 years at 4% interest compounded annually?

13. Find the cost of insuring a house worth \$5000 for $\frac{4}{5}$ of its value at 30¢ per \$100.

14. Find the tax on a house assessed at \$7500 when the tax rate is 23 mills.

Standards: *A, 11.* *B, 9.* *C, 7.*

CHAPTER IX

INVESTMENTS

Forming a Company

The pupils in the Winter School thought it would be good experience to run a school store. They appointed a committee to investigate the cost of supplies; the committee decided that about $300 would be needed to open the store. Acting under the advice of a banker, they formed a company called the Winter Coöperative Society. They raised the $300, called their *capital*, and issued 100 *shares of stock* at $3 each. They said the *par value* or *face value* of each share was $3.

(1) The pupils who intended to buy stock elected 5 pupils to act as a *board of directors*. What do you think the duties of the board of directors were?

(2) The board of directors had *certificates of stock* printed in the school printing shop. Each stockholder was given one of these certificates, signed by the president and the secretary of the company, telling how many shares of stock he had bought. (See next page.) All the shares were sold.

What does the certificate on the next page show? How does it differ from a promissory note?

(3) When Frank Jones bought 15 shares of stock, he became a *stockholder* in the company. The shares cost him $__?__.

(4) Ruth Fisher paid $__?__ for 5 shares.

(5) When one of the stockholders, Agnes Scott, moved to another school, she decided to sell her 4 shares. She sold them to John Ott for $2.90 each, or at 10 cents *below par*. At what per cent below par did she sell them?

(6) Profits were small at first, but after a few months the company began to do a good business. John Ott thought he

CERTIFICATE OF STOCK

Winter Coöperative Society

This certifies that....Frank Jones.....is the owner of..fifteen...fully-paid shares of the Capital Stock of the Winter Coöperative Society.

Transferable only on the books of the Company by the holder in person.

...John Ott...President.
....Ruth Fisher.Secretary.

Par value $3. The Winter School, March 1, 1936

would buy 10 more shares, but no one was anxious to sell. How do you account for this? Finally he paid $3.50 a share. This was 50 cents *above par*. What per cent above par was it?

(7) At the end of the first year the manager reported a net profit of $62.50. The board of directors decided to set aside $32.50 to be used in enlarging the business, and to distribute $30 among the stockholders as a *dividend*. Each holder of one share of stock was entitled to $\frac{1}{100}$ of the *dividend*, or $\frac{?}{}$ cents. What was the rate of the dividend?

(8) Frank Jones owned 15 shares of stock. What was the amount of the dividend he received?

(9) How much was the dividend on Helen's 8 shares?

(10) Do you think that the stockholders received a good return on their investment?

(11) How would the ability of this company to declare a dividend be affected if the manager were decidedly inefficient or dishonest?

(12) One girl's father objected to her buying stock in the company. He knew that many stocks are worthless. Would you buy stock of a company about which you knew very little?

(13) The Winter Coöperative Society was conducting its business in much the same way that large businesses are carried on today. The Pennsylvania Railroad, for example, belongs to more than 143,000 stockholders. There is about $500,000,000 worth of property, represented by 10,000,000 shares of *stock*, the par value of each share being $50.

(14) An association of stockholders is called a *corporation*. A corporation receives permission from the state to carry on its business. The license which a state issues to a corporation is called the *charter* of the corporation.

(15) Stocks may be bought and sold like any other property. The price at which a share of stock sells is its *market value*. If a share of stock has a par value of $100 and sells for more than $100, it sells *above par*. If it sells for less than $100, it is *below par*.

(16) The amount of money to be distributed from the earnings of a company among the owners of stock is determined by the *board of directors*. The money distributed is the *dividend*. The *rate of dividend* is always computed on the *par value* of the stock. A "6% dividend" means 6% of the par value of a share, whether the market value is above par, at par, or below par. For example, a man paid $85 for a share of stock having a par value of $100. He received a 6% dividend; that is, 6% of $100, or $6.

(17) The money, property, bills receivable, etc., are the *assets* of a company; the debts of a company are its *liabilities*. The value of the stock of a company is the amount by which its assets exceed its liabilities.

(18) Companies often issue both *common* and *preferred* stock. The rate of the dividend on preferred stock is definitely stated in the certificates. The dividends on all preferred stock are paid before any dividend is paid on common stock. Owners of preferred stock are more certain, therefore, to get dividends on their stock. The rate of income on

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THE NEW YORK STOCK EXCHANGE

the common stock may be very large or very small. It depends on the amount of profits remaining after the dividends on preferred stock have been paid.

(19) Shares of stock are usually bought and sold through an agent called a *broker*, in an association of stockbrokers called a *stock exchange*. The compensation which the broker receives for each transaction is called *brokerage*. This is usually a certain sum for each share bought or sold. The fees charged recently by brokers on the New York Stock Exchange were as follows:

Market price per share ($)	10–24⅞	25–49⅞	50–74⅞	75–99⅞	100 up
Brokerage per share (¢) . .	12½	15	17½	20	25

(20) The brokerage for selling 75 shares of National Railroad stock (par value $50) at $67⅜ a share is $75 \times \$.17\frac{1}{2}$, or $13.13. What is the brokerage on 120 shares at $69⅛ a share?

Millions of persons invest their money in stocks. The market prices of many stocks are published in the daily newspapers. This is the way a newspaper reports the prices at which stocks were sold during one day on the New York Stock Exchange, the world's largest stock market:

Sales	Name of Stock	Previous Close	Open	High	Low	Close	Net Change
5715	General Electric	$19\frac{3}{4}$	$19\frac{3}{4}$	$20\frac{3}{8}$	$19\frac{3}{4}$	$20\frac{3}{8}$	$+\frac{5}{8}$
6300	Chrysler Motors	$39\frac{3}{4}$	$39\frac{3}{4}$	$40\frac{7}{8}$	$39\frac{3}{4}$	$40\frac{3}{8}$	$+\frac{5}{8}$
12,000	U. S. Steel.....	$39\frac{1}{2}$	$39\frac{1}{4}$	$40\frac{3}{8}$	39	$39\frac{5}{8}$	$+\frac{1}{8}$
500	do preferred.....	85	84	84	$83\frac{1}{2}$	84	-1

1. We read the first line in the table as follows:

(1) 5715 shares of common stock of the General Electric Company were sold during the day.

(2) The price of this stock at the time the market closed the day before (Previous Close) was $\$19\frac{3}{4}$ a share.

(3) The price of this stock when the market opened (Open) was $\$19\frac{3}{4}$.

(4) The highest price at which the stock sold that day (High) was $\$20\frac{3}{8}$ a share; the lowest price (Low) was $\$19\frac{3}{4}$.

(5) The price when the market closed (Close) was $\$20\frac{3}{8}$ a share. How much higher was this than the price at which the market closed the previous day? See column headed "Net Change." Why is it marked "$+\frac{5}{8}$"?

(6) How many shares of Chrysler Motors were sold? What was the highest price for the day? the lowest price? the net change? How much higher did it close than it closed the previous day?

(7) The third line in the table refers to U. S. Steel Corporation common stock; the fourth line refers to U. S. Steel preferred stock. (*do* means "ditto" or "the same.")

(8) What were the sales of U. S. Steel common stock? U. S. Steel preferred?

Which is better for the buyer, to purchase a stock paying an annual dividend of \$6 a share for \$91, or a stock paying a dividend of \$7 a year for \$102? (Assume the stocks to be equally desirable in other respects.)

To answer this question, one must know how to compute the rate of income on investments.

$$\text{Rate of income} = \frac{\text{dividend}}{\text{cost price}}$$

SOLUTION. $\frac{6}{91} = 6.6\%$ $\frac{7}{102} = 6.9\%$

Hence the stock at \$102 a share is the better buy.

1. Smith & Co. 7% preferred stock (par value \$100), bought for \$95, makes the owner ___?___ % on his investment.

2. Would you get 8% on your investment if you bought 6% stock at \$75 a share? (Par value \$100.)

3. What rate of income would I get on 8% stock bought at \$110 a share? (Par value \$100.)

4. What is the rate of income on 5% stock bought at 80? (80% of par value.)

5. Mr. Paxton owned some preferred stock that paid \$3 annually a share. It was worth (could be sold for) \$50 a share. What was the rate of income on the stock?

6. Mr. Paxton was advised to sell the preferred stock (Ex. 5) and buy some common stock paying a dividend of \$6 a share, at \$75 a share. Compare the rates of income on the two stocks.

7. Mr. Paxton said: "Oh, no. The rate of income on my stock is low, but it is more certain to continue. You know that preferred stock dividends are paid before any common stock dividends are paid. That makes the value of the stock more secure. To be sure, the common stock pays a higher rate, but it is more likely to drop in value. I prefer security to high income." What do you think of this reasoning?

1. The Jackson Oil Company was authorized by its charter to issue 10,000 shares of stock at \$100 each. What was the amount of the company's capital stock in dollars?

2. Mr. Hoke owns 10 shares of stock in the Farmers Bank. On his stock certificate is the statement, "Capital \$50,000; shares \$100 each." What part of the stock does he own?

3. I directed my broker to buy for me on the New York Stock Exchange 10 shares of National Railroad Company stock at $\$38\frac{3}{8}$ a share. What did they cost me, including brokerage?

4. Mr. Hassler owned 5 shares of stock in the Newberry Elevator Company, for which he paid \$90 a share. At the end of the year the company declared a dividend of \$10 a share. What was the rate of income on his investment?

5. Mrs. Penman bought 25 shares of Texo Oil at par (\$50). No dividends were paid, and at the end of two years the market value of the stock was \$23.50 a share. If she had sold her stock then, what would her loss (brokerage cost not counted) have been?

6. The Bedford Stone Corporation increased its dividend from \$4 to \$6 a share. What effect would this probably have on the market value of the stock?

7. When the brokerage for purchasing any number of shares of a stock, at the rate shown in the table on page 285, is less than \$5, brokers usually make a minimum charge of \$5. Keeping this fact in mind, find the total cost of—

(*a*) 10 shares of American Can at $106\frac{1}{8}$
(*b*) 20 shares of American Telephone and Telegraph at $110\frac{1}{2}$
(*c*) 30 shares of Allied Chemical at $140\frac{7}{8}$
(*d*) 22 shares of U. S. Steel pfd at $86\frac{3}{4}$
(*e*) 18 shares of Eastman Kodak at $112\frac{1}{2}$
(*f*) 30 shares of Air Reduction at $110\frac{1}{2}$
(*g*) 25 shares of Woolworth at $52\frac{7}{8}$

1. Mr. Reid had $2500 with which he wished to buy stock in a mining company. The stock was selling for approximately $50 a share. How many shares could he pay for? (Disregard brokerage in these exercises.)

2. He wished to buy 150 shares, or ___?___ dollars' worth of stock. His broker bought the stock, accepting the $2500 as a part payment. How much did Mr. Reid still owe?

3. What per cent of the purchase price ($7500) of the 150 shares of stock did Mr. Reid pay in cash?

We say that Mr. Reid was buying on a $33\frac{1}{3}\%$ *margin.* ($\frac{2500}{7500} = 33\frac{1}{3}\%$.)

"Buying stocks on margin" means paying cash for only a portion of the cost price of the stocks. The broker pays the rest and the buyer owes the broker the amount the broker has thus put up.

4. Soon after Mr. Reid bought the mining stock, he was distressed to learn that the price had dropped $10 (to $40) a share. What were his 150 shares then worth? If he had sold the stock, what would his loss have been?

5. The broker informed Mr. Reid that he owed $5000 on the $6000 worth of stock. What per cent of the value of the stock did he owe?

6. The broker demanded that Mr. Reid make a payment of $1000 at once. If he made the payment, how much would he still owe the broker? (7500 − 2500 − 1000 = ?)

7. Mr. Reid was unable to pay the broker any more money. So the broker then sold the 150 shares of stock for $38 a share. How much of the proceeds should be sent to Mr. Reid? What was Mr. Reid's loss?

Buying on margin is hazardous!

When an individual wishes to borrow money, he usually gives a promissory note. Sometimes the lender may require the borrower to give some kind of *security* in addition to his promise contained in the note. Such security for a loan is a pledge to the lender of some kind of property which gives the lender the right to sell the property if interest and principal are not paid when due.

When a corporation or a government wishes to borrow large sums of money for special purposes, it may *issue bonds* instead of giving promissory notes.

A bond is a form of written promise to pay a specified sum of money on a specified date, with interest at a specified rate.

Banks and individuals lend a corporation money in exchange for these written promises or bonds. This is called "selling bonds." The purchaser of a bond may in turn sell it to someone else at any time.

1. Bonds that are secured by a pledge of some kind of property are called *mortgage bonds*. Bonds that are not secured are called *debenture bonds*. Why do you suppose that mortgage bonds are more attractive to investors than debenture bonds?

Bonds usually have a long time to run; for example, 5, 10, 20, or 50 years.

2. In what way would you say that a bond resembles a promissory note?

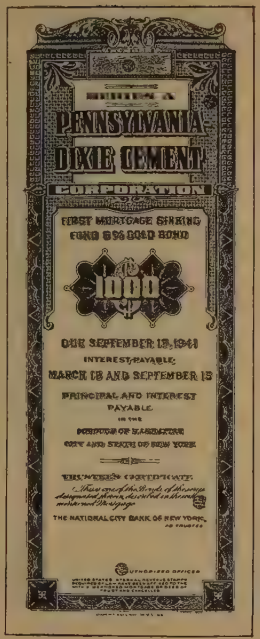
3. In what way does a bond differ from a share of common stock? from a share of preferred stock?

4. What corporation issued the bond shown on the next page? What is the *denomination* (face value) of the bond? What rate of interest does it pay? On what dates is the interest payable?

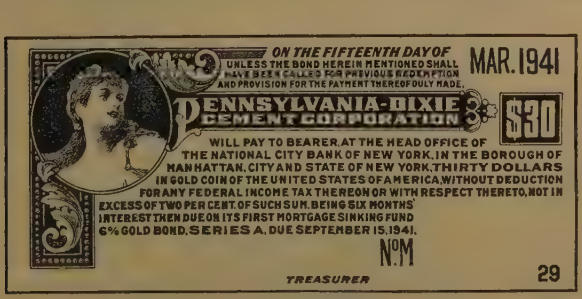
The coupon bond shown here is like many corporation and government bonds. It consists of two parts. The first part is the promise to pay principal and interest.

The second part consists of a series of *coupons*. Each of these coupons is a promise to pay six months' interest on a certain date. In order to collect interest, the owner clips a coupon when due and deposits it in a bank. The bank sends the coupon to the company, gets the cash, and places it in the bondholder's account.

Two important items on a coupon are (1) a statement of the amount of the interest and (2) the date when this interest is due. It will be helpful if the class can arrange to have a bond brought to a recitation. The important features of a bond can then be studied at first hand.



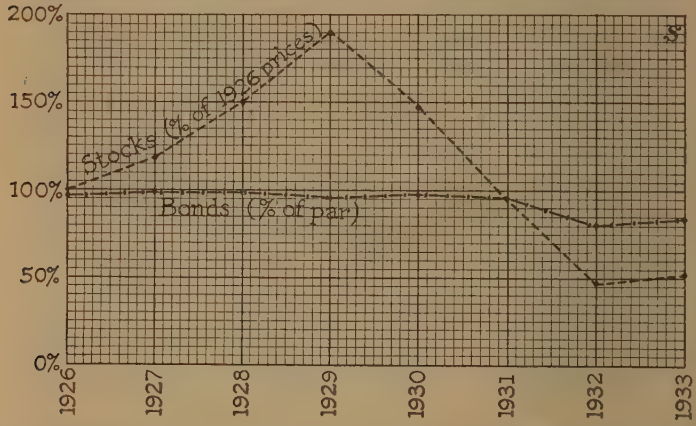
A Bond (About $\frac{1}{3}$ length)


ON THE FIFTEENTH DAY OF
MAR. 1941
UNLESS THE BOND HEREIN MENTIONED SHALL
HAVE BEEN CALLED FOR PREVIOUS REDEMPTION
AND PROVISION FOR THE PAYMENT THEREOF DULY MADE,
PENNSYLVANIA-DIXIE
CEMENT CORPORATION
$30
WILL PAY TO BEARER AT THE HEAD OFFICE OF
THE NATIONAL CITY BANK OF NEW YORK, IN THE BOROUGH OF
MANHATTAN, CITY AND STATE OF NEW YORK, THIRTY DOLLARS
IN GOLD COIN OF THE UNITED STATES OF AMERICA, WITHOUT DEDUCTION
FOR ANY FEDERAL INCOME TAX THEREON OR WITH RESPECT THERETO, NOT IN
EXCESS OF TWO PER CENT. OF SUCH SUM, BEING SIX MONTHS'
INTEREST THEN DUE ON ITS FIRST MORTGAGE SINKING FUND
6% GOLD BOND, SERIES A, DUE SEPTEMBER 15, 1941.
No M
TREASURER
29

A Coupon (Full size)

1. This graph shows the fluctuations (ups and downs) in the average prices of 421 stocks and 60 bonds during the

FLUCTUATIONS IN AVERAGE PRICES OF 60 BONDS AND 421 STOCKS



years 1926 to 1933. It should be read: In 1926 the average price of the 60 bonds was 97% of their par value; in 1927 the average price was 99% of their par value, and so on. In 1927 the average price of the 421 stocks was 118% of their 1926 price; in 1929 it was 190% of their 1926 price, and so on.

(*a*) In 1932 the average price of stocks was __?__ of their 1926 price.

(*b*) An average stock that was worth \$100 in 1926 was worth how much in 1929? in 1932?

(*c*) An average \$1000 bond was worth how much in 1929? in 1932?

(*d*) What is the most striking fact shown by this graph?

2. On April 1 Mr. Nelson bought at par a \$100 bond drawing 6% interest payable January 1 and July 1. The owner charged him \$100 + \$1.50. The \$1.50 was for interest *accrued* since January 1. Was this fair to both?

Mr. Smith: Yes, Brown, I'll let you have $500 for a year if you will pay me interest on it at 6% and give me a *mortgage* on your house and lot as security.

Mr. Brown: You will be perfectly safe in doing so. My property is worth over $2000. If I don't pay the $500, you certainly can get your money back from the property.

These men were talking about the oldest form of investment—first-mortgage loans on real estate. Mr. Smith was lending $500 to Mr. Brown and was receiving a mortgage, which consists of—

(1) A promise to pay a fixed amount of money, at a definite rate of interest, at a specified time; and

(2) A written pledge of real estate to guarantee prompt payment of principal and interest when due.

The laws of practically all countries give the owner of a mortgage the privilege of selling the property (under the direction of a court) if the loan is not repaid. In case Mr. Brown does not repay the loan, Mr. Smith can have the property sold and thereby receive the money due him out of the proceeds of the sale, if sufficient. Mr. Brown gets what is left.

1. Would Smith probably have lent Brown as much money as Brown's property was worth, or more? Why?

2. Who will keep the mortgage?

3. Why will Mr. Brown be particularly concerned about paying this debt promptly?

4. Mortgages are popular investments. They usually carry a fairly high rate of interest and have a reasonable degree of safety.

5. Mortgages are usually less readily converted into cash than are stocks and bonds — especially stocks and bonds that are sold on stock exchanges.

1. If a person wishes to borrow money by giving a first mortgage on his property as security—

(*a*) He cannot borrow usually more than from 50% to 60% of the market value of the property which he offers to pledge as security. Why?

(*b*) He has to show that the property is free from debt. Why?

(*c*) He has to pay all the unpaid taxes on the property. Why?

(*d*) If buildings are part of the security, he has to insure them in favor of the owner of the mortgage. Why?

2. What is the difference between a mortgage and a promissory note? How does a mortgage resemble a promissory note?

3. What does it mean to say that Mr. Johnson's house "has a mortgage on it"?

4. Mr. Waggoner owns a $5\frac{1}{2}\%$ mortgage for \$2500, a 6% mortgage for \$3250, and a $5\frac{3}{4}\%$ mortgage for \$2725. What is his annual income from the three?

5. A farmer bought 160 acres of land at \$175 an acre. He gave a $5\frac{1}{2}\%$ mortgage for 60% of the cost price and paid the remainder, \$__?__, in cash. The yearly interest on the mortgage was __?__.

6. A large part of the real estate in the United States is pledged as security for mortgage loans. It has been estimated that over \$10,000,000,000 is invested in mortgages. Savings banks and insurance companies invest a large part of their funds in mortgages.

7. If some property is mortgaged for less than 50% of its value, do you suppose that the owner could give a "second mortgage" on it to secure an additional loan? Why do you suppose that investments in second mortgages are considered hazardous?

1. Recently 2385 adults, one third of them women, living in 102 cities scattered through 36 states, were asked by personal interviewers:

> "If you were advising a younger brother or a friend as to the most satisfactory way of investing his savings, where would you advise him to put his money after provision for current expenses and a reserve for emergencies?"

Table 1 shows the results of the interviews. What kind of investment was rated highest? next highest?

Order of Preference
1. Insurance
2. United States Bonds
3. Bank Savings Account
4. Postal Savings
5. Building and Loan
6. Stocks (common and preferred)
7. Real Estate
8. Other Bonds
9. All other types of investment

Table 1

Order of Preference	
Banks	43.6%
Building & Loan	28.9%
Private Lender	16.8%
Mortgage Company	5.7%
Federal Government	2.3%
Insurance Company	1.8%
All others	0.9%

Table 2

2. It would be profitable to ask your parents to rate (arrange in order of preference) the different kinds of investments mentioned in Table 1.

3. The interviewers mentioned in Ex. 1 also obtained answers to the question:

> "Under normal conditions, if you wanted to borrow money to buy or build a home, to what type of institution would you go?"

Table 2 shows what institutions were most highly favored by the 2385 persons.

4. It would be interesting to get your parents' views as to the question in Ex. 3.

1. Mr. Norton had saved regularly until he had accumulated eight hundred dollars. His savings were deposited in the First National Bank at 3% interest compounded semi-annually. Do you think his savings were earning a good rate of interest for him?

2. He had been urged by salesmen to invest his savings in real estate, oil stocks, mining stocks, railroad bonds, shares in a new rat-trap company, and the like. They argued that he should place his money where it would earn more than 3%. Do you think he should have invested his savings in one of these ways?

3. Mr. Norton knew very little about how to make investments, and was very cautious, as his neighbor, Mr. Short, had recently lost all his savings through an unwise investment. He decided to consult Mr. Reed, with whom he had become acquainted in the First National Bank. Mr. Reed was the cashier of the bank. He had had long experience with investments and was known to be a man of *integrity*. What do you think of Mr. Norton's decision? Do you know anyone whom you would consult if you had money to invest?

4. Mr. Reed advised Mr. Norton as follows:

(*a*) Perfectly safe investments usually earn interest at low rates. State bonds and United States Government bonds are examples of this kind of investment. Ordinarily they pay from $2\frac{1}{2}$% to 4% per annum.

(*b*) Good investments may pay from about 5% to 6% per annum, but the element of risk should be considered. Examples of good investments are promissory notes secured by mortgages on real estate, and county and municipal bonds.

(*c*) Whenever a high rate of interest (say 8% to 10%) is offered on an investment, there is almost always a large element of risk, and it should be regarded as a speculation rather than an investment. Examples of such

investments are stock in new patents and in undeveloped oil wells and mines. Inexperienced investors are advised not to buy these.

(*d*) A good rule for you to remember is:

The higher the rate of income, the greater the risk.

Discuss what Mr. Norton should do with his savings.

5. The following are investment advertisements taken from some magazines and investment circulars. Select the one which you think is safest, the next safest, and so on.

(*a*) "Would you invest $100 or more *now* in an article that you thought better, from the standpoint of financial return, than the Gillette Safety Razor? Its stock is reported to be worth $22,000 for every $100 originally invested. If you have funds for *immediate investment*, send in your order *today* — otherwise don't bother. Thirty days from now will be too late. This is one of the real opportunities everyone dreams of, but only those who have enough courage to back their own opinion profit by it." (An advertisement of stock in a "newly patented self-feeding, refillable toothbrush.")

(*b*) "Thrift is Power." "Save and Succeed." United States of America 4% Treasury Bonds, maturing 1944 to 1954. Priced to yield about 3.23%.

(*c*) "6% First Mortgage Real Estate Bonds on Improved Chicago Real Estate."

(*d*) "Make Your Idle Money Earn 10%. Buy 8% Lift-Jack Bonds at 80."

(*e*) "City of Baltimore, Maryland, General Improvement, 5% Water Bonds. Tax exempt in Maryland. Exempt from all Federal income taxes. Due serially April 1, 1942, to 1957. Priced to yield about 4.15%."

(*f*) "Commonwealth of Australia, 4½% Bonds. Due 1957. Priced to yield about 5.37%."

Tell which of the following statements you believe to be true and which false. Be prepared to defend your belief.

1. If you buy a bond, you have confidence in the company, city, state, or nation which issues the bond.

2. If you buy common stock in a corporation, you are a part owner of the corporation.

3. Bonds as a rule are safer than stocks.

4. An inexperienced investor should consult his local banker before buying stocks, bonds, or mortgages.

5. A 6% stock pays 6% of its market price.

6. A 6% bond pays 6% of its face value.

7. The chief concern of an investor is "safety of principal."

8. Mortgage bonds usually pay a lower rate of income than debenture bonds.

9. U. S. Government bonds are the safest investment you can make.

10. If two mortgages are believed to be equally safe, the one paying the higher rate of interest is preferable.

11. Most investors pay too little attention to the rate of income on the investments which they make.

12. The Federal government should see to it that all securities that are sold within the country are sound.

13. A savings account, in which regular deposits are made, is one of the best kinds of investment for young people.

14. Most persons who try to get a rate of income greater than 3% or 4% on their investments "come to grief."

15. It is easier to save money than it is to invest money wisely.

If you were traveling in Europe, you would need to know the value of the coins used in the various countries which you visited. The value of foreign money, in terms of United States money, varies from time to time. This table shows the normal value of certain coins used in England, France, and Germany.

English Money
1 penny (*d.*) = about 2¢ (U. S. money)
12 pence (plural of "penny") = 1 shilling (*s.*) = about 25¢
20 shillings = 1 pound (£) = about $4.86
French Money
1 centime (*sahn-teem*) = about $\frac{1}{25}$¢
100 centimes = 1 franc (fr.) = about 4¢
German Money
1 pfennig (pf.) = about $\frac{1}{4}$¢
100 pfennigs = 1 mark (M.) = about 24¢

1. According to the table, how many pounds, shillings, and pence could you get for $10?

2. How many francs could you get for a dollar?

3. How many marks and pfennigs could you get for a dollar?

4. A suit bought in London for £4 10*s.* would cost ___?___ dollars.

5. A book priced at 50 fr. in Paris would cost ___?___ dollars.

6. A certain kind of German camera sells for 20 M. or ___?___ dollars.

7. When Elizabeth was abroad she spent £8 6*s.* in England, 250 fr. 50 centimes in France, and 80 M. 20 pfennigs in Germany. All together, that was equivalent to how much in U. S. money?

8. A salary of $20 a week in U. S. money would be equivalent to what in English money?

By Ewing Galloway, N. Y.

Mr. Skinner: Well, Mr. Everyman, here we are at the lot you decided on for your house.

Mr. Everyman: Yes, this is the place Mrs. Everyman and I picked, Lot 23. It's a smaller lot and a little more expensive than either of the others we had considered, but it's in a better community and has more improvements.

Mr. Skinner: I think you have made a wise choice, Mr. Everyman. The *location* of real estate, the character of the adjoining property, and the probable future developments of the district determine the value of any parcel of real estate. The Jones Addition is a very desirable home site.

Mr. Everyman: I will give you my personal check for $50 as a deposit on the lot. I'll pay the rest of the money ($750) as soon as you make out the papers.

Mr. Skinner: You may make the check for the full amount. We can go over to my office and I'll write the *deed* now. Our company gives you a *guaranteed title* to the property. [*Later, at the office, hands Mr. Everyman a deed.*] You should take this deed to the Recorder's Office to have a public record made of the transfer of ownership of this property.

A *deed* is a written agreement covering the transfer of ownership of property. It contains a description of the land.

A guaranteed title is a certificate supplied by a company which has investigated all previous transfers and which guarantees that the property legally belongs to the person who is selling it.

Mr. Everyman: Thank you. I am happy, indeed, to be a *freeholder* [owner of real estate]. Now Mrs. Everyman and I shall look forward to building our new home.

Mr. Everyman has gone to the office of a Building and Loan Association to borrow money to build his house.

Mr. Everyman: Good morning, Mr. Brown. I want to arrange for a loan. We are planning to build this spring.

Mr. Brown: How much do you need, Mr. Everyman?

Mr. Everyman: The type of house we want is estimated to cost about $6000. I have $2000.

Mr. Brown: You have the lot *clear?*

Mr. Everyman: Yes, it's all paid for.

Mr. Brown: Then our Building and Loan Association will lend you $4000 at 6% interest, payable January 1 and July 1. We shall expect you to pay 1% of the $4000, or $__?__, each month until the loan is all paid. Part of this $40 pays interest, and part goes to reduce the principal. In a little over eleven years you will have it all paid.

Mr. Everyman: That would be easy. We want to *let the contract* for the house as soon as we can. When can we get part of the money?

Mr. Brown: The Directors of the Association will have to pass on the application. I am quite certain their action will be favorable. If so, the Association will take your note for $4000 and a *mortgage* on the property. You will have to insure the property in favor of the Association, so that, in case of loss by fire or tornado, the company would get the money more easily.

Mr. Everyman: I understand about the insurance, but will you explain more fully about the mortgage?

Mr. Brown: The mortgage is a written agreement by which you give the Association a claim against the property in case you fail to pay the note when due. If you should fail to pay the note when due, the Building and Loan Association has the right to *foreclose* the mortgage. This means that the court would order the property sold to pay the debt. In case of such sale, you would have one year from the date of foreclosure in which to *redeem* [get back, legally] the property. You could do this by paying the amount of the debt, interest, and court costs. If you failed to do so, you would lose the property but would get whatever money was left after the debt was paid.

Mr. Everyman: That is fair enough. I think I can pay the note when due.

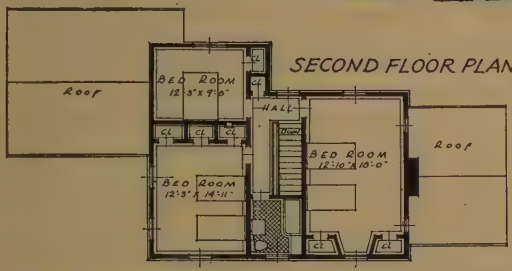
House Plans

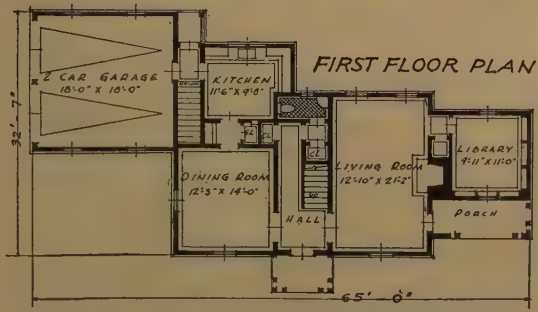
1. On the following page are the sketch and the first and second floor plans of Mr. and Mrs. Everyman's house.

What rooms are on the first floor? the second floor?

2. What are the dimensions of the living room? the dining room? the library? the bedrooms?

3. What is the over-all length of the house? What is the over-all width?


SECOND FLOOR PLAN
Roof
BED ROOM
12'-3" X 9'-0"
HALL
BED ROOM
12'-10" X 18'-0"
Roof
BED ROOM
12'-3" X 14'-11"


FIRST FLOOR PLAN
2 CAR GARAGE
18'-0" X 18'-0"
KITCHEN
11'-6" X 9'-8"
DINING ROOM
12'-3" X 14'-0"
HALL
LIVING ROOM
12'-10" X 21'-2"
LIBRARY
9'-11" X 11'-0"
PORCH
32'-7"
65'-0"

(1) At $40 a month, how long would it take Mr. Everyman to pay off the $4000 if he did not have to pay interest? How does the fact that he pays interest affect the length of time?

(2) To compute the length of time it would take Mr. Everyman to pay off the loan is a long problem. Only a few eighth-grade pupils should try to solve it. Here is the beginning of the solution:

(Assume $4000 borrowed July 1, at 6 per cent interest payable January 1 and July 1, and monthly payments of $40.)

(*a*) On January 1 of the next year, without the monthly payments Mr. Everyman would owe $4000 + 6 mo. interest on $4000, or $4120.

(*b*) But he has paid $40 on August 1 and on the first of each succeeding month, making $240. This is to be subtracted from $4120.

(*c*) The Association allows him interest on the payments. On the first $40 payment (August 1) he is allowed 5 months' interest (why 5 months?); on the second $40 payment (September 1) he is allowed 4 months' interest; 3 months' on the next, 2 months' on the next, and one on the next, making 15 months' interest on $40, which is $3.

(*d*) On January 1 he owes $4120 − $240 − $3, or $3877.

Letting the Contract for Building the House

The Everymans, having bought a lot, chosen plans for the house, and arranged to borrow money, are now ready to come to some agreement with a builder. They have decided to have Mr. Strong build their house, for $6000. He is called a *contractor*, meaning that he does a specified job, in a specified way, for a fixed price. The written agreement made by Mr. Everyman and Mr. Strong is called a ***contract***. A copy of the contract appears on the next page.

ARTICLES OF AGREEMENT, Made this 15th day of April in the year one thousand nine hundred and thirty-six Between J. M. Everyman party of the first part, and Mason Strong party of the second part.

FIRST.—The said party of the second part does hereby covenant, promise and agree to and with the said party of the first part, that the said party of the second part, shall and will, for the consideration hereinafter mentioned, on or before the first day of Aug. 1936 well and satisfactorily erect and finish a new residence on lot 23 in the Jones Addition agreeably to the accompanying Drawings and Specifications made by W. C. Smith, architect, and signed by the said parties and hereunto annexed, within the time aforesaid, in a good, workmanlike and substantial manner, and also shall and will find and provide such good, proper and sufficient materials, of all kinds whatsoever, as shall be proper and sufficient for completing the said Building for the sum of six thousand dollars ($6000.00)

SECOND.—And the said party of the first part does hereby covenant, promise and agree, to and with the said party of the second part, that the said party of the first part shall and will, in consideration of the covenants and agreements being strictly performed and kept by the said party of the second part, as specified, pay, or cause to be paid unto the said party of the second part, within ten days after said Building is ready for occupancy, provided satisfactory evidence that receipted bills for all labor and materials for said house shall have been submitted, the sum of six thousand dollars lawful money of the United States of America:

In Witness Whereof, the said parties to these presents have hereunto set their hands and seals, the day and year above written.

WITNESSES:

B. R. Bartlet — J. M. Everyman

Frank Drake — Mason Strong

1. How much did the Everymans' house and lot cost?

2. The tax rate on their property was \$3.60 per \$100. The property was assessed at \$5000. Compute the tax.

3. The total investment of the Everymans in their house was \$6800. What is one year's interest on \$6800 at 6%? (Why should this item be included in figuring the cost of owning the home during the first year?)

4. If a house lasts 40 years, it is said to *depreciate* (shrink in value) $\frac{1}{40}$ or $2\frac{1}{2}\%$ a year, on the average. At this rate how much was the annual depreciation for the Everymans' \$6000 house? (Why should this item of \$150 be included in figuring the cost of owning the home during the year?)

5. What was the value of the Everymans' house and lot at the end of one year? To figure the interest cost of owning their home the second year, Mr. Everyman found the interest on \$6800 − \$150, or \$6650, at 6%. The next year the interest charge was 6% of \$6650 − \$150, etc.

6. The Everymans insured their 6000-dollar house for $\frac{4}{5}$ of its cost price. The annual rate was 22 cents per \$100. What was the annual cost of insurance? (Should this item be included in figuring the cost of owning the home?)

7. If the cost of painting and other repairs averaged \$50 a year, what was the total cost of owning the Everymans' home the first year? (See Exs. 2, 3, 4, 6, and 7.)

8. Before the Everymans moved into their new home, they had been paying \$60 a month rent. How much more than a year's rent did it cost them to own their home the first year? Can you think of any reasons why the Everymans would prefer to "own their own home"?

Yearly expense of living in a home = taxes + interest on investment + depreciation + insurance + upkeep.

Mr. Everyman (12 years later): Mother, now that our house has been paid for, I've been wondering what we should do with our savings. It seems that while we were paying for our house, we got into the habit of saving.

Mrs. Everyman: I was just looking in our savings-bank book. Our balance is over a thousand dollars.

Johnny: Let's take a trip to Europe. Several of my friends have been abroad. It would be a great vacation, and it costs only a few hundred dollars to go to Europe.

Mr. Everyman: We've managed to keep well and happy with our two weeks at the lake each year. I think that we should invest our money now so that it will earn more than $2\frac{1}{2}\%$.

Mrs. Everyman: Yes. Mrs. Wise told me that they bought a mortgage that paid 6%. You know that mortgages are considered a safe investment.

Mr. Everyman: That's right. Mortgages are good investments. I'll telephone to Mr. Wise at the bank and ask whether they know anyone who wants to build a home, as we did twelve years ago. [*He telephones.*] Mr. Wise says that we have an opportunity to lend $1000 to a young couple who are buying a 3000-dollar home. They have $2000. He thinks a 1000-dollar mortgage from them would be a good investment.

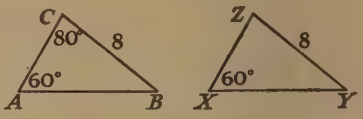
Johnny: Why don't you buy stocks and make a lot of money? That's the way to get rich quick. You folks are too conservative.

Mrs. Everyman: No, no, no. We've worked hard and been thrifty. Let's play safe. You know that many of our friends have lost their money in unwise investments. Here, Father, take this pass book over to the savings bank and transfer $1000 to our checking account. That mortgage is a good, safe investment.

(Mr. Everyman bought the mortgage.)

1. Mr. Terry bought a 1000-dollar 6% bond for \$900. What rate of income did the investment pay?

2. Complete: These triangles are congruent if $\angle Y$ = ___?___ degrees.



3. Find the hypotenuse of a right triangle whose base is 8 and whose altitude is 6.

4. Find the interest on \$750 for 3 mo. at $4\frac{1}{2}$%.

5. What is the yearly premium on a 5000-dollar life insurance policy if the yearly premium on a 1000-dollar policy is \$22.32?

6. If the assessed value of the property in Allerton is \$245,600 and the budget shows that \$4912 must be raised in taxes, what will be the tax rate that year?

7. When 9 is subtracted from 5 times a certain number, the result is 21. Find the number.

8. What is $\frac{1}{2}$% of 200?

9. 9 is 3% of what number?

10. What is the volume of a cone if the radius of the base is 6 inches and the altitude is 10 inches?

11. Allowing 10% for shrinkage, how many yards of material should Mrs. Allen buy so that she will have 18 yards of material left after it has shrunk?

12. Jane has \$6.50 in the school bank. If she deposits 25¢ a week, in how many weeks will she have \$10, not counting interest? (Use an equation.)

Standards: A, 10. B, 8. C, 6.

CHAPTER X

GENERAL REVIEW

Using Per Cents*

1. The assessed value of property in Middletown is $16,240,000. The estimated expenses of the town for next year are $450,000. What will be the tax rate?

2. Find the amount to which $400 will grow in two years if it is compounded semiannually at 4%.

3. Find the yearly premium on a fire insurance policy which covers a house worth $8500, insured for 60% of its value at 35¢ per $100.

4. A 5% bond whose par value is $1000 is selling for $875. If you buy the bond, what is the rate of income it will yield?

5. When the tax rate is 2.5%, what is the tax on a piece of property assessed at $3800?

6. The People's Bank holds a promissory note signed by Mr. Long. The face of the note is $2000 and the time is 3 months. The bank discounted the note at 6%. How much money did Mr. Long receive from the bank?

7. The population of Brown's Mill is 14,700. The *Brown's Mill News* says that the population has increased 200% in the last five years. What was it five years ago?

8. The Neighborhood Grocery bought a carton of wafers for $4.50. The estimated overhead expenses are equal to 12% of the cost of goods. What must the carton of wafers bring to net a profit of 10% of the cost?

9. Mr. Ray received a bill for $3.71 for electricity. On the bill was the following notice: "This statement shows 3% added to bill previously rendered for failure to pay within 21 days." If Mr. Ray had paid the bill when it was sent the first time, how much would he have saved?

10. What is the interest on \$840 for 126 days at 6%?

11. Using the formula $i = prt$, find the interest on \$185 for 90 days at $4\frac{1}{2}\%$.

12. A weather report stated that the amount of rainfall for May of this year was .4 inch less than it was during the preceding May. If you wished to find the per cent of decrease in rainfall, what information would you have to get from the weather bureau?

13. If we increase the length of the sides of a square 100%, by what per cent do we increase the area of the square?

14. Find the total cost to an importer of a set of dishes valued at \$125 if the duty is 45%.

15. Mrs. Stark is considering buying some new living-room furniture. She can get the furniture she wants for \$200 cash, or by paying \$25 down and \$25 a month for 8 months. Which plan of buying the furniture is cheaper?

16. Mrs. Stark, in Ex. 15, has enough money to pay cash for the furniture, but it is in her savings account, drawing interest at 3% compounded semiannually. It is five months since her interest was credited. She will lose interest for five months if she pays cash for the furniture. Would you advise her to withdraw \$200 to pay cash for the furniture or to withdraw the amounts as needed to meet the installment-plan price? Give reasons.

17. Tom Barry has borrowed \$2000, giving a note secured by a mortgage at 5%. How much will it cost him to carry the mortgage for 5 years if he does not reduce the principal?

18. An agent has been offered a position at \$12 a week and 3% of the amount of his sales. How much will his sales have to amount to if he wishes his income to be \$25 a week?

19. The Speed Motorboat Company advertised: "Our Regular 700-Dollar Speed Boats Now \$650." What rate of discount was being offered on the boats?

1. Which of the following statements express the relation between the circumference and the diameter of a circle?

(1) $\frac{c}{d} = \pi$ (2) $c = \pi d$ (3) $\frac{c}{d} = \frac{22}{7}$ (4) $d = 3.14$

2. The ratio of the width to the length of an American flag should be 10 to 19. This means that (1) the length is ___?___ of the width, and (2) the width is ___?___ of the length.

3. What should be the length of a flag (Ex. 2) that is to be 3 ft. wide? 5.4 ft. wide?

4. Which of these statements apply to a rectangle whose length is 5 in. and whose width is 3 in.?

(1) $\frac{w}{l} = .6$ (2) $\frac{l}{w} = 1.67$ (3) $w = \frac{3}{5}l$ (4) $l = \frac{5}{3}w$

5. With your metric scale measure a side and the diagonal of this square. What is the ratio of d to s? (If carefully done, your ratio d should be very close to 1.41.)

6. What is the ratio of s to d in Ex. 5?

7. What is the ratio of the side opposite a 30° angle in a right triangle to the side next to it? of the side opposite a 45° angle in a right triangle to the side next to it? of the side opposite a 60° angle to the side next to it?

8. The ratio of x to 8 equals the ratio of 3 to 5. Find x.

9. The ratio of the sides of two squares is 1 to 2. What is the ratio of their areas? (Think of the sides of the squares as 1 inch and 2 inches, then as 2 inches and 4 inches.)

10. The ratio of the radii of two circles is 3 to 1. What is the ratio of their areas? (You might think of the radii as 3 inches and 1 inch, then as 6 inches and 2 inches.)

11. The ratio of two numbers is 2 to 5. The smaller number is 12. Find the larger number.

Tell whether these statements are true or false:

1. The formula for the volume of a sphere is $V = \frac{4}{3}\pi r^3$.

2. If a rectangular field is n rods long and p rods wide, the value of the field at \$100 an acre is $\frac{100\ np}{160}$ dollars.

3. An isosceles triangle is one which has two sides equal.

4. A right triangle always contains one 90-degree angle and two acute angles.

5. The volume of a cone with a radius of 4 ft. and a height of 7 ft. is 293.1 cu. ft.

6. The total area of a cylinder which has a radius of 4 ft. and a height of 8 ft. is 96π sq. ft.

7. When the length of a line is given as 6.8 inches, this means that its true length is believed to be between 6.75 and 6.85 inches.

8. 2,785,096 expressed to the nearest million is 2,000,000.

9. A square is always a rectangle.

10. A rectangle is always a square.

11. A rectangle is always a parallelogram.

12. A parallelogram is always a rectangle.

13. If a prism and a pyramid have equal bases and equal heights, the volume of the prism is three times the volume of the pyramid.

14. If a cone and a cylinder have equal bases and equal altitudes, the volumes of the cone and the cylinder are equal.

15. The volume of a rectangular box, when measured in cubic inches, is larger than when measured in cubic feet.

16. The formula $V = Bh$ may be used for finding either the volume of a cylinder or the volume of a prism.

1. If each pupil in a schoolroom should have 350 cubic feet of air space, how many pupils may be assigned to your classroom?

2. Your great-grandparents spent approximately 25% of their school time on mathematics. What per cent of your school day is spent on mathematics?

3. The window area of a well-lighted schoolroom should be approximately 25% of the floor area. Compare the window and the floor areas of your classroom.

4. The average number of days in a school year is 180. In your school is the length of the school year above or below the average? What per cent?

5. How far do you live from your school? If you walk, how long does it take you? What is your rate of walking, in miles per hour?

6. Pupils of your age grow in height, on the average, about 2% in a year. If your height increases at that rate, what should your height be a year from now?

7. Pupils of your age grow in weight, on the average, about 6% in a year. If your weight increases at that rate, what should your weight be a year from now?

8. Pupils of your age are urged to sleep 40% of a 24-hour day. What per cent of a 24-hour day do you sleep?

9. The length of average pace of pupils your age is $28\frac{1}{2}$ inches. What is the length of your average pace?

10. The average 14-year-old boy can run 100 yards in 15 seconds; the average 14-year-old girl can run 100 yards in 16 seconds. Compare your speed with these averages.

11. The average pupil your age can concentrate his attention on a problem in a school subject for approximately 30 minutes. Are you an average pupil in the power to use your mind?

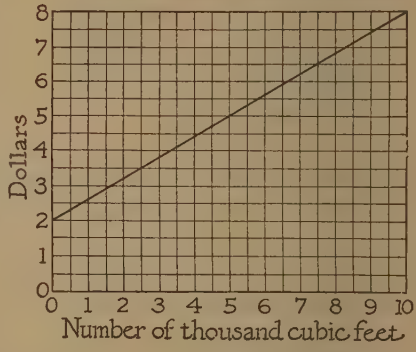
1. The yearly charge for water in a certain city is 60 cents per 1000 cubic feet, plus a service fee of $2. Copy and complete this table, showing the charge in dollars (c) for any number of cubic feet (f).

If f is	1000	2000	3000	4000	5000			20,000
Then c is..	2.60	3.20				8.00	9.20	

2. Which of these formulas agrees with the table above?
(a) $c = .60 \times \frac{1}{1000} f + 2$ (b) $c = 2.60 f$ (c) $c = 200 + 60 f$

3. This graph was made from the data given in the table for Ex. 1. Using the graph, find the cost of 8000 cu. ft. of water; the cost of 10,000 cu. ft.; the number of cubic feet that could be used at a cost of $4.50.

GRAPH SHOWING COST OF WATER



4. Solve each of the following equations:

(a) $\frac{x}{2} = \frac{3}{5}$ (b) $\frac{y}{3} = \frac{4}{5}$ (c) $\frac{n}{4} = \frac{1}{2}$

(d) $\frac{y}{2} = \frac{3}{8}$ (e) $.8\,n = 16$ (f) $.9\,x = 18$ (g) $1.2\,y = 24$

(h) $1.5\,p = 30$ (i) $2\,x + 1 = 18$ (j) $3\,y - 2 = 31$ (k) $4\,p + 5 = 17$ (l) $5\,n - 3 = 42$

5. If $ax + b = c$, does $x = \frac{c - b}{a}$? (If you have difficulty in deciding, think: "If $ax + b = c$, then $ax = ?$ Then $x = ?$")

6. If $ax - b = c$, does $x = \frac{c + b}{a}$?

1. Make a formula for finding the cost, C, of any number of oranges, n, at 30 cents a dozen; at c cents a dozen.

2. Make a formula for finding the number of cubic feet, F, in a rectangular box that is l inches long, w inches wide, and h inches deep.

3. Make a formula for finding the number of acres, A, in a rectangular field that is l rods long and w rods wide.

4. Grace wished to draw two lines on a sheet of paper, to form three columns as shown here. She wanted Column II to be 1 inch wider than Column I, and Column III to be 1 inch wider than Column II. The sheet of paper was 9 inches wide. Can you find the width of each of the columns?

x	$x+1$	$x+2$
I	II	III

5. Albert wished to enlarge a kodak picture 3 in. wide and 4 in. long so that it would be 15 inches long. How wide would the enlarged picture be?

6. A merchant estimated that his overhead expenses were 20% of his sales. He wished to make a net profit of 5% of his sales. How much could he afford to pay for a pair of shoes if they were to sell for $5?

7. How much could the merchant (Ex. 6) pay for articles to sell on his One Dollar counter?

8. A clothing dealer paid $10 for a suit. He marked it to sell at an advance of 50% over the cost. Later he sold the suit at a discount of 40% of the marked price. How much did he receive for it?

9. George read in a magazine that the horse power of an automobile may be found by using the formula $H.P. = \frac{D^2N}{2.5}$, in which D represents the diameter of a cylinder of the engine in inches and N the number of cylinders. In his car, D equaled $3\frac{1}{2}$ and N equaled 6. What was the horse power?

1. Are two right triangles similar if an acute angle of one triangle is equal to an acute angle of the other triangle?

2. Is a triangle a right triangle if the sum of the squares of two of the sides is equal to the square of the third side?

3. Are the corresponding sides of similar triangles equal?

4. Is a scale drawing similar to the figure it is made to represent?

5. What are the two ways of making a scale drawing of a triangle?

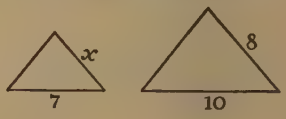
6. In a right triangle, is the sum of the squares of the two sides always equal to the square of the hypotenuse?

7. What is the length of the hypotenuse (to the nearest tenth) of a right triangle whose sides are 4 and 6?

8. In triangle ABC, angle $C = 90°$ and $AC = 20$ ft. What would you have to know to find the length of BC?

9. Are two triangles necessarily congruent if the three angles of one triangle are equal to the three angles of the other triangle?

10. These triangles are similar. Find the length of side x.



11. Solve and check the equation: $\frac{x}{5} = \frac{9}{12}$.

12. In these triangles, angle C = angle D. Are the two triangles necessarily similar? Are they necessarily congruent?

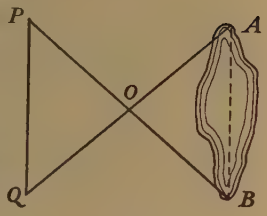
13. Would you say that similar triangles could also be congruent? If so, under what condition?

14. In right triangle ABC, AB (the hypotenuse) is 10 and AC is 8. How long is BC?

15. What is the diagonal of a square in which $s = 20$?

16. Find the side of a square whose area is 200 sq. in.

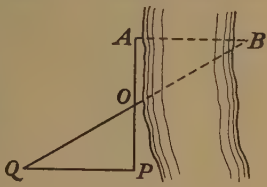
17. To find the length AB of a pond, a stake was set at O. Then AO was extended, making · · · (Tell how to complete the problem.)



18. How tall is a tree whose shadow is 48 ft. long at the same time that a post 8 ft. tall casts a shadow 6 ft. long?

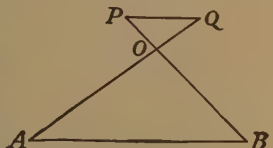
19. What is the angle of elevation of the sun when a post 5 ft. high casts a shadow 8 ft. 7 in. long? (Use the table of tangents.)

20. How could you use congruent triangles to find the length of AB, the distance across a river? (How would you locate point P? point Q?)



21. Complete: If two triangles are known to be similar, then the ratios of · · ·

22. If AO of a triangle AOB is extended so that $OQ = \frac{1}{3}\,AO$, and BO is extended so that $OP = \frac{1}{3}\,BO$, would triangle PQO then be similar to triangle BOA? Why? Would PQ then equal $\frac{1}{3}\,AB$?



23. In a rectangle that is 10 ft. long and 8 ft. wide, the diagonal is how much less than the sum of the base and the altitude of the rectangle?

24. To measure indirectly the distance XY across a lake, a pupil constructed (1) XP perpendicular to XY, (2) YQ perpendicular to XY, and (3) YQ and XP equal. Then he thought that he could find the length of XY by measuring PQ. Do you think that he could? Why?

1. The cost of running the Parker School includes such items as *salaries*, *maintenance*, *supplies*, *interest on investment*, and *depreciation*. Explain the meaning of each of these items.

2. The school building and grounds cost \$126,500. The yearly depreciation is estimated at $3\frac{1}{2}\%$ of the whole cost, and the interest on the cost is figured at 5% per annum. What is the amount of these two items for a year?

3. Salaries of janitors, teachers, and principal together are \$3480 a month for 10 months; the janitors' salaries are \$265 a month for the remaining 2 months. The yearly cost of salaries is ___?___ dollars.

4. During a year, the school used 86 tons of coal, which costs \$7.80 a ton; various other expenses amount to \$846.75. The cost of these items is ___?___ dollars a year.

5 The total yearly cost for all the items (Exs. 2, 3, and 4) is ___?___ dollars.

6. The average attendance at the school is 425 pupils; therefore the average yearly cost per pupil is ___?___ dollars.

7. How much money is your community spending for public schools (elementary and high) this year? How much does it cost your community to provide this year's schooling for you?

8. About 29,000,000 pupils were enrolled in the schools of our country in a recent year. The expenses for education in that year were about two billion dollars, or, on an average, about ___?___ dollars for every person enrolled in the schools.

9. In democratic countries a larger per cent of young people under 18 years of age go through high school than in autocratic countries. Why do you suppose that this is true?

1. The Board of Education of a city in Illinois issued recently $100,000 worth of 4 per cent bonds for the purpose of erecting a new junior high school building. These bonds were to be paid in ten equal annual installments. The bonds sold at par. How much did the Board of Education receive from the sale of the bonds, the brokerage rate being $2 per thousand dollars?

2. The interest on the bonds was payable semiannually. How much will the board have to pay the first year, including interest? the second year? the third year? the fifth year? the tenth year?

3. What will be the total amount of interest paid on these bonds in ten years?

4. The school building was planned to accommodate 600 pupils. What was the per-pupil cost of the building, neglecting interest?

5. It is estimated that the life of the building will be 25 years, after which it will be practically worthless. Neglecting interest, what is the average depreciation per year in the value of the school building?

6. What was the cost of constructing your school building? What was the cost per pupil? (Use as a basis present number of pupils enrolled.) What is the expected period of use of the school? How was the money raised to pay for it?

7. The Board of Education levied a *building tax* of 15 mills on the dollar to raise the money that would be needed at the end of the first year. The assessed value of the property in the city that year was $1,000,000. How much money would be raised by this tax?

8. What is the tax rate for schools in your city or township? What is the assessed value of the property? How much money is raised by your tax?

The national government is urging boys and girls to continue in school longer. State governments are giving the same advice. In a report sent out several years ago by the United States Office of Education, these facts are given:

(*a*) Boys who remain in school until they are 18 (go through high school) receive on the average the following wages:

at age 18, $10 a week,
at age 22, $20 a week,
at age 25, $31 a week.
Total earnings at age 25, $7337.50.

(*b*) Boys who go to work at 14 (do not go to high school) receive on the average the following wages:

at age 14, $4 a week,
at age 18, $7 a week,
at age 21, $9.50 a week,
at age 25, $13 a week.
Total earnings at age 25, $5112.50.

1. Why do you suppose the Office of Education sent out these data? Remember that the longer pupils remain in school the more it costs the government to finance the school. In some countries the government does not provide schools for all children.

2. Make line graphs to help show these facts.

3. At 25 the boy who remained in school until he was 18 was earning ___?___ times as much as the boy who left school at 14.

4. Which boy would probably advance the more rapidly after 25?

5. Many persons assert that a high school education is the best investment any boy or girl can make. Can you offer more than one reason for this?

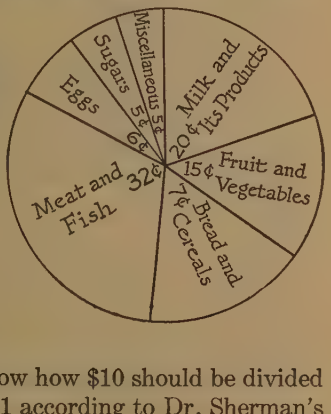
1. Out of $10 your family spends for food, what proportion do you think should be spent for each of the following?

(*a*) Milk and its products
(*b*) Fruits and vegetables
(*c*) Breads and cereals
(*d*) Meat and fish
(*e*) Eggs
(*f*) Sugar, molasses, and syrup
(*g*) Various other foods

Make your estimates before you read the next paragraph.

2. The average American family now divides $10 that is spent for food as shown in the accompanying graph. What per cent of the $10 is spent for each kind of food?

How the Food Dollar Is Spent



3. Dr. Sherman of Columbia University and other noted authorities recommend that the $10 be spent as follows:

for item (*a*), 44%, or $__?__
for item (*b*), 17%, or $__?__
for item (*c*), 13%, or $__?__
for item (*d*), 12%, or $__?__
for item (*e*), 6%, or $__?__
for item (*f*), 5%, or $__?__
for item (*g*), 3%, or $__?__

4. Make a circle graph to show how $10 should be divided for the foods mentioned in Ex. 1 according to Dr. Sherman's recommendations.

5. In one school the eighth-grade pupils kept a record of their families' expenses for food for a month, so that they could determine whether the money had been spent wisely. At the end of the month graphs were made showing how the actual expenses compared with the recommendations of the experts. The principal, teachers, and parents considered the project very valuable.

There was a small family named Hess,
Who ate chocolates and sweets to excess.
They made out a budget
And found that their fudge, it
Had kept them too shabby in dress.

In the Hess family were Mr. and Mrs. Hess and their two children. Mr. Hess earned $45 a week. Toward the end of each week Mrs. Hess would say: "No, we can't buy any candy today. There are two more days before pay day, and my pocketbook is almost empty. I do declare our candy eating keeps us poor."

Mrs. Hess was right—their candy eating kept them poor. One day Mrs. Hess read a magazine article entitled "The Family Budget," in which she learned that her pocketbook need not be empty before the end of the week if she planned her spending wisely. The article included the suggested budgets shown on the next page for the weekly division of income.

1. Which of the suggested budgets do you think would meet the needs of the Hess family, knowing as you do their weekly income and the number in the family?

2. According to the budget that fits the Hess family, how much should they save each week? How much can they afford to spend for food? clothing? shelter? personal expenses? advancement?

3. Mrs. Hess sent for some budget and expense-account blanks and, using suggested Budget No. (9), the family made out a budget and expense account of their own. (See Fig. 1, page 324.) Note how they filled in the first line of the blank. In what respects does it differ from Budget No. (9) (the budget suggested for a family in their circumstances)?

Family: Man and Wife and One Child

Budget No.	Weekly Income	Savings & Insurance	Food	Clothing	Shelter	Personal	Advancement
(1)	$30.00	$1.50	$7.00	$5.00	$11.00	$1.50	$4.00
(2)	35.00	4.50	7.00	6.00	11.00	1.50	5.00
(3)	40.00	7.50	8.00	6.00	12.00	1.50	5.00
(4)	45.00	8.00	8.00	7.00	14.00	2.00	6.00
(5)	50.00	9.00	9.00	7.00	15.00	2.00	7.00

Family: Man and Wife and Two Children

	Weekly Income	Savings & Insurance	Food	Clothing	Shelter	Personal	Advancement
(6)	$30.00	$1.00	$8.00	$7.00	$10.00	$1.50	$2.50
(7)	35.00	3.00	9.00	7.00	11.00	1.50	3.50
(8)	40.00	4.50	10.00	7.00	12.00	1.50	5.00
(9)	45.00	6.00	10.00	8.00	13.00	2.00	6.00
(10)	50.00	8.00	10.00	8.00	15.00	2.00	7.00

Family: Man and Wife and Three Children

	Weekly Income	Savings & Insurance	Food	Clothing	Shelter	Personal	Advancement
(11)	$30.00	$0.50	$10.00	$6.50	$10.00	$1.50	$1.50
(12)	35.00	2.50	10.00	7.00	11.00	1.50	3.00
(13)	40.00	2.50	11.00	8.00	12.00	1.50	5.00
(14)	45.00	4.00	11.00	8.00	14.00	2.00	6.00
(15)	50.00	5.00	12.00	9.00	16.00	2.00	6.00

WEEKLY BUDGET AND EXPENSE ACCOUNT

BUDGET FOR WEEK		45.00	6.00	10.00	6.00	13.00	2.00	5.00	3.00
Carried forward									
Total		45.00	6.00	10.00	6.00	13.00	2.00	5.00	3.00
DATE	INCOME	EXPENSES	SAVINGS	FOOD	CLOTHING	SHELTER	PERSONAL	ADV'MENT	
Nov.	Salary, Commission, Interest	Daily Totals	Sav. Acct., Insurance, Investment, Payments on Home	Groceries, Meats, Dairy Products, Meals out	Wearing Apparel, Materials, Cleaning, Pressing	Rent, Gas, Coal, Electricity, Water, Laundry	Carfare, Taxi, Toilet Articles, Barber	Recreation, Education, Travel, Church, Charity	Rays Allowance
1	45.00	9.95	6.00	.95					3.00
2				1.80		1.00		.25	
3				1.40				.75	
4				1.15	3.25				
5				90				.15	
6				3.55		2.00		1.00	
7				.20			.50	.50	
Total	45.00	28.35	6.00	9.95	3.25	3.00	.50	2.65	3.00
Balance carried forward		16.65		.05	2.75	10.00	1.50	2.35	

FIG. 1

BUDGET FOR WEEK	45.00	6.00	10.00	6.00	13.00	2.00	5.00	3.00
Carried forward	16.65		.05	2.75	10.00	1.50	2.35	
Total	61.65	6.00	10.05	8.75	23.00	3.50	7.35	3.00

FIG. 2

4. Notice that they included a heading "Ray's Allowance." That accounts for some of the changes in the budget. They took $2 from the *suggested* Clothing budget and $1 from the suggested Advancement budget to make up Ray's allowance. (Ray will pay for his own clothing from now on.) Do you consider that a wise change? Why?

5. How closely did they follow their budget the first week they tried to live within it?

6. During the week they spent how much for clothing? Is it likely that they would frequently spend *exactly* $6 for clothing in one week? The $2.75 goes into a reserve fund. Explain how Mrs. Hess's budget might permit her to buy a 17-dollar coat next fall.

7. At the end of each month Mr. Hess pays $40 rent. The expense account given here shows only $3 spent for shelter (heat, light, etc.), but ?? dollars was put in the shelter reserve account. When the Hess family has kept the budget ?? weeks, there will be enough money in the shelter reserve to pay a month's rent.

8. Show how they should have filled in the "Daily Totals" column.

9. What were their total expenditures for the week? In what three ways can this amount be arrived at on the expense sheet?

10. How did Mrs. Hess arrive at the amounts she entered opposite "Balance carried forward"?

11. The top of Mrs. Hess's Weekly Budget and Expense Account for the next week was as shown in Figure 2 on the preceding page. Explain the entries she made.

Suggested budgets to meet a variety of needs may be had by writing to the Society for Savings, Cleveland, Ohio.

1. During one month a school lunchroom used 16,760 cubic feet of gas. What was the cost of the gas at $1.15 per thousand cubic feet?

2. At 68 cents a bushel, what is the value of a load of corn weighing 1840 pounds? (A bushel of corn weighs 70 pounds.)

3. It is frequently estimated that 20 % of the population of a city is of school age. There are 5420 persons of school age in a certain town. What is the estimated population of the town?

4. An empty milk can weighs 16.4 pounds. When filled with milk it weighs 81.7 pounds. What is the value of 10 cans of milk at 12.4 cents a gallon? Milk weighs 8.6 pounds a gallon.

5. Mr. Wilson earned $3600 during a certain year. He spent $900 of his income for rent and fuel, $350 for clothing, $250 for recreation, and $1075 for all other expenses. He saved the remainder. He wished to save 30 % of his income. Did he?

6. A real-estate dealer sold a farm for $6500. His commission was $4\frac{1}{2}$ % on the first $1000 and $2\frac{1}{2}$ % on the balance. How much money should the person who owned the farm receive?

7. Harry wanted to earn some money during vacation. His father offered to let him plant a rectangular field in corn. The field was 20 rods by 40 rods. Harry's net profit was $37.50, or ___?___ dollars an acre.

8. Mr. Scott had $12,000 invested in a hardware store. During a certain year his profit was $2220. If his own work was worth $100 a month, what rate of interest did he make on his investment?

1. An old farmer left a will bequeathing his 17 horses to his three sons, John, James, and William, in the following proportions: John, the eldest, was to have one half, James one third, and William one ninth. How do you think they divided the horses? In their perplexity they consulted a clever lawyer, who hit on a scheme of borrowing a horse from a friend. He then gave John $\frac{1}{2}$ of them, or 9; James $\frac{1}{3}$ of them, or 6; William $\frac{1}{9}$ of them, or 2. Thus he distributed the 17 horses. Then he returned the borrowed horse. Can you explain?

2. "Football is a glorious game," an enthusiast remarked. "At the close of last season, of the football players of my acquaintance, four had broken their left arms, five had broken their right arms, two had sound right arms, and three had sound left arms." What is the smallest number of players that the speaker could have had in mind?

3. *Magician:* Here are six pennies. I have placed them so as to have two rows with three pennies in each row. When I feel a headache coming on, I arrange them in two rows with four in each row. This always cures my pain.

Astonished Spectator: I admit that you can do tricks, but you can't make eight out of six.

Magician: Watch me carefully. It's like this · · ·

A good eighth-grade pupil can easily discover how the magician arranged the coins.

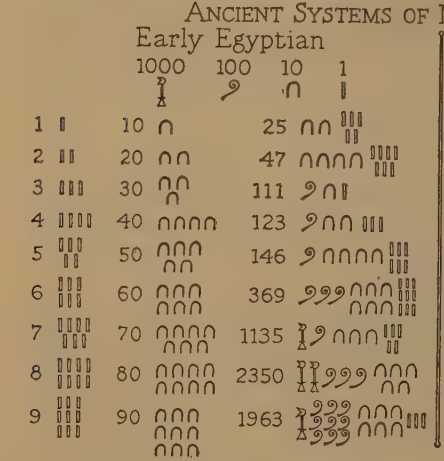
4. It took Dixie Davis two minutes to do this one: I have a rectangular cardboard box. The top has an area of 120 square inches, the side 96 square inches, and the end 80 square inches. What are the exact dimensions of the box?

5. Can you arrange three sevens (777) so as to equal 20? (HINT. Use a decimal point.)

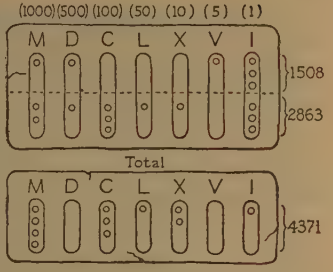
The figures that we use, 1, 2, 3, etc., are called *Arabic numerals* because they have come from similar numbers that the ancient Arabs used. Our Roman numerals, which are quite different, are like those that the Romans used centuries ago. Long ago, peoples in other countries used different kinds of figures, or symbols for numbers.

The ancient Egyptians had no symbols for 2, 3, 4, 5, 6, 7, 8, 9, or 0. They had only four symbols, which represented the numbers 1, 10, 100, and 1000. To represent 2, they used two 1's; to represent 3, three 1's; and so on as shown in the accompanying table.

ANCIENT SYSTEMS OF NOTATION



The early Greeks had a similar system, with symbols for 1, 10, 100, and 1000, but with an additional symbol for 5, which, combined with the symbols for 10 and 100, gave the symbols for 50 and 500. These symbols also are shown in the table above, together with various numbers as they were written in this early Greek notation.



Have you ever tried to add two Roman numerals; for example, MDVIII plus MMDCCCLXIII?

On a flat stone the Romans made grooves as shown in the figure: one groove to represent units (I), one to represent 5's (V), one 10's (X), one 50's (L), etc. They placed in the grooves small stones or counters called *calculi* (our word "calculate" comes from this). To record the number MDVIII (1508), a Roman would place one counter in the M groove, one in the D groove, one in the V groove, and three in the I groove, as shown in the upper part of the diagram. If he wanted to add to this MMDCCCLXIII (2863), he would place other counters for this number as shown in the lower part of the figure and then proceed to add the counters in the following manner:

He would first see that there are six counters in the units (I) groove, which would make one V and one I over. So he would take five of the I counters off the units groove and instead put one counter in the V groove (this leaves one counter in the I groove). He would then have two counters in the V groove. Two V counters equal one X counter; so he would take them off and put one counter in the X groove beside the one that was already there. Two X counters do not make one L counter; so he would leave the two in the X groove. In the D groove he would notice two counters, which combined make one M counter. So he would take these two off and lay one counter in the M groove. That completed the addition, and the total as shown in the illustration is MMMMCCCLXXI. This number is 4371, which is the sum of 1508 and 2863.

1. Mr. Thomas paid \$6000 for a house, which he rented to a tenant for \$55 a month. During the first year he paid \$140 for upkeep, \$30 for insurance, and \$50 for taxes. What per cent did he realize on his investment?

2. During the second year Mr. Thomas spent \$90 for repairs, \$30 for insurance, and \$55 for taxes. What per cent did he make on his investment the second year?

3. At the end of the second year he sold the house for \$6750. In figuring his profit, he deducted 12% of \$6000. Why should he do this? What was his profit?

4. Is it cheaper for Mr. Jones to pay \$6 a month for garage rent or to build a 600-dollar garage on his own lot, even though he would have to borrow the money at 6% and allow \$20 a year for insurance, depreciation, etc.?

5. Which provides the higher rate of income on the investment, a 6% bond at \$80 or an 8% bond at \$105? (Par value of each, \$100.)

6. If you buy a 1000-dollar 6% bond for \$900, you receive __?__ dollars' interest annually. What per cent would you make on your investment?

7. Solve and check these equations:

(*a*) $\frac{y}{3} = 18$ (*b*) $4x + 5 = 45$ (*c*) $8 = \frac{2x}{5}$

8. Find the area of an isosceles right triangle in which the equal sides are each 4 inches. Draw the figure, making the triangle stand on one of its equal sides.

9. Find the property tax on a property assessed at \$3500 when the tax rate is 35 mills.

10. Find the cost of insuring a house worth \$8000 for $\frac{3}{4}$ of its value at \$.35 per \$100.

11. Find the duty on 18 rugs valued at \$75 each if the tariff rate is $16\frac{2}{3}$%.

1. Supply the missing numbers in the following table:

Article	Cost	Margin		Selling Price
		In Per Cent	In Dollars	
(*a*) Suit	$30	30% of cost	?	?
(*b*) Piano	$250	25% of s.p.	?	?
(*c*) Auto	$750	15% of s.p.	?	?
(*d*) Shoes	?	25% of cost	$2	?
(*e*) Tire	?	25% of s.p.	?	$15
(*f*) Radio	?	20% of s.p.	?	$84

2. Would you rather pay $85 for a 6% bond (par value $100) or $45 for a 7% bond (par value $50), if both were equally safe? Which offers the higher rate of income?

3. If the owner of a share of stock in the National Harvester Company (par value $100) receives $8 in dividends, is he likely to wish to sell the stock for less than par? Why? For par? For what should the stock sell in order to yield the purchaser 6% on his investment?

4. Recently this stock (Ex. 3) paid a dividend of $6 a year and sold for $150 a share; it then yielded what per cent on the market price?

5. Mr. Harrison bought 500 shares of stock (par value $25) in the Excelsior Oil Company at $19\frac{3}{4}$, plus brokerage of 15 cents a share. If the stock pays 4% a year, what rate does he receive on his investment?

6. Find the volume of a sphere which has a radius of 6 in.

7. Solve and check: $\frac{2}{3}y + 4 = 8$

8. Find the length of the diagonal of a square whose side is 8 in.

9. Solve the equation $d^2 = 2s^2$ for d, when $s = 10$.

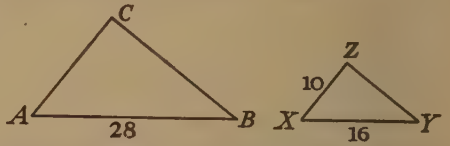
1. Mr. Jones bought 200 shares of National Motors stock at $47\frac{3}{8}$ and sold it at $52\frac{1}{4}$. What was his profit if he paid 15 cents a share brokerage on the purchase and $17\frac{1}{2}$ cents a share on the sale, and a tax of \$34?

2. Mr. Thompson bought 50 shares of Brace National Bank stock at $62\frac{1}{8}$ and sold it at $57\frac{3}{4}$. What was his loss, not counting interest, if he paid brokerage at $17\frac{1}{2}$ cents a share on both the purchase and the sale of the stock, and a tax of \$34?

3. Mr. Smith bought a 1000-dollar 6% bond for \$920. After one year he sold it for \$940. He figured that the rate of income on his investment was $\frac{\$60 + \$20}{\$920}$ of 100%, or $\frac{?}{}$%. Do you agree?

4. Solve this problem by an equation, and check: A merchant said that by selling a chair for \$60 he was making a margin of 25%. What did the chair cost?

5. In these two triangles $\angle A = \angle X$ and $\angle B = \angle Y$. Find AC.



6. Are these formulas true of a right triangle in which h is the hypotenuse and a and b are the sides?

$$b = \sqrt{h^2 - a^2} \qquad a = \sqrt{h^2 - b^2}$$

7. What number increased by 115% of itself equals 1505?

8. A ferryboat charges 25¢ for the transportation of an automobile and 5¢ for each person in the car. Write the formula which represents the transportation charge for a car in which there are n persons.

9. Is it true that the formula for the area of a figure always includes the product of two dimensions, or one dimension squared?

APPENDIX

PRACTICE TESTS FOR ACCURACY AND SPEED

To the Pupil. The following tests are designed to help you become more *accurate* and *rapid* in the fundamentals of mathematics. You should practice these tests until you can write the correct answers to each test on pages 334 to 337 in 3 minutes and each test on pages 338 to 346 in 4 minutes. Place folded paper below each row of examples so that it will not be necessary to copy the figures or to write the answers in the book. Your motto should be, "*Accuracy* first, then *speed*."

To the Teacher. Careful investigation has demonstrated the value of regular *timed practice* exercises in which each pupil tries to excel his own previous achievement in his effort to attain a definite goal. Studies in the psychology of learning emphasize the great importance of having the *learner* become interested in his *improvement*. These practice exercises utilize this principle: (1) by setting up specific goals for the pupil, (2) by having the pupil keep a record of his daily achievement as suggested by the record blank on page 42, and (3) by providing each pupil with an opportunity to concentrate upon those processes which present peculiar difficulties to him. They permit pupils to progress according to their varying skills and abilities.

Teachers who have used standardized, timed practice exercises are gratified at the unusual interest which children develop.

How to Give the Tests

(1) Start all pupils at the same time. Allow all the pupils exactly 4 minutes for each test. (2) Have pupils score their work by referring to the answers which follow the last test. (3) Some pupils need to be taught how to keep their records. (4) The pupils' progress records should be inspected at intervals by the teacher to discover any pupils who may need special instruction. (5) It stimulates the class to have the teacher collect each day the papers of all pupils who have succeeded in reaching the standard to which they aspire. (6) The tests should be given at the beginning of the recitation period. (7) Frequently pupils will ask questions just before beginning a test. This is an opportune time in which to explain and give illustrations of the point in question. Similarly, pupils often want to discuss their errors after they have corrected their papers. It is profitable to discuss *type errors* at this time. The time given to such discussion should be brief.

Test 1. 100 Addition Facts

See if you can write the answers to all these addition examples on folded paper in 3 minutes. If you make an error in any fact, study that fact and then take the whole test again. Keep practicing until you can answer every fact correctly.

1.	7	2	2	2	4	4	3	4	1	9
	1	9	5	8	4	1	1	0	8	9
2.	7	2	6	5	3	1	9	8	1	1
	7	0	1	4	3	1	2	8	3	6
3.	1	2	0	5	2	0	7	1	0	8
	7	1	0	5	2	1	2	9	5	1
4.	6	0	3	0	2	3	4	0	6	8
	2	4	6	2	4	0	5	8	0	3
5.	8	6	1	9	5	6	3	4	1	9
	2	4	4	1	0	6	2	3	5	0
6.	2	9	0	6	3	3	3	2	3	6
	6	3	6	5	8	4	9	3	5	3
7.	7	2	8	4	8	1	5	4	1	5
	3	7	7	8	0	0	2	2	2	3
8.	0	0	5	7	8	7	5	9	6	6
	3	9	1	0	5	9	8	7	8	9
9.	5	7	8	9	5	8	8	4	7	4
	7	8	4	6	9	9	6	7	5	9
10.	9	9	6	0	5	4	7	7	9	3
	5	4	7	7	6	6	6	4	8	7

See the directions at the top of page 334.

1.	6	2	5	6	3	7	4	5	8	6
	1	1	2	3	1	1	3	5	4	5
2.	9	4	9	9	5	5	1	8	4	6
	8	4	1	9	3	4	1	1	2	6
3.	3	3	7	5	0	3	5	9	8	10
	3	2	7	1	0	0	0	6	2	4
4.	8	9	7	6	9	7	12	10	8	11
	8	3	3	0	0	2	6	7	6	2
5.	6	10	10	7	9	8	4	2	7	6
	2	1	2	6	5	7	1	2	5	4
6.	8	13	15	11	11	11	9	10	10	14
	5	6	8	7	5	8	2	3	9	7
7.	7	10	12	12	1	8	7	12	10	10
	4	8	4	8	0	3	0	9	5	6
8.	8	2	15	12	4	14	13	16	14	17
	0	0	9	3	0	9	4	9	5	9
9.	9	17	15	13	13	16	13	13	11	14
	7	8	7	9	5	7	7	8	3	6
10.	9	15	12	14	12	11	11	16	18	11
	4	6	5	8	7	4	6	8	9	9

See the directions at the top of page 334.

1.	6	0	3	0	2	3	4	0	6	8
	2	4	6	2	4	0	5	8	0	3
2.	8	6	1	9	5	6	3	4	1	9
	2	4	4	1	0	6	2	3	5	0
3.	2	9	0	6	3	3	3	2	3	6
	6	3	6	5	8	4	9	3	5	3
4.	7	2	8	4	8	1	5	4	1	5
	3	7	7	8	0	0	2	2	2	3
5.	0	0	5	7	8	7	5	9	6	6
	3	9	1	0	5	9	8	7	8	9
6.	7	2	2	2	4	4	3	4	1	9
	1	9	5	8	4	1	1	0	8	9
7.	7	2	6	5	3	1	9	8	1	1
	7	0	1	4	3	1	2	8	3	6
8.	1	2	0	5	2	0	7	1	0	8
	7	1	0	5	2	1	2	9	5	1
9.	9	9	6	0	5	4	7	7	9	3
	5	4	7	7	6	6	6	4	8	7
10.	5	7	8	9	5	8	8	4	7	4
	7	8	4	6	9	9	6	7	5	9

See the directions at the top of page 334.

1.	$5\overline{)30}$	$2\overline{)18}$	$1\overline{)4}$	$5\overline{)25}$	$3\overline{)0}$	$7\overline{)35}$	$1\overline{)5}$	$4\overline{)8}$	$9\overline{)0}$
2.	$1\overline{)3}$	$5\overline{)35}$	$3\overline{)3}$	$9\overline{)81}$	$7\overline{)42}$	$8\overline{)8}$	$7\overline{)28}$	$8\overline{)0}$	$4\overline{)12}$
3.	$3\overline{)6}$	$2\overline{)16}$	$5\overline{)40}$	$7\overline{)49}$	$1\overline{)2}$	$8\overline{)16}$	$4\overline{)16}$	$9\overline{)9}$	$7\overline{)21}$
4.	$2\overline{)14}$	$3\overline{)9}$	$7\overline{)56}$	$5\overline{)45}$	$9\overline{)72}$	$6\overline{)0}$	$8\overline{)24}$	$7\overline{)14}$	$5\overline{)20}$
5.	$1\overline{)1}$	$7\overline{)63}$	$3\overline{)12}$	$5\overline{)15}$	$4\overline{)20}$	$7\overline{)7}$	$9\overline{)63}$	$6\overline{)6}$	$8\overline{)32}$
6.	$8\overline{)56}$	$2\overline{)12}$	$5\overline{)10}$	$4\overline{)24}$	$7\overline{)0}$	$9\overline{)54}$	$6\overline{)12}$	$3\overline{)15}$	$8\overline{)40}$
7.	$2\overline{)10}$	$5\overline{)5}$	$4\overline{)28}$	$6\overline{)54}$	$9\overline{)45}$	$6\overline{)18}$	$4\overline{)4}$	$3\overline{)18}$	$8\overline{)48}$
8.	$5\overline{)0}$	$4\overline{)32}$	$6\overline{)48}$	$9\overline{)36}$	$6\overline{)24}$	$1\overline{)6}$	$3\overline{)21}$	$2\overline{)2}$	$1\overline{)0}$
9.	$4\overline{)36}$	$6\overline{)42}$	$9\overline{)27}$	$6\overline{)30}$	$2\overline{)8}$	$4\overline{)0}$	$8\overline{)64}$	$2\overline{)0}$	$3\overline{)24}$
10.	$6\overline{)36}$	$2\overline{)6}$	$1\overline{)7}$	$9\overline{)18}$	$1\overline{)8}$	$8\overline{)72}$	$2\overline{)4}$	$1\overline{)9}$	$3\overline{)27}$

Test 5. Addition

7	4	5	8	5	9	2	8
9	8	8	0	6	4	8	6
8	9	6	7	9	8	7	6
7	9	2	8	0	7	8	6
4	6	9	3	7	5	9	7 (8)

27 + 9	55 + 8	14 + 9	36 + 8	45 + 9 (13)
44 + 7	69 + 6	49 + 7	78 + 5	89 + 8 (18)

6	9	5	3		
5	6	9	7		
4	2	5	9	7579	8768
8	9	9	3	375	7685
8	4	1	1	67	6996
4	7	7	8	34867	5558
5	5	5	6	96	7893 (24)

Standards: A, 23. B, 20. C, 18.

Test 6. Multiplication

49	56	70	48	37	19
7	8	8	9	6	8 (6)

327	290	68	46	85	77
7	4	16	19	37	49 (12)

806	703	500	970	560	5937
40	78	900	40	98	78 (13)

Standards: A, 17. B, 15. C, 13.

54	81	73	54	93	86	
9	8	4	8	7	8	(6)
213	411	94	88	62	71	
5	5	36	79	57	63	(12)
95	73	87	609	406	963	
7	69	37	44	39	408	(18)
973	300	2000	700	87160		
869	74	408	378	39096		(23)
926798	355222	527831	156151			
752854	78671	96906	58725			(27)

Standards: A, 26. B, 24. C, 22.

Test 8. Division

$8\overline{)76}$	$4\overline{)26}$	$9\overline{)60}$	$6\overline{)44}$	$8\overline{)57}$	$9\overline{)57}$	(6)
$3\overline{)19}$	$8\overline{)81}$	$9\overline{)40}$	$6\overline{)58}$	$4\overline{)37}$	$8\overline{)69}$	(12)
$7\overline{)81}$	$3\overline{)40}$	$3\overline{)538}$	$5\overline{)539}$	$8\overline{)630}$	$8\overline{)680}$	(18)
$9\overline{)835}$	$7\overline{)399}$	$97\overline{)3935}$	$86\overline{)6364}$			(22)
$92\overline{)3496}$	$39\overline{)2223}$	$72\overline{)2818}$	$31\overline{)1271}$			(26)

Standards: A, 25. B, 23. C, 20.

Test 9. Addition of Fractions

$\frac{1}{2}$	$\frac{7}{8}$	$\frac{1}{4}$	$\frac{1}{6}$	$\frac{1}{2} + \frac{7}{8} =$	$\frac{5}{8} + \frac{7}{12} =$
$\frac{1}{4}$	$\frac{3}{16}$	$\frac{4}{5}$	$\frac{7}{12}$	$\frac{3}{4} + \frac{5}{16} =$	$\frac{3}{8} + \frac{3}{4} =$ (8)

$\frac{1}{2}$	$\frac{3}{4}$	$\frac{1}{2}$		
$\frac{9}{10}$	$\frac{5}{8}$	$\frac{1}{4}$	$\frac{1}{2} + \frac{3}{5} + \frac{7}{10} =$	$9\frac{3}{16}$ $8\frac{2}{5}$
$\frac{3}{5}$	$\frac{5}{16}$	$\frac{7}{8}$	$\frac{1}{2} + \frac{5}{6} + \frac{3}{8} =$	$5\frac{7}{8}$ $7\frac{3}{4}$ (15)

Standards: A, 14. B, 13. C, 12.

Test 10. Subtraction of Fractions

$\frac{1}{2}$	$\frac{3}{5}$	$\frac{2}{3}$	$\frac{3}{4}$	$\frac{1}{2}$	$\frac{5}{8}$	$\frac{11}{12}$	$\frac{15}{16}$
$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{6}$	$\frac{2}{3}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{5}{6}$	$\frac{3}{4}$ (8)

$\frac{5}{6} - \frac{7}{10} =$	$\frac{7}{8} - \frac{1}{12} =$	$\frac{5}{8} - \frac{1}{16} =$	$\frac{11}{16} - \frac{5}{8} =$ (12)
$\frac{5}{6} - \frac{7}{12} =$	$\frac{7}{12} - \frac{3}{8} =$	$\frac{9}{10} - \frac{3}{10} =$	$\frac{5}{8} - \frac{5}{16} =$ (16)

$9\frac{2}{5}$	$7\frac{7}{8}$	$8\frac{3}{8}$	$9\frac{1}{4}$
$5\frac{1}{2}$	$6\frac{5}{12}$	$3\frac{7}{16}$	$4\frac{5}{6}$ (20)

Standards: A, 19. B, 18. C, 17.

Test 11.

Multiplication and Division of Fractions

1. $10 \times \frac{2}{5} =$
2. $\frac{2}{3} \times \frac{3}{8} =$
3. $\frac{5}{8}$ of $30 =$
4. $2\frac{1}{2} \times 4\frac{1}{2} =$
5. $\frac{3}{4}$ of $52 =$
6. $4\frac{1}{2} \times 5\frac{1}{2} =$
7. $2\frac{3}{4} \times 2\frac{1}{2} =$
8. $2\frac{4}{5} \div 4 =$
9. $3\frac{3}{5} \div \frac{2}{5} =$
10. $3\frac{1}{2} \div 6 =$
11. $\frac{2}{3} \div \frac{1}{6} =$
12. $\frac{1}{3} \div \frac{3}{4} =$
13. $2\frac{3}{4} \div \frac{1}{4} =$
14. $2\frac{7}{12} \div \frac{3}{4} =$
15. $\frac{5}{8} \div 4 =$
16. $2\frac{4}{5} \div 3\frac{1}{2} =$
17. $5\frac{1}{2} \times 7\frac{3}{8} =$

Standards: A, 16. B, 15. C, 14.

Test 12. Decimals

Place the decimal point in each of these answers:

1. $100 \times .03 = 300$
2. $33.528 \div .132 = 254$
3. $30 \times .20 = 600$
4. $33.528 \div 13.2 = 254$
5. $.50 \times 6.0 = 3000$
6. $3352.8 \div 132 = 254$
7. $1.8 \times 0.4 = 72$
8. $335.28 \div .132 = 254$
9. $3.0 \times .2 = 60$
10. $33.528 \div 132 = 254$
11. $3.3528 \div 1.32 = 254$
12. $4.0 \times 2.0 = 800$
13. $5.0 \times .6 = 300$
14. $335.28 \div 13.2 = 254$
15. $335.28 \div 1.32 = 254$
16. $0.7 \times 5.0 = 350$
17. $2345 \times .2 = 4690$
18. $33.528 \div 1.32 = 254$
19. $4.35 \times .3 = 1305$
20. $.33528 \div .132 = 254$
21. $1.73 \times .3 = 519$
22. $45.64 \times .06 = 27384$
23. $.33528 \div .0132 = 254$
24. $.33528 \div 132 = 254$

Standards: A, 23. B, 22. C, 21.

Test 13.

Equivalent Fractions, Decimals, and Per Cents

Write each of the following in three forms—as a common fraction, as a decimal fraction, and as a per cent.

1. $\frac{1}{10}$
2. $12\frac{1}{2}\%$
3. .8
4. $\frac{7}{8}$
5. 2%
6. 1.25
7. $\frac{2}{5}$
8. 25%
9. .2
10. $2\frac{5}{8}$
11. $37\frac{1}{2}\%$
12. .0625
13. $\frac{3}{4}$
14. 60%
15. .9
16. $\frac{2}{50}$
17. 70%
18. .035
19. $\frac{9}{25}$
20. $137\frac{1}{2}\%$
21. $\frac{1}{4}\%$
22. .5
23. $2\frac{1}{4}\%$
24. 110%
25. .16
26. $\frac{1}{2}\%$
27. .625
28. .2%
29. 8.3%
30. $6\frac{2}{5}\%$

Standards: A, 29. B, 24. C, 20.

Test 14. Per Cents

Complete each of the following:

1. 25% of 60 = ?
2. ? % of 24 = 8
3. .6% of 40 = ?
4. 8% of 900 = ?
5. ? % of 60 = 6
6. $12\frac{1}{2}\%$ of 32 = ?
7. ? % of 20 = 6
8. 5 = ? % of 15
9. 40% of 12 = ?
10. $62\frac{1}{2}\%$ of 24 = ?
11. 30 = ? % of 300
12. 8% of 8 = ?
13. 8 = ? % of 8
14. 120% of 20 = ?
15. 106% of 50 = ?
16. ? % of 5 = 15
17. 80% of 8 = ?
18. 9 = ? % of 150
19. $83\frac{1}{3}\%$ of 36 = ?
20. ? % of 36 = 12
21. 500% of 7 = ?
22. ? % of 24 = 9
23. 9 = ? % of 18
24. ? % of 50 = 9
25. 3.1% of 20 = ?
26. 1 = ? % of $\frac{1}{3}$

Standards: A, 25. B, 22. C, 20.

Test 15. Decimals

1. $6.0 \div \frac{1}{3}$	8. $.6\overline{)3.0}$	15. $0.5\overline{)0.10}$
2. $\frac{1}{3} \div 6.0$	9. 2.0×7.5	16. $0.05\overline{)0.1}$
3. $6 \times \frac{1}{3}$	10. $.2 \times 0.75$	17. $0.5\overline{)1.0}$
4. $\frac{1}{3} \times 6$	11. 0.02×75.0	18. 5.0×0.10
5. $.3\overline{)6.0}$	12. 2.0×0.75	19. $5.0 + 0.01$
6. $6\overline{)0.3}$	13. $5.0\overline{)1.00}$	20. $0.4 - 0.05$
7. $3\overline{)0.6}$	14. $0.5\overline{)10.0}$	21. $0.01\overline{)100.0}$

Standards: A, 20. B, 18. C, 15.

Test 16. Mixed Fundamentals

1.	2.	3.	4.	5.
$9 + 8$	$9\overline{)70}$	$.29 \times 3$	$43 - 18$	$9 + 18$

6.	7.	8.	9.
726×8	$17 \div 8$	$19 + 8 + 45 + 7$	$6 - 2\frac{3}{4}$

10.	11.	12.	13.
$8 + 14 + 6 + 17$	$7\overline{)1316}$	$54 - 9\frac{2}{3}$	$\frac{2}{5} \div 4$

14.	15.	16.	17.
$493 \times .06$	$\frac{3}{4}$ of 17	$9.318 - 4.004$	$4\overline{)17 \text{ lb. } 4 \text{ oz.}}$

Standards: A, 16. B, 14. C, 12.

Test 17. Adding Common Units of Measure

1. 4 qt. 1 pt.
   1 pt.

2. 8 bu. 2 pk.
   12 bu. 3 pk.

3. 1 gal. 2 pt.
   3 gal. 7 pt.

4. 3 yd. 2 ft.
   4 yd. 1 ft.

5. 5 gal. 3 qt.
   2 gal. 3 qt.

6. 4 mi. 200 rd.
   3 mi. 312 rd.

7. 12 lb. 15 oz.
   4 lb. 12 oz.

8. 2 ft. 8 in.
   1 ft. 2 in.

9. 8 tons 1800 lb.
   2 tons 1000 lb.

10. 3 pk. 7 qt.
    5 qt.

11. 3 hr. 10 min.
    5 hr. 50 min.

12. 25 min. 25 sec.
    30 min. 40 sec.

Standards: *A, 12.* *B, 10.* *C, 9.*

Test 18.
Multiplying and Dividing Common Units of Measure

Multiply:

1. 3 qt. 1 pt. × 3

2. 3 tons 400 lb. × 7

3. 2 pk. 7 qt. × 4

4. 9 yd. 2 ft. × 7

5. 5 hr. 15 min. × 6

Divide:

6. 5)18 ft. 4 in.

7. 2)7 gal. 2 qt.

8. 3)12 bu. 3 pk.

9. 4)6 lb. 4 oz.

10. 5)17 min. 30 sec.

Standards: *A, 10.* *B, 9.* *C, 7.*

Find the cost of—

1. $4\frac{1}{2}$ lb. at 20¢ a pound.
2. 36 things at 12 for 25¢.
3. 7 bananas at 2 for 5¢.
4. $3\frac{3}{4}$ yd. at 12¢ a yard.
5. $4\frac{1}{4}$ yd. at 24¢ a yard.
6. 21 things at 3 for 10¢.
7. 9 things at 40¢ a dozen.
8. 38 articles at 2 for 1¢.
9. 1 pk. of potatoes at $1.60 a bushel.
10. 5 things at 72¢ a dozen.

Standards: A, 10. B, 9. C, 7.

Test 20.

Finding Costs to the Nearest Tenth of a Cent

ILLUSTRATION. At 3 for 5¢, the cost of 1 banana, to the nearest tenth of a cent, is \$0.017. We know that \$0.016 is less accurate than \$0.017, for $\frac{1}{3}$ of \$0.05 is \$.0166+. Notice that we have to find the number in the fourth decimal place to get the nearest three-decimal-place answer.

1. At 4 for 5¢, the cost of 1 article is ___?___.
2. At 12 for 35¢, the cost of 1 article is ___?___.
3. At 7 for 20¢, the cost of 1 article is ___?___.
4. At 7 for 50¢, the cost of 1 article is ___?___.
5. At 4 for 25¢, the cost of 1 article is ___?___.
6. At 3 for 25¢, the cost of 1 article is ___?___.
7. At 75¢ a dozen, the cost of 1 article is ___?___.
8. At \$1.75 a dozen, the cost of 1 article is ___?___.
9. At 70¢ a pound, the cost of 1 ounce is ___?___.
10. At \$6 a gross, the cost of 1 article is ___?___.

Standards: A, 10. B, 9. C, 8.

Test 21. Discounts

What do you pay for goods listed at—

1. \$70 less 10%?
2. \$7.00 less 10%?
3. \$2.00 less $\frac{1}{4}$?
4. \$24 less $33\frac{1}{3}$%?
5. 60¢ less $\frac{1}{5}$?
6. \$7.20 less $12\frac{1}{2}$%?
7. \$7.20 less $\frac{1}{8}$?
8. \$40 less 50%?
9. \$7.50 less 2%?
10. \$6.00 less 40%?
11. \$1.60 less 5%?
12. \$1.60 less $\frac{1}{5}$?

Standards: A, 11. B, 9. C, 7.

Test 22. General Review

1. $\frac{2}{3} + \frac{1}{4} =$
2. $4\frac{2}{5} + 8\frac{7}{10} =$
3. $\frac{3}{4} - \frac{7}{16} =$
4. $8\frac{1}{6} - 3\frac{3}{4} =$
5. $\frac{2}{3} \times \frac{3}{5} =$
6. $\frac{3}{8}$ of 42 =
7. $12 \times 3\frac{3}{4} =$
8. $3\frac{1}{2} \times 2\frac{1}{2} =$
9. $\frac{1}{2} \div \frac{1}{3} =$
10. $7\frac{1}{2} \div \frac{3}{4} =$
11. Add: 35.07 + 2.93 + 25.008 =
12. Subtract: 9.37 − 4.693 =
13. Multiply: 6.58 × 3.5 =
14. Divide: 57.318 ÷ 1.23 =
15. 12 is what part of 20?
16. Write as per cent: $\frac{1}{2}$, $\frac{2}{5}$, $\frac{3}{8}$.
17. Write as a common fraction: 60%, $87\frac{1}{2}$%, 25%.
18. Write as a decimal and as per cent: $\frac{2}{3}$, $\frac{5}{8}$, $\frac{4}{5}$.
19. 45% of 40 =
20. ___?___% of 90 = 60.

Standards: A, 19. B, 17. C, 15.

ANSWERS TO TESTS

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Group I		Group II		Group III		Group IV	
40	38	59	53	$4.84	$5.56	681	579

Group V		Group VI		Group VII	
$35.60	$33.57	27776	40162	$1859.23	$501.87

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Practice Group I

(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)
42	40	43	41	38	46	39	42

Practice Group II

62	62	50	62	73	73	62	65

Practice Group III

(a)	(b)	(c)	(d)	(e)	(f)
$5.34	$5.99	$6.44	$4.40	$5.25	$4.44

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Practice Group IV

(a)	(b)	(c)	(d)	(e)	(f)
517	500	583	580	566	553

Practice Group V

(a)	(b)	(c)	(d)	(e)
$35.62	$31.54	$24.32	$35.62	$33.24

Practice Group VI

32476	19031	28813	36407	26863

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PRACTICE GROUP VII

$952.31 $1813.96 $1075.04 $1984.52 $1323.53

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GROUP I		GROUP II		GROUP III	
326	227	203	306	8417	4447
GROUP IV		**GROUP V**		**GROUP VI**	
231	173	5106	511	10065	650

GROUP VII

$5069.09 $5952.08 $6616.01

GROUP VIII

$4100.26 $5129.08 $4010.95

PRACTICE GROUP I

257 35 286 174 327

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PRACTICE GROUP II

(*a*) 407 (*b*) 306 (*c*) 309 (*d*) 305 (*e*) 409

PRACTICE GROUP III

(*a*) 7449 (*b*) 9068 (*c*) 7789 (*d*) 4900

PRACTICE GROUP IV

(*a*) 133 (*b*) 18 (*c*) 521 (*d*) 101 (*e*) 219

PRACTICE GROUP V

(*a*) 5235 (*b*) 3121 (*c*) 3136 (*d*) 1417

PRACTICE GROUP VI

(*a*) 362 (*b*) 21 (*c*) 217 (*d*) 557

PRACTICE GROUP VII

(*a*) $6193.24 (*b*) $4562.25 (*c*) $5859.50 (*d*) $5860.25

PRACTICE GROUP VIII

(*a*) $5152.38 (*b*) $5123.25 (*c*) $1512.75 (*d*) $3445.28

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	(a)	(b)		(a)	(b)		(a)	(b)
I.	2925	2150	II.	99242	255970	III.	2097748	2821892
IV.	107800	206550	V.	1466800	2086000	VI.	269700	93420

	(a)	(b)		(a)	(b)
VII.	173602	157488	VIII.	92587	259722
IX.	8050072	28062030	X.	$10881.78	$8742.25

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	(a)	(b)	(c)		(a)	(b)	(c)
I.	3306	3450	3276	II.	109824	237325	145126
III.	1104384	624764	1430457	IV.	167400	115560	108080

	(a)	(b)	(c)	(d)	(e)
V.	867500	2530800	1661700	4531800	8174700
VI.	96880	178940	435240	132020	702210
VII.	140208	158283	45368	250084	465156
VIII.	112442	87312	35425	231963	613590
IX.	12059072	8014005	12030012	30082056	56114054

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X. $24800.50 $7691.64 $23975.49 $36814.50 $68619.33

	(*a*)	(*b*)		(*a*)	(*b*)
I.	879	754	II.	21	22
III.	82	45	IV.	36.5	25.9
V.	901.1	708.8	VI.	70.1	61.0
VII.	97.7	89.5	VIII.	9008	9008

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	(*a*)	(*b*)	(*c*)	(*d*)
I.	876	985	789.7	867.8
II.	22.1	43.1	21.1	34.3
III.	71.2	81	99.6	89.1
IV.	34	25.7	62.3	53.7
V.	801	901	905.3	902.6
VI.	90.7	70.6	80.5	70.2
VII.	93.5	97.6	84.1	61.7
VIII.	8009	9007	8009	8007

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	Part I		Part II		
I	II	III	X	XI	XII
(*a*) $\frac{7}{8}$	$1\frac{3}{5}$	$14\frac{4}{5}$	(*a*) $\frac{4}{15}$	26	$1\frac{3}{4}$
(*b*) $\frac{11}{12}$	$1\frac{17}{24}$	$9\frac{2}{3}$	(*b*) $\frac{5}{18}$	21	$1\frac{7}{18}$
IV	V	VI	XIII	XIV	XV
(*a*) $8\frac{11}{12}$	$\frac{13}{24}$	$4\frac{1}{24}$	(*a*) $8\frac{1}{2}$	$\frac{2}{3}$	9
(*b*) $11\frac{5}{8}$	$\frac{11}{20}$	$3\frac{1}{3}$	(*b*) 6	$\frac{1}{5}$	$\frac{1}{12}$
VII	VIII	IX	XVI	XVII	XVIII
(*a*) $3\frac{11}{12}$	$16\frac{1}{6}$	$12\frac{3}{8}$	(*a*) $\frac{1}{2}$	$\frac{8}{25}$	$\frac{15}{22}$
(*b*) $8\frac{5}{8}$	$3\frac{1}{3}$	$2\frac{5}{8}$	(*b*) $8\frac{3}{4}$	4	$1\frac{3}{11}$

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	I	II	III
(*a*)	$\frac{5}{6}$	$2\frac{1}{8}$	$16\frac{3}{10}$
(*b*)	$\frac{3}{4}$	$1\frac{17}{20}$	$16\frac{11}{20}$
(*c*)	$\frac{7}{8}$	2	$12\frac{5}{8}$
(*d*)	$\frac{11}{20}$	$2\frac{1}{4}$	$12\frac{3}{4}$
(*e*)	$\frac{7}{20}$	$1\frac{11}{24}$	$15\frac{2}{3}$

	IV	V	VI
(*a*)	$11\frac{11}{24}$	$\frac{1}{8}$	$4\frac{7}{15}$
(*b*)	$12\frac{1}{4}$	$\frac{7}{20}$	$4\frac{1}{24}$
(*c*)	11	$\frac{7}{12}$	$7\frac{1}{12}$
(*d*)	$20\frac{7}{20}$	$\frac{7}{24}$	$7\frac{3}{20}$
(*e*)	$14\frac{1}{8}$	$\frac{3}{10}$	$1\frac{13}{24}$

	VII	VIII	IX
(*a*)	$3\frac{2}{3}$	$17\frac{1}{4}$	$13\frac{1}{8}$
(*b*)	$\frac{23}{24}$	$19\frac{1}{8}$	$16\frac{1}{4}$
(*c*)	$\frac{7}{10}$	$15\frac{1}{3}$	$15\frac{7}{8}$
(*d*)	$2\frac{7}{20}$	$3\frac{1}{6}$	$3\frac{2}{3}$
(*e*)	$4\frac{7}{8}$	$7\frac{5}{8}$	$5\frac{3}{5}$

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	X	XI	XII
(*a*)	$\frac{5}{8}$	27	$2\frac{1}{2}$
(*b*)	$\frac{1}{2}$	$16\frac{1}{3}$	$4\frac{2}{5}$
(*c*)	$\frac{7}{10}$	$40\frac{1}{2}$	$1\frac{3}{4}$
(*d*)	$\frac{2}{7}$	12	$3\frac{19}{24}$
(*e*)	$\frac{5}{12}$	28	$5\frac{3}{7}$

	XIII	XIV	XV
(*a*)	$2\frac{11}{12}$	$1\frac{1}{2}$	14
(*b*)	$9\frac{1}{6}$	$1\frac{1}{6}$	12
(*c*)	$8\frac{1}{3}$	$1\frac{1}{4}$	$\frac{1}{20}$
(*d*)	$8\frac{1}{6}$	$\frac{7}{16}$	$\frac{7}{16}$
(*e*)	2	$\frac{4}{5}$	12

	XVI	XVII	XVIII
(*a*)	$\frac{1}{4}$	$\frac{1}{3}$	3
(*b*)	$\frac{1}{3}$	$\frac{2}{3}$	$1\frac{1}{3}$
(*c*)	$6\frac{3}{4}$	6	$\frac{2}{5}$
(*d*)	11	6	$\frac{2}{3}$
(*e*)	$\frac{1}{6}$	$1\frac{5}{6}$	$1\frac{4}{5}$

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	I	II	III	IV
(*a*)	3.61	33.64	10.49	1.48
(*b*)	2.76	30.16	28.7	6.5

	V	VI	VII	VIII
(*a*)	.34	.385	.4725	32.25
(*b*)	.32	16.26	.2940	5.248

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	IX	X	XI	XII	XIII	XIV	XV
(*a*)	.0175	.0056	32.3	.63	.008	35.12	.88
(*b*)	.0375	.0066	6.519	7.5	.06	842.86	.89

	(*a*)	(*b*)	(*c*)	(*d*)	(*e*)
I.	4.06	3.06	3.47	2.69	3.94
II.	22.67	20.05	23.62	27.65	

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	(*a*)	(*b*)	(*c*)	(*d*)	(*e*)	(*f*)
III.	11.91	25.15	11.84	63.40		
IV.	.7	1.57	6.4	2.62	5.76	.25
V.	.49	.26	.01	.07	.08	
VI.	.604	7.564	15.86	.16		
VII.	.5355	.2688	.310	.56		
VIII.	14.08	2.850	4.620	28.08		
IX.	.060	.0150	.088	.0245		
X.	.0054	.0077	.0040	.0028		

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	(*a*)	(*b*)	(*c*)	(*d*)
XI.	6.2	6.94	.5065	300.2
XII.	.96	8.5	64	3.8
XIII.	.022	.075	.028	.006

	(*a*)	(*b*)	(*c*)	(*d*)	(*e*)	(*f*)	(*g*)	(*h*)
XIV.	6.86	7.71	.37	2.00	18.71	1.37	1.48	267.86
XV.	.73	.78	.62	.78	.59	.25	.60	.77

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PART I

I. (*a*)	18.72	II. (*a*)	1.9	III. (*a*)	56
(*b*)	5.12	(*b*)	3.04	(*b*)	156
IV. (*a*)	18.2	V. (*a*)	15	VI. (*a*)	.09
(*b*)	124.8	(*b*)	12	(*b*)	.044
VII. (*a*)	1.275	VIII. (*a*)	.152	IX. (*a*)	2.1
(*b*)	2.53	(*b*)	.145	(*b*)	2.448

PART II

I. (*a*) 60	II. (*a*) 120	III. (*a*) 300	IV. (*a*) .5
(*b*) $36\frac{4}{11}$	(*b*) 125	(*b*) 400	(*b*) .8

PART III

I. (*a*) 120	II. (*a*) 96	III. (*a*) 44	IV. (*a*) 40
(*b*) 80	(*b*) 40	(*b*) 30	(*b*) 400

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PART I

	I	II	III	IV	V
(*a*)	17.52	28.8	112	74.4	36
(*b*)	4.32	8.2	72	81.6	18
(*c*)	21.28	.29	117	311.5	63
(*d*)	26.32	2.52	420	123.75	16
(*e*)	4.68	2.24	378	78.75	100

	VI	VII	VIII	IX
(*a*)	1.4	1.8375	.39	.7
(*b*)	.08	5.33	.384	2.176
(*c*)	.92	1.47	.156	1.872
(*d*)	.9	5.9375	.083	10.875
(*e*)	.046	3.78	.05	3.825

PART II

	I	II	III	IV
(*a*)	80%	125	400%	.5
(*b*)	58.3%	120	200%	.4
(*c*)	46.2%	183.3	300%	.6
(*d*)	$33\frac{1}{3}$%	128.6	800%	.7
(*e*)	44.4%	115.4	1000%	.2

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PART III

	I	II	III	IV
(*a*)	480	30	120	900
(*b*)	240	20	400	900
(*c*)	600	15	400	800
(*d*)	150	96	200	400
(*e*)	200	80	80	1200
(*f*)	50	40		

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Test 1. 100 Addition Facts

1.	8	11	7	10	8	5	4	4	9	18
2.	14	2	7	9	6	2	11	16	4	7
3.	8	3	0	10	4	1	9	10	5	9
4.	8	4	9	2	6	3	9	8	6	11
5.	10	10	5	10	5	12	5	7	6	9
6.	8	12	6	11	11	7	12	5	8	9
7.	10	9	15	12	8	1	7	6	3	8
8.	3	9	6	7	13	16	13	16	14	15
9.	12	15	12	15	14	17	14	11	12	13
10.	14	13	13	7	11	10	13	11	17	10

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Test 2. 100 Subtraction Facts

1.	5	1	3	3	2	6	1	0	4	1
2.	1	0	8	0	2	1	0	7	2	0
3.	0	1	0	4	0	3	5	3	6	6
4.	0	6	4	6	9	5	6	3	2	9
5.	4	9	8	1	4	1	3	0	2	2
6.	3	7	7	4	6	3	7	7	1	7
7.	3	2	8	4	1	5	7	3	5	4
8.	8	2	6	9	4	5	9	7	9	8
9.	2	9	8	4	8	9	6	5	8	8
10.	5	9	7	6	5	7	5	8	9	2

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Test 3. 100 Multiplication Facts

1.	12	0	18	0	8	0	20	0	0	24
2.	16	24	4	9	0	36	6	12	5	0
3.	12	27	0	30	24	12	27	6	15	18
4.	21	14	56	32	0	0	10	8	2	15
5.	0	0	5	0	40	63	40	63	48	54
6.	7	18	10	16	16	4	3	0	8	81
7.	49	0	6	20	9	1	18	64	3	6
8.	7	2	0	25	4	0	14	9	0	8
9.	45	36	42	0	30	24	42	28	72	21
10.	35	56	32	54	45	72	48	28	35	36

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Test 4. 90 Division Facts

1.	6	9	4	5	0	5	5	2	0
2.	3	7	1	9	6	1	4	0	3
3.	2	8	8	7	2	2	4	1	3
4.	7	3	8	9	8	0	3	2	4
5.	1	9	4	3	5	1	7	1	4
6.	7	6	2	6	0	6	2	5	5
7.	5	1	7	9	5	3	1	6	6
8.	0	8	8	4	4	6	7	1	0
9.	9	7	3	5	4	0	8	0	8
10.	6	3	7	2	8	9	2	9	9

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Test 5. Addition

35	36	30	26	27	33	34	33 (8)
36		63	23		44	54	(13)
51		75	56		83	97	(18)
40	42	41	37	42984		36900	(24)

Test 6. Multiplication

343	448	560	432	222	152 (6)
2289	1160	1088	874	3145	3773 (12)
32,240	54,834	450,000	38,800	54,880	463,086 (18)

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Test 7. Subtraction

45	73	69	46	86	78 (6)
208	406	58	9	5	8 (12)
88	4	50	565	367	555 (18)
104	226	1592	322		48064 (23)
173944		276551	430925		97426 (27)

Test 8. Division

$9\frac{1}{2}$	$6\frac{1}{2}$	$6\frac{2}{3}$	$7\frac{1}{3}$	$7\frac{1}{8}$	$6\frac{1}{3}$ (6)
$6\frac{1}{3}$	$10\frac{1}{8}$	$4\frac{4}{9}$	$9\frac{2}{3}$	$9\frac{1}{4}$	$8\frac{5}{8}$ (12)
$11\frac{4}{7}$	$13\frac{1}{3}$	$179\frac{1}{3}$	$107\frac{4}{5}$	$78\frac{3}{4}$	85 (18)
$92\frac{7}{9}$		57	40.57		74 (22)
38		57	39.14		41 (26)

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Test 9. Addition of Fractions

$\frac{3}{4}$ $1\frac{1}{16}$ $1\frac{1}{20}$ $\frac{3}{4}$ $1\frac{3}{8}$ $1\frac{1}{16}$ $1\frac{5}{24}$ $1\frac{1}{8}$ (8)

2 $1\frac{11}{16}$ $1\frac{5}{8}$ $1\frac{4}{5}$ $1\frac{17}{24}$ $15\frac{1}{16}$ $16\frac{3}{20}$ (16)

Test 10. Subtraction of Fractions

$\frac{1}{4}$ $\frac{1}{10}$ $\frac{1}{2}$ $\frac{1}{12}$ $\frac{1}{6}$ $\frac{3}{8}$ $\frac{1}{12}$ $\frac{3}{16}$ (8)

$\frac{2}{15}$ $\frac{19}{24}$ $\frac{9}{16}$ $\frac{1}{16}$ (12) $\frac{1}{4}$ $\frac{5}{24}$ $\frac{3}{5}$ $\frac{5}{16}$ (16)

$3\frac{9}{10}$ $1\frac{11}{24}$ $4\frac{15}{16}$ $4\frac{5}{12}$ (20)

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Test 11. Multiplication and Division of Fractions

1. 4 4. $11\frac{1}{4}$ 7. $6\frac{7}{8}$ 10. $\frac{7}{12}$ 13. 11 16. $\frac{4}{5}$
2. $\frac{1}{4}$ 5. 39 8. $\frac{7}{10}$ 11. 4 14. $3\frac{4}{9}$ 17. $40\frac{9}{16}$
3. $18\frac{3}{4}$ 6. $24\frac{3}{4}$ 9. 9 12. $\frac{4}{9}$ 15. $\frac{5}{32}$

Test 12. Decimals

1. 3.00	5. 3.000	9. .60	13. 3.00	17. 469.0	21. .519	
2. 254.	6. 25.4	10. .254	14. 25.4	18. 25.4	22. 2.7384	
3. 6.00	7. .72	11. 2.54	15. 254.	19. 1.305	23. 25.4	
4. 2.54	8. 2540.	12. 8.00	16. 3.50	20. 2.54	24. .00254	

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Test 13. Equivalent Fractions, Decimals, and Per Cents

1. $\frac{1}{10}$	.1	10%	9. $\frac{1}{5}$	.2	20%
2. $\frac{1}{8}$	.125	$12\frac{1}{2}\%$	10. $2\frac{5}{8}$	2.625	$262\frac{1}{2}\%$
3. $\frac{4}{5}$	.8	80%	11. $\frac{3}{8}$	.375	$37\frac{1}{2}\%$
4. $\frac{7}{8}$	.875	$87\frac{1}{2}\%$	12. $\frac{1}{16}$	.0625	$6\frac{1}{4}\%$
5. $\frac{1}{50}$	.02	2%	13. $\frac{3}{4}$	.75	75%
6. $1\frac{1}{4}$	1.25	125%	14. $\frac{3}{5}$	.60	60%
7. $\frac{2}{5}$	.4	40%	15. $\frac{9}{10}$	.9	90%
8. $\frac{1}{4}$	.25	25%	16. $\frac{2}{50}$	.04	4%

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17.	$\frac{7}{10}$	.7	70%	25.	$\frac{4}{25}$	.16	16%
18.	$\frac{7}{200}$	.035	$3\frac{1}{2}\%$	26.	$\frac{1}{200}$	.005	$\frac{1}{2}\%$
19.	$\frac{9}{25}$	.36	36%	27.	$\frac{5}{8}$	.625	$62\frac{1}{2}\%$
20.	$1\frac{3}{8}$	1.375	$137\frac{1}{2}\%$	28.	$\frac{1}{500}$	.002	.2%
21.	$\frac{1}{400}$	.0025	$\frac{1}{4}\%$	29.	$\frac{83}{1000}$	.083	8.3%
22.	$\frac{1}{2}$	.5	50%	30.	$\frac{8}{125}$	.064	$6\frac{2}{5}\%$
23.	$\frac{9}{400}$	.0225	$2\frac{1}{4}\%$				
24.	$1\frac{1}{10}$	1.1	110%				

Test 14. Per Cents

1. 15	8. $33\frac{1}{3}$	14. 24	21. 35
2. $33\frac{1}{3}$	9. 4.8	15. 53	22. $37\frac{1}{2}$
3. .24	10. 15	16. 300	23. 50
4. 72	11. 10	17. 6.4	24. 18
5. 10	12. .64	18. 6	25. .62
6. 4	13. 100	19. 30	26. 300
7. 30		20. $33\frac{1}{3}$	

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Test 15. Decimals

1. 18 2. $\frac{1}{18}$ 3. 2 4. 2 5. 20 6. .05 7. .2
8. 5 9. 15 10. .15 11. 1.5 12. 1.5 13. .2 14. 20
15. .2 16. 2 17. 2 18. .5 19. 5.01 20. .35 21. 10,000

Test 16. Mixed Fundamentals

1. 17 2. $7\frac{7}{9}$ 3. .87 4. 25 5. 27 6. 5808
7. $2\frac{1}{8}$ 8. 79 9. $3\frac{1}{4}$ 10. 45 11. 188 12. $44\frac{1}{3}$
13. $\frac{1}{10}$ 14. 29.58 15. $12\frac{3}{4}$ 16. 5.314 17. 4 lb. 5 oz.

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Test 17. Adding Common Units of Measure

1. 5 qt. 0 pt.
2. 21 bu. 1 pk.
3. 5 gal. 1 pt.
4. 8 yd. 0 ft.
5. 8 gal. 2 qt.
6. 8 mi. 192 rd.
7. 17 lb. 11 oz.
8. 3 ft. 10 in.
9. 11 tons 800 lb.
10. 4 pk. 4 qt.
11. 9 hr. 0 min.
12. 56 min. 5 sec.

Test 18. Multiplying and Dividing Common Units of Measure

1. 10 qt. 1 pt.
2. 22 tons 800 lb.
3. 11 pk. 4 qt.
4. 67 yd. 2 ft.
5. 31 hr. 30 min.
6. 3 ft. 8 in.
7. 3 gal. 3 qt.
8. 4 bu. 1 pk.
9. 1 lb. 9 oz.
10. 3 min. 30 sec.

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Test 19. Mental Arithmetic

1. 90¢
2. 75¢
3. 18¢
4. 45¢
5. $1.02
6. 70¢
7. 30¢
8. 19¢
9. 40¢
10. 30¢

Test 20. Finding Costs to the Nearest Tenth of a Cent

1. 1.3¢
2. 2.9¢
3. 2.9¢
4. 7.1¢
5. 6.3¢
6. 8.3¢
7. 6.3¢
8. 14.6¢
9. 4.4¢
10. 4.2

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Test 21. Discounts

1. $63
2. $6.30
3. $1.50
4. $16.00
5. 48¢
6. $6.30
7. $6.30
8. $20
9. $7.35
10. $3.60
11. $1.52
12. $1.28

Test 22. General Review

1. $1\frac{1}{2}$
2. $13\frac{1}{10}$
3. $\frac{5}{16}$
4. $4\frac{5}{12}$
5. $\frac{2}{5}$
6. $15\frac{3}{4}$
7. 45
8. $8\frac{3}{4}$
9. $1\frac{1}{2}$
10. 10
11. 63.008
12. 4.677
13. 23.03
14. 46.6
15. $\frac{3}{5}$
16. 50%, 40%, $37\frac{1}{2}$%
17. $\frac{3}{5}$, $\frac{7}{8}$, $\frac{1}{4}$
18. .66$\frac{2}{3}$, 66$\frac{2}{3}$%; .62$\frac{1}{2}$, 62$\frac{1}{2}$%; .8, 80%
19. 18
20. 66$\frac{2}{3}$

TABLES FOR REFERENCE

LINEAR MEASURE OR MEASURES OF LENGTH

12 inches (in.)	= 1 foot (ft.)
3 feet	= 1 yard (yd.)
5.5 yards	= 1 rod (rd.)
320 rods	= 1 mile (mi.)

A mile = 320 rd. = 1760 yd. = 5280 ft. = 1.61 kilometers

SQUARE MEASURE OR MEASURES OF SURFACE

144 square inches (sq. in.)	= 1 square foot (sq. ft.)
9 square feet	= 1 square yard (sq. yd.)
30.25 square yards	= 1 square rod (sq. rd.)
160 square rods	= 1 acre (A.)
640 acres	= 1 square mile (sq. mi.)
36 square miles	= a township (tp.)

An acre = 160 sq. rd. = 4840 sq. yd. = 43,560 sq. ft.

CUBIC MEASURE OR MEASURES OF VOLUME

1728 cubic inches (cu. in.)	= 1 cubic foot (cu. ft.)
27 cubic feet	= 1 cubic yard (cu. yd.)
128 cubic feet	= 1 cord of wood of 4-ft. length

IMPORTANT VOLUME EQUIVALENTS

1 cubic foot	=	7.5 gallons (gal.)
1 cubic foot of water	=	62.5 pounds (lb.)
1 cubic foot	=	0.8 bushel (bu.)
1 gallon	=	231 cubic inches (cu. in.)
1 gallon of water	=	$8\frac{1}{3}$ pounds (lb.)
1 cubic foot of ice	=	$56\frac{1}{4}$ pounds (lb.)

MEASURES OF WEIGHT

16 ounces (oz.)	= 1 pound (lb.)
100 pounds	= 1 hundredweight (cwt.)
2000 pounds	= 1 ton (T.)
2240 pounds	= 1 long ton
One pound Avoirdupois	= 7000 grains (gr.)

MEASURES OF TIME

60 seconds (sec.)	= 1 minute (min.)
60 minutes	= 1 hour (hr.)
24 hours	= 1 day (da.)
7 days	= 1 week (wk.)
4 weeks (approximately)	= 1 month (mo.)
12 months	= 1 year (yr.)
365 days	= a common year
366 days	= a leap year
10 years	= a decade
100 years	= a century

Century years divisible by 400 are leap years and all other years divisible by 4, except century years not divisible by 400, are leap years.

MEASURES OF LIQUIDS

4 gills (gi.)	= 1 pint (pt.)
2 pints	= 1 quart (qt.)
4 quarts	= 1 gallon (gal.)
31.5 gallons	= 1 barrel (bbl.)
2 barrels	= 1 hogshead (hhd.)

MEASURES OF ANGLES AND ARCS

60 seconds (60″)	= 1 minute (1′)
60 minutes	= 1 degree (1°)
360° (of arc)	= a circumference
360° (of angle)	= a complete rotation
90°	= a right angle

An acute angle is less than 90°.
An obtuse angle is greater than 90° but less than 180°.
A reflex angle is greater than 180° but less than 360°.

OUR MONEY UNITS

10 mills	= 1 cent (¢ or ct.)
10 cents	= 1 dime (d.)
10 dimes	= 1 dollar ($)
10 dollars	= 1 eagle

COUNTING UNITS

12 things	= 1 dozen (doz.)
20 things	= 1 score
12 dozen	= 1 gross (gr.)
12 gross	= 1 great gross

DRY MEASURE

2 pints (pt.)	= 1 quart (qt.)
4 quarts	= 1 gallon (gal.)
8 quarts	= 1 peck (pk.)
4 pecks	= 1 bushel (bu.)

PAPER

24 sheets	= 1 quire
480 sheets (sometimes 500)	= 1 ream

WEIGHT PER BUSHEL

Corn in ear = 70 pounds
Alfalfa seed, beans, beets, clover seed, potatoes, wheat = 60 pounds
Flaxseed, rye, shelled corn = 56 pounds
Sweet potatoes = 55 pounds
Apples, barley, buckwheat, peaches = 48 pounds
Onions vary from 48 to 57 pounds
Timothy seed = 45 pounds
Oats = 32 pounds

Pupils should verify the above weights to see if these are the legal weights for the state. The legal weights vary in different states.

CONVENIENT UNITS FOR THE KITCHEN

3 teaspoonfuls (tsp.)	= 1 tablespoonful (tbsp.)
2 gills	= 1 cup
4 cups flour	= 1 pound (lb.)
2 tablespoonfuls sugar	= 1 ounce (oz.)
2 cups granulated sugar	= 1 pound (lb.)
2 cups butter, solid	= 1 pound (lb.)

CONVENIENT RULES FOR THE FARMER

To find the number of —

(1) Bushels of corn in a crib: For old and dry corn take $\frac{4}{9}$ of the product of the three dimensions in feet.

(2) Bushels of wheat in a bin: Take .8 (or $\frac{4}{5}$) of the product of the three dimensions in feet. Another rule is divide the volume in cubic inches by 2150.

(3) Tons of hay in a mow: Measure the three dimensions in yards, find the product, and divide by 15 for well-packed hay. For new hay, which is less well packed, find the product of the three dimensions in yards and divide by 18.

MEASURES CONVENIENT FOR PRACTICAL ESTIMATES

1 ton hard coal	= 35	cubic feet
1 ton soft coal	= 38	cubic feet
1 cubic foot	= .8	bushel
4000 cubic inches of ear corn	= 1	bushel of shelled corn
7.5 gallons	= 1	cubic foot
1 gallon water	= $8\frac{1}{3}$	pounds
196 pounds flour	= 1	barrel

IMPORTANT METRIC UNITS

LENGTH

10 millimeters (mm.)	= 1 centimeter (cm.), about .4 inch
10 centimeters	= 1 decimeter (dm.), about 4 inches
10 decimeters	= 1 meter (m.), nearly 40 inches (39.37)
1000 meters	= 1 kilometer (km.), about $\frac{5}{8}$ mile

AREA OR SURFACE

100 square millimeters (sq. mm.)	= 1 square centimeter (sq. cm.)
100 square centimeters	= 1 square decimeter (sq. dm.)
100 square decimeters	= 1 square meter (sq. m.)

VOLUME

1000 cubic millimeters (cu. mm.)	= 1 cubic centimeter (cu. cm.)
1000 cubic centimeters	= 1 cubic decimeter (cu. dm.)
1000 cubic decimeters	= 1 cubic meter (cu. m.)

CAPACITY

10 milliliters (ml.)	= 1 centiliter (cl.)
10 centiliters	= 1 deciliter (dl.)
10 deciliters	= 1 liter (l.), about 1 liquid quart

FORMULAS

Rectangle	$A = bh$	Circle	$c = \pi d = 2\pi r$
Parallelogram	$A = bh$	Circle	$A = 3\frac{1}{7} r^2$
Square	$A = s^2$	Rectangular prism	$V = lwh$
Triangle	$A = \frac{1}{2} bh$	Cube	$V = s^3$
Trapezoid	$A = \frac{h \times (B + b)}{2}$	Interest	$i = prt$
		Distance	$d = rt$

WEIGHT

10 milligrams (mg.)	= 1 centigram (cg.)
10 centigrams	= 1 decigram (dg.)
10 decigrams	= 1 gram (g.)
A gram	= weight of 1 cu. cm. of water
1000 grams	= 1 kilogram (kg.), about 2.2 pounds

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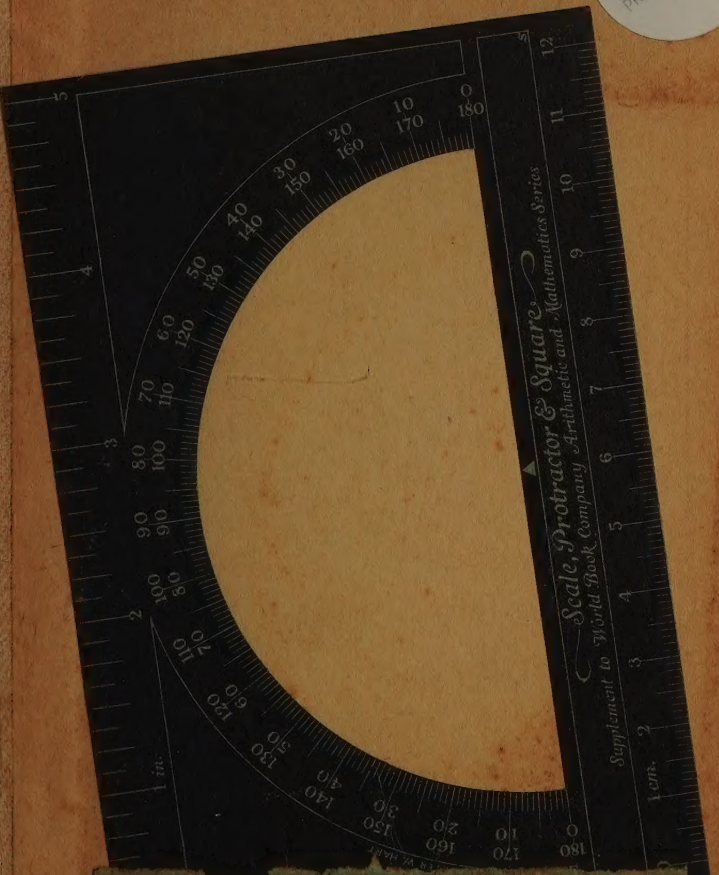

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